Experimental study on the influence of flexible bearing clearance on natural frequency of harmonic reducer

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Abstract

Harmonic reducer is a transmission device with high transmission ratio and high accuracy, which is widely used in robotics and other instruments. Backlash reflects the positioning accuracy of harmonic drive, which is one of the key performances of harmonic reducer. Because the flexible bearing is a wave generator of harmonic reducer, its clearance has a great influence on the accuracy of harmonic drive. To obtain the influence of clearance on the backlash of harmonic reducer, the clearance of flexible bearing is equivalent to the backlash of gear teeth to carry out the theoretical value of the backlash of harmonic reducer. Then a new measuring instrument was set up to measure the backlash of harmonic reducer. The results show that the test value is larger than the theoretical value. The research finds that the torsional stiffness of the harmonic drive has non-linear characteristics. Then the hysteresis curve of the flexible bearing is drawn, and the four-segment stiffness values of the flexible bearing under different clearances are analyzed. Finally, the vibration equation is established based on the time-varying stiffness, and the calculation shows that the clearance also has influence on the natural frequency of the flexible bearing system and test results show that the rotation position has a great influence on the natural frequency.

Keywords : Backlash, Flexible bearing clearance, Hysteresis curve, Natural frequency

1. Introduction

Harmonic gear drive achieves mechanical transmission by using a flexible component, breaking through the rigid component mechanism mode of mechanical transmission. As a new type of high precision transmission mechanism, harmonic reducer is widely used in the wrist and hand of robot (E. Brassitos et al., 2017). With the rapid development of science and technology, industrial robots have higher requirements on transmission performance (Wang et al., 2021). The backlash performance of harmonic reducer, as show in Fig.1, has an important influence on the repetitive positioning accuracy of industrial robot (Jia et al., 2021). Rached et al. (2003) proposed that the hysteresis phenomenon of harmonic reducer is the result of the interaction of its own non-linear stiffness element and nonlinear dynamic element. Michael et al. (2009) analyzed the hysteretic characteristics of elastic robot joint, established the hysteretic model of harmonic reducer based on dynamic friction model, and carried out experimental research. Curt et al. (2012) used Preisach model to study the hysteretic characteristics of harmonic reducer and Maxwell resistive capacitor model to identify the weight function. In order to fully simulate the harmonic drive, Zhang et al. (2015) obtained a harmonic drive model which can describe the hysteresis loss by modeling the flexibility of the wave generator. Huang et al. (2019) set up the torsional vibration test-bed of harmonic reducer, and the torsional vibration signal of harmonic reducer is analyzed. Ehrich et al. (2008) described the nonlinear phenomenon caused by bearing clearance. Jiang et al. (2017) analyzed the load distribution and contact stress of flexible bearing. Tan et al. (2019) tested the torsional stiffness of two kinds of harmonic reducers under static torsional load and dynamic torsional load respectively.

To sum up, the current research on harmonic reducer mainly focuses on structural strength and dynamic characteristics. However, the effect of flexible bearing clearance on the backlash and natural frequency of harmonic drive is less. Therefore, the backlash and natural frequency of harmonic gear drive are analyzed theoretically. A new measuring device is established to verify the theoretical results. The research results are of great significance to improve the vibration stability and positioning accuracy of harmonic drive.

Fig.1 Structure of harmonic reducer

2. Theoretical calculation model

2.1 Analysis of transmission error

As the core component of industrial robot, the performance of harmonic reducer directly affects the performance of the whole machine. Generally speaking, the analysis of transmission error is mainly composed of the motion error and assembly error of harmonic reducer, and the error caused by the external load torque related to the stiffness and backlash of harmonic reducer.

The wave generator (WG) as the input end, the flex spline (FS) as the output end and the circular spline (CS) as the fixed end is the most common form. This type of harmonic drive can be seen as a two-degree-of-freedom system as shown in Fig. 2, similar to a planetary gear train. The driving relationship can be expressed as:

$$\theta_{WG} = -\frac{Z_{FS}}{Z_{CS}-Z_{FS}}\theta_{FS} = -N\theta_{FS} \quad (1)$$

where N is the gear reduction ratio, θ_{WG} is the angular position of the WG, θ_{FS} is the angle position of the FS.

It can be seen from formula (1) that the harmonic transmission is linearly correlated. However, the actual test results show that the output is not linearly correlated with the input. From the angle of error mechanism, transmission error is mainly composed of lag component caused by clearance and nonlinear torsional stiffness of harmonic reducer.

Suppose B and C are two points on the neutral line of CS and FS respectively. At first, B and C coincide. After the WG rotates an angle counterclockwise, in an ideal case, point B on the CS rotates to point B' , and point C on the FS rotates to point C' . In fact, due to assembly error and machining error, point C actually moves to point C'' , as shown in Fig.3. The harmonic transmission error is simplified here, among which θ_{me} is the motion error of the harmonic, θ_{le} is the lag component, and the sum of the two is the total transmission error of harmonic reducer.

Fig.2 Harmonic drive diagram

Fig.3 Harmonic drive error

Then the modified motion relation can be expressed as:

$$\theta_{WG} = -N(\theta_{FS} + \theta_{me} + \theta_{le}) \quad (2)$$

where: θ_{me} is motion error, θ_{le} is the lag component.

2.2 Theoretical calculation of the backlash

The existence of backlash makes a non-linear relationship between input shaft and output shaft of harmonic drive system. This paper mainly studies the influence of flexible bearing clearance on backlash of harmonic drive. The measuring principle of clearance of flexible bearing is shown in Fig.4. It is assumed that the flexible bearing with different clearances is installed in the Shaft end cam, and the cam eliminates all radial clearance at the long shaft of the flexible bearing. Because of the existence of clearance, the amount of deformation after the outer ring of the flexible bearing is different. Therefore, the dial gauge readings under different clearances is the corresponding clearance of the flexible bearing. Fig.5 shows the test device for clearance measurement.

In order to study the effect of clearance on the backlash, the method used in this paper is to calculate the change of backlash by equivalent radial clearance of flexible bearing to the change of center distance of internal gear mesh.

Assuming that the teeth of FS are outer teeth 1 and the teeth of CS are inner teeth 2, the gear pair is meshed with no backlash when there is no clearance in the flexible bearing. If the clearance is considered, it is equivalent to reducing the center distance between the outer gear and the inner gear, causing the backlash of the gear pair. The dashed line in Fig.6 represents the base circle of outer gear 1 before the center distance is changed, and the solid line represents the base circle of outer gear 1 in the presence of clearance. When the center distance is reduced by Δa , the mesh angle of the FS and CS will change accordingly. According to the diagram, it can be directly deduced that the size of the mesh angle under the gap is:

$$\alpha' = \arccos \frac{r_{b2} - r_{b1}}{a - \Delta a} \quad (3)$$

where r_{b1} is the radius of the base circle of the FS, r_{b2} is the radius of the base circle of the CS.

As the center distance decreases, the normal side gap of J'_{bn} increases symmetrically distributed left and right along the common tangent of the base circle. According to Fig.7, the size of the normal side gap J'_{bn} is:

$$J'_{bn} = (MP + NP) - (KP + LP) \quad (4)$$

Rewind the straight line of MP and NP to the base circle, then M will follow the left tooth profile of the internal gear back to the M' point on the base circle of the internal gear, P will fall on the P_3 point on the base circle of the inner gear along the asymptote shown by the dashed line. According to the nature of the asymptotes, $MP = \widehat{M'P_3}$, $NP = \widehat{N'P_4}$. Then, $MP + NP = \widehat{M'N'} - \widehat{P_3P_4}$. Similarly, $KP + LP = \widehat{K'L'} - \widehat{P_1P_2}$, the following formula can be obtained:

$$MP + NP = e_{b2} - 2r_{b2} \text{inv} \alpha' \quad (5)$$

$$KP + LP = s_{b1} - 2r_{b1} \text{inv} \alpha' \quad (6)$$

where, e_{b2} is the width of the base circle of the internal gear 2, s_{b1} is the base gear thickness of outer gear 1. According to the calculation formula of the base circle gear thickness:

$$e_{b2} = \pi m - \left(\frac{1}{2} \pi m \cos \alpha - m z_2 \cos \alpha \text{inv} \alpha \right) \quad (7)$$

$$s_{b1} = \frac{1}{2} \pi m \cos \alpha + m z_1 \cos \alpha \text{inv} \alpha \quad (8)$$

Through the above solution, it can be known that the clearance caused by the radial clearance of the flexible bearing is:

$$J'_{bn} = \frac{e_{b2} - 2r_{b2} \text{inv}\alpha' - s_{b1} + 2r_{b1} \text{inv}\alpha'}{mz_1 \cos \alpha} \quad (9)$$

The relationship between the center distance reduction Δa and the clearance c of the flexible bearing is as follows:

$$\Delta a = \frac{c}{2} \quad (10)$$

The parameters of a harmonic decelerator used in this paper are: number of flexible teeth $z_1=160$, number of rigid teeth $z_2=162$, modulus 0.4, meshing angle 20° , and the main parameters of flexible bearing are shown in Table 1. The clearance range of flexible bearings is from 10um to 15um. By substituting the above parameters into the formula (9), it is known that the backlash angle caused by radial clearance of flexible bearing is $0.152^\circ \sim 0.155^\circ$. The theoretical calculation only considers the geometric error caused by the clearance of the flexible bearing, and other cases such as the non-linear stiffness are not considered in the solution process.

Table 1 Main parameters of flexible bearing

Parameters	Value
Outer diameter	61.3 mm
Inner diameter	45.1 mm
Wall thickness of inner ring	1.5 mm
Wall thickness of outer ring	1.5 mm
Curvature coefficient of inner groove	0.515
Curvature coefficient of outer groove	0.515
Roller diameter	5.556 mm
Number of rollers	23
Long axis of wave generator	45.9 mm
Short axis of wave generator	44.3 mm

Fig.4 Principle of bearing clearance measurement

Fig.5 Test device for clearance measurement

Fig.6 Diagram of base circle change

Fig.7 Theoretical analysis of influence of clearance on flexible bearing

3. Experiment and analysis

In order to measure the backlash of harmonic reducer, a new measuring instrument is designed in this paper, which mainly include the rotating arm, output cover, clamp, connecting flange, frame and dial gauge. Fig.8 shows the test diagram of the experimental device.

The test condition of backlash test for harmonic reducer designed in this paper is to measure the angle of inner ring to outer ring by restraining outer ring, locking output, and then applying torque to inner ring. The torque is controlled by the force applied to the force arm. It is found that the difference of the input shaft rotation displacement is still reduced when the flexible bearing is applied with a torque of 1 Nm. Therefore, the torsional stiffness of bearings with 2 and 3 Nm torques was tested at the same clearance. It is found that the difference of input shaft rotation displacement tends to be stable and constant gradually under the 3Nm torque of the flexible bearing. It is known that the applied torque is within the limit of the torque range, so the torque applied during the test cannot exceed the critical torque of 3Nm to avoid bearing damage.

According to the circumferential displacement of the rotating arm measured by the dial gauge, it is converted into the corresponding rotation angle of the flexible bearing, and the hysteric curve is obtained after the average value of 12 groups of experimental data is processed. The hysteric curves under different clearances can be expressed as shown in Fig.9, Fig.10 and Fig.11. The data are shown in Table 2.

Fig.8 The experiment measuring instrument

Fig. 9 Hysteresis curves of different torques with clearance 11 μm

Fig.10 Hysteresis curves of different torques at clearance 13 μm

Fig.11 Hysteresis curves of different torques with clearance of 15 μm

Table 2 Backlash of flexible bearing with different clearance under 3Nm

Clearance (μm)	average value (mm)	Backlash (rad)	Angle ($^{\circ}$)
11	0.34	3.4×10^{-3}	0.195°
13	0.3767	3.767×10^{-3}	0.216°
15	0.4133	4.133×10^{-3}	0.237°

Table 3 Four-section stiffness values of flexible bearings with different clearances at 3Nm

Clearance (μm)	K_1 (Nm/rad)	K_2 (Nm/rad)	K_3 (Nm/rad)	K_4 (Nm/rad)
11	81.08	101.35	90.09	115.83
13	74.07	94.94	81.97	108.30
15	73.71	91.19	79.58	100.33

Fig.12 Stiffness curves of flexible bearings with different clearances at 3Nm

The test results show that the backlash angle of the flexible bearing is $0.195^{\circ} \sim 0.237^{\circ}$. It is larger than the theoretical solution and increases with the increase of the clearance. Through analysis, it is found that the error comes from the nonlinearity of the structural stiffness. Therefore, the average stiffness of each stage is approximately described by connecting the four points with zero torque on the hysteretic curve of the flexible bearing with the maximum point, so as to obtain the average stiffness of forward loading (K_1), forward unloading (K_2), reverse loading (K_3) and reverse unloading (K_4) to reflect the overall stiffness value of the flexible bearing in the four sections. The calculation results are shown in Table 3. The average stiffness values of four parts of flexible bearing with different clearance are compared in Fig. 12. It can be seen that the stiffness value of reverse unloading is the largest, followed by forward unloading, followed by reverse loading, and the stiffness value of forward loading is the smallest. It is also found that the larger the clearance of the flexible bearing, the smaller the stiffness.

4. Torsional vibration simulation and experimental analysis

4.1 Vibration modeling and calculation

Based on the time-varying characteristics of torsional stiffness, the vibration equation of flexible bearing system is established. The Runge-Kutta method is used to solve the response, and the steady-state response of the system under

variable stiffness is analyzed. The stiffness of flexible bearing is different under forward loading, forward unloading, reverse loading and reverse unloading. The torsional stiffness of the flexible bearing is analyzed by four segments:

$$K = K_1(K_1, K_2, K_3, K_4) \quad (11)$$

where K_1 is the stiffness of forward loading, K_2 is the stiffness of forward unloading, K_3 is the stiffness of reverse loading, and K_4 is the stiffness of reverse unloading, so the torsional vibration equation of wave generator is as follows:

$$J\ddot{\theta}(t) + C\dot{\theta}(t) + K\theta(t) = M \quad (12)$$

$$\frac{d^2\theta}{dt^2} + 2\xi\omega_n \frac{d\theta}{dt} + \omega_n^2\theta = A \cos \omega_p t \quad (13)$$

where J is the moment of inertia of the system, C is the damping of the system, $\ddot{\theta}(t)$ is the angular acceleration of rotation, $\dot{\theta}(t)$ is the angular velocity of rotation, $\theta(t)$ is the rotation angle, ξ is the damping ratio, ω_n is the natural frequency of the system, ω_p is the frequency of simple harmonic excitation, A is the amplitude of simple harmonic excitation.

According to the stiffness test results, the system parameters are set: system damping ratio $\xi = 0.03$, simple harmonic frequency $\omega_p = 100\text{Hz}$, amplitude of harmonic excitation $A = 1$, moment of inertia $J = 2.9624 \times 10^{-3}\text{kgm}^2$.

Fig.13 Time Domain of Flexible Bearing Torsional Vibration Simulation at 11um

Fig.14 Response Spectrum of Flexible Bearing Torsional Vibration Simulation at 11um

Fig.15 Time Domain of Flexible Bearing Torsional Vibration Simulation at 13um

Fig.16 Response Spectrum of Flexible Bearing Torsional Vibration Simulation at 13um

Fig.17 Time Domain of Flexible Bearing Torsional Vibration Simulation at 15um

Fig.18 Response Spectrum of Flexible Bearing Torsional Vibration Simulation at 15um

The torsional stiffness of flexible bearing with different clearance is tested. Firstly, the influence of clearance on the backlash of flexible bearing is analyzed. Then, the flexible bearings of 11um, 13um and 15um are tested under different torques, and the results under critical torque are relatively stable. Finally, based on the critical torque, the hysteresis curve of flexible bearing under different clearance is analyzed. The vibration response of flexible bearing is analyzed under critical torque. The system vibration response is divided into transient response and steady-state response. Figs. 13, 15 and 17 show excitation signals with a frequency of 100Hz. About 1 second ago is the free response waveform of the system transient stage, and then the forced response waveform of the system steady-state stage. The time domain waveform and response spectrum of the system are shown in Figs. 13 -18. Through the simulation analysis of torsional vibration, it can be obtained that when the clearance of flexible bearing is 11um, 13um and 15um, the first-order natural frequencies of the system are 32.5Hz, 31.7Hz and 31.6Hz respectively. It shows that clearance also has certain influence on first-order natural frequency of flexible bearing system.

4.2 Experimental analysis of natural frequency of bearing system

In order to verify the first-order natural frequency of flexible bearing system under different working conditions, the test device is built, as shown in Fig.19. The device is mainly composed of motor, bearing system, vibration sensor and exciter. The test condition is that the test bench rotates driven by the motor. The moment of inertia used in the test device is $J=2.9624 \times 10^{-3} \text{ kgm}^2$, which is consistent with the simulation. To eliminate the randomness of the test, the flexible bearings used in the test come from China and Japan, and the same test measurement conditions are adopted. The main experimental content is to collect the vibration signal of the bearing system in the process of excitation. The test principle is that under the action of external excitation, the vibration value of components with natural frequency suddenly increases at a certain frequency, and the frequency corresponding to this value is the natural frequency.

Fig.19 Torsional vibration measuring device

Fig.20Time domain diagram of vibration response

Fig.21Time domain diagram of local vibration response

The vibration acceleration time domain signal collected after the excitation signal is applied at a rotating position at a constant rotation speed as show in Fig.20. The signal when the excitation signal occurs are intercepted for analysis due to the factor studied is the natural frequency. This part includes a complete excitation response process, as shown in Fig.21.

Fig.22 Static power spectrum analysis

Fig.23 Power spectrum analysis at 300 rpm

Table 4 First order natural frequency values at different rotational speeds and positions at clearance is 15um (unit/Hz)

Angular position/°	Speed=0 rpm	Speed=100 rpm	Speed=300 rpm
0	26.25	26.23	26.24
45	23.54	23.54	23.54
90	32.50	32.35	32.52
135	30.10	30.12	30.11
180	26.25	26.35	26.21
225	31.25	31.25	31.24
270	31.24	31.15	31.23
315	30.11	30.15	30.15
360	26.35	26.25	26.15

Table 5 First order natural frequency values at different rotational speeds and positions at clearance is 11um (unit/Hz)

Angular position/°	Speed=0 rpm	Speed=100 rpm	Speed=300 rpm
0	31.21	31.19	31.19
45	30.32	30.31	30.19
90	31.50	31.45	31.44
135	30.17	30.19	30.16
180	31.42	31.41	31.43
225	33.27	33.24	33.25
270	33.45	33.45	33.43
315	30.64	30.62	30.63
360	31.23	31.19	31.21

Fig.24 The natural frequency of bearing system at 15um

Fig.25 The natural frequency of bearing system at 11um

Then, the vibration signal is analyzed by the short-time Fourier transform method to obtain the results in Fig.22 and Fig.23. Among them, the power spectrum of the bearing system when it is stationary at a certain position is shown in Fig.22. It can be concluded that the frequency corresponding to the maximum response is 23.54 Hz, which is the first-order natural frequency of the bearing system. Similarly, the natural frequency obtained at the same position at different speeds is basically the same as 23.54 Hz. However, we were surprised to find that the natural frequencies at different positions were significantly different, and we carried out correlation tests on multiple models of flexible bearings and obtained the same conclusion.

Table 4 shows the first-order natural frequency values of a certain type of bearing at different rotation speeds and positions when bearing clearance is 15um. The results show that the natural frequency values at the same position are basically the same, and the change of rotation speed has little effect on the natural frequency, as shown in Fig.24. However, at different locations, the value of natural frequency varies greatly. After data statistics, the overall variation range of the first-order natural frequency is 23-33 Hz. When clearance is 11um, the first natural frequency value is shown in Table 5, which variation range of the first-order natural frequency is 30-34 Hz, as shown in Fig.25. Through test and comparison, it is found that the smaller the bearing clearance, the smaller the frequency variation range of the measured results.

5. Conclusions

(1) The theoretical model is established through the research and analysis of the backlash of harmonic reducer. The geometric backlash caused by the clearance of flexible bearing is equivalent to the gear meshing backlash, and the calculation formula of backlash caused by the clearance is obtained. The calculation results show that when the clearance range of the flexible bearing is 10um ~ 15um, and the backlash value is 0.152° ~ 0.155°. However the test results of the

flexible bearing measuring instrument are greater than the theoretical value. It is found that the main reason is the non-linearity of torsional stiffness through analysis and verification. After test comparison, when the maximum torque reaches 3Nm, the influence reaches the maximum and stable.

(2) The test results show that reducing the clearance can reduce the backlash difference of the harmonic reducer. At the same time, through qualitative analysis, the larger the clearance of flexible bearing, the smaller the torsional stiffness. In other words, the larger the clearance of flexible bearing, the greater the nonlinear torsional stiffness.

(3) The simulation and test results show that the natural frequency has little relationship with the speed. However, compared with the rotation speed, the influence of rotational position is more significant. In addition, it is found that the clearance also has certain influence on the natural frequency of the system. When the clearance is 15 μ m, the variation range of the first natural frequency is 23-33Hz. When the clearance is 11 μ m, its variation range is 30-34Hz. Therefore, reducing the clearance can improve the stability of natural frequency variation and the overall natural frequency has been improved. The research of this paper has guiding significance for improving the optimization of harmonic drive, and provides a reference for the selection of flexible bearing in harmonic drive.

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