On Mechanical Finger Design for Expanded Prescribed Grasping Poses*

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Abstract

This work formulates and demonstrates a motion generation method for the synthesis of a multi-loop planar six-bar mechanism comparable to the Belgrade/USC robotic hand ⁽¹³⁾⁽¹⁴⁾. The method presented in this work can accommodate numbers of prescribed rigid-body poses beyond the maximum number of poses allowable using conventional planar six-bar mechanism synthesis models ⁽¹⁰⁾. The example included demonstrates the design of a mechanical hand to achieve twice the maximum number of displacements allowable with conventional synthesis models.

Key words: Motion Generation, Planar Six-Bar Mechanism, Mechanical Finger

1. Introduction

Six-bar and multi-loop mechanism design includes the work of Simionescu and Smith ⁽¹⁾ who presented a proof of the existence of function cognates of Stephenson II and Stephenson III mechanisms. Xiaohong and Ting ⁽²⁾ developed an approach for the branch identification of a variety of multi-loop linkages (including Watt and Stephenson chains). Todorov ⁽³⁾ presented a method for dimensional synthesis of symmetrical six-bar Watt mechanisms. Shiakolas, Koladiya and Kebrle ⁽⁴⁾ presented a methodology for the synthesis of six-bar mechanisms that combines differential evolution, an evolutionary optimization scheme and the geometric centroid of precision position technique. Watanabe and Katoh ⁽⁵⁾ considered every relationship between the input and output displacement (the domain of motion of the driving link and the circuit) of the Stephenson III mechanism. Pramanik ⁽⁶⁾ synthesized a six-bar mechanism to achieve five precision points of an automotive steering mechanism. Baker ⁽⁷⁾⁽⁸⁾ investigated two specific groups of six-bar mechanisms and demonstrated a method for generating a family of six-bar folding mechanisms. Foster and Cipra ⁽⁹⁾ examined the problem of identifying assembly configurations of planar multi-loop mechanisms with joint limits, link interference, link collision and constraint regions.

A method to synthesize a particular type of planar six-bar motion generator to approximate prescribed rigid-body poses beyond conventional six-bar mechanism synthesis models ⁽¹⁰⁾ is presented in this work. The method and mechanism are effective in the design of finger mechanisms for robotic hands ⁽¹¹⁾⁽¹²⁾⁽¹⁴⁾. The six-bar mechanism design this work considers has two closed loops, one degree of freedom and is essentially a “stacked” arrangement of two planar four-bar mechanisms. The structure of the six-bar mechanism is similar to the structure of the Belgrade/USC robotic hand ⁽¹³⁾⁽¹⁴⁾.

2. The Six-Bar Mechanism, Design Application and Synthesis Equations

2.1 The Planar Six-Bar Mechanism

Planar six-bar linkages are often utilized in applications where planar four-bar linkages do not provide the required type of performance. Figure 1 illustrates Watt and Stephenson chains (two single degree of freedom six-bar mechanism chain types).

The specific planar six-bar mechanism design this work considers is illustrated in Fig. 2. With the exception of the two ternary links containing pivots $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{e}_1$, and pivots $\mathbf{a}_1$, $\mathbf{b}_1$ and $\mathbf{c}_1$ respectively, the remaining links of the six-bar mechanism design are binary (like Watt I and Stephenson I and II mechanisms). The driving link will include pivots $\mathbf{b}_0$, $\mathbf{b}_1$ and $\mathbf{e}_1$. The six-bar mechanism is comprised of two four-bar mechanisms (mechanism $\mathbf{a}_0$ - $\mathbf{a}_1$ - $\mathbf{b}_1$ - $\mathbf{b}_0$ and mechanism $\mathbf{c}_1$ - $\mathbf{d}_1$ - $\mathbf{e}_1$ - $\mathbf{b}_1$) that are stacked in a manner where the coupler of mechanism $\mathbf{a}_0$ - $\mathbf{a}_1$ - $\mathbf{b}_1$ - $\mathbf{b}_0$ is the moving ground of mechanism $\mathbf{c}_1$ - $\mathbf{d}_1$ - $\mathbf{e}_1$ - $\mathbf{b}_1$. The mechanism in Fig. 3 has two closed loops and a single degree of freedom. The two-coupler design of this six-bar mechanism is also useful for approximating two distinct groups of prescribed rigid-body poses in a single crank displacement sequence.

Fig. 1 Planar six-bar mechanism classifications (Watt and Stephenson)

Fig. 2 The planar six-bar mechanism design considered for synthesis

2.2 Finger Mechanism Application

Figure 3 illustrates a mechanical finger incorporating the planar six-bar mechanism design considered in this work. For decades, empirical design methods have been used for this mechanism^{(11)~(14)}. Like the human finger, the mechanical finger has a proximal, middle

and distal phalanx (and associated joints) and is capable of achieving extended and grasping poses (Fig. 4). Affixing several mechanical fingers to a rigid palm and thumb would result in a mechanical hand.

Fig. 3 The planar six-bar finger mechanism

Fig. 4 Six-bar finger mechanism in extended and arbitrary grasping poses

2.3 Conventional Planar Six-Bar Mechanism Synthesis

Equation (1)⁽¹⁰⁾ is used to calculate four of the five unknowns for the link defined by parameters $\mathbf{a}_0$, $\mathbf{a}_1$ and R_1 (see Fig. 2) given the parameters for four prescribed rigid-body poses (defined by variables $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$). In Eq. (1) $\mathbf{a}_0 = (a_{0x}, a_{0y}, 1)^T$ and $\mathbf{a}_1 = (a_{1x}, a_{1y}, 1)^T$. Equation (6)⁽¹⁰⁾ is a displacement matrix (from pose 1 to pose “j”) for the rigid-body with variables $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.

$$([\mathbf{D}_{1j}]\mathbf{a}_1 - \mathbf{a}_0)^T ([\mathbf{D}_{1j}]\mathbf{a}_1 - \mathbf{a}_0) - R_1^2 = 0, \quad j = 1, 2, 3, 4 \quad (1)$$

Equations (2)~(7)⁽¹⁰⁾ are used to calculate unknown parameters $\mathbf{b}_0$, $\mathbf{b}_1$, $\mathbf{c}_1$, $\mathbf{d}_1$, $\mathbf{e}_1$, R_2 , R_3 and R_4 (see Fig. 2) given the parameters for the four prescribed rigid-body poses (defined by variables $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ and variables $\mathbf{p}^*$, $\mathbf{q}^*$ and $\mathbf{r}^*$). In Eqs. (2)~(4), $\mathbf{b}_0 = (b_{0x}, b_{0y}, 1)^T$, $\mathbf{b}_1 = (b_{1x}, b_{1y}, 1)^T$, $\mathbf{c}_1 = (c_{1x}, c_{1y}, 1)^T$, $\mathbf{d}_1 = (d_{1x}, d_{1y}, 1)^T$ and $\mathbf{e}_1 = (e_{1x}, e_{1y}, 1)^T$. Like Eq. (6), Eq. (7)⁽¹⁰⁾ is the displacement matrix for the rigid-body with variables $\mathbf{p}^*$, $\mathbf{q}^*$ and $\mathbf{r}^*$. Matrix $[\mathbf{R}_{1j}]$ in Eq. (4) is a pure z-axis rotation matrix of joints $\mathbf{b}_1$ and $\mathbf{e}_1$ about joint $\mathbf{b}_0$ with the following conditions:

$$\cos \theta_{1j} = \frac{(\mathbf{b}_1 - \mathbf{b}_0) \cdot ([\mathbf{D}_{1j}]\mathbf{b}_1 - \mathbf{b}_0)}{|\mathbf{b}_1 - \mathbf{b}_0| \cdot |[\mathbf{D}_{1j}]\mathbf{b}_1 - \mathbf{b}_0|} \quad \sin \theta_{1j} = \frac{\det[\mathbf{M}_{1j}]}{|\mathbf{b}_1 - \mathbf{b}_0| \cdot |[\mathbf{D}_{1j}]\mathbf{b}_1 - \mathbf{b}_0|}$$

$$\mathbf{M}_{1j} = \begin{bmatrix} 1 & | & | \\ 1 & \mathbf{b}_1 - \mathbf{b}_0 & | \\ 1 & | & | \end{bmatrix} \begin{bmatrix} [\mathbf{D}_{1j}] \mathbf{b}_1 - \mathbf{b}_0 \end{bmatrix}, \quad j = 1, 2, 3, 4$$

The determinant constraint in Eq. (5) ensures that the mechanical finger is calculated in the extended pose.

$$([\mathbf{D}_{1j}] \mathbf{b}_1 - \mathbf{b}_0)^T ([\mathbf{D}_{1j}] \mathbf{b}_1 - \mathbf{b}_0) - R_2^2 = 0 \quad (2)$$

$$([\mathbf{S}_{1j}] \mathbf{c}_1 - [\mathbf{D}_{1j}] \mathbf{b}_1)^T ([\mathbf{S}_{1j}] \mathbf{c}_1 - [\mathbf{D}_{1j}] \mathbf{b}_1) - R_3^2 = 0 \quad (3)$$

$$([\mathbf{S}_{1j}] \mathbf{d}_1 - [\mathbf{R}_{1j}] (\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0)^T ([\mathbf{S}_{1j}] \mathbf{d}_1 - [\mathbf{R}_{1j}] (\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0) - R_4^2 = 0 \quad (4)$$

$$\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1y} \\ 1 & 1 & 1 \end{pmatrix} = 0 \quad (5)$$

$$[\mathbf{D}_{1j}] = \begin{bmatrix} p_{jx} & q_{jx} & r_{jx} \\ p_{jy} & q_{jy} & r_{jy} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{1x} & q_{1x} & r_{1x} \\ p_{1y} & q_{1y} & r_{1y} \\ 1 & 1 & 1 \end{bmatrix}^{-1} \quad (6)$$

$$[\mathbf{S}_{1j}] = \begin{bmatrix} p_{jx}^* & q_{jx}^* & r_{jx}^* \\ p_{jy}^* & q_{jy}^* & r_{jy}^* \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} p_{1x}^* & q_{1x}^* & r_{1x}^* \\ p_{1y}^* & q_{1y}^* & r_{1y}^* \\ 1 & 1 & 1 \end{bmatrix}^{-1} \quad (7)$$

where $j = 1, 2, 3, 4$

2.4 Planar Six-Bar Mechanism Synthesis for Expanded Prescribed Poses

Using Eqs. (1)~(7), one is limited to a maximum of four prescribed rigid-body poses and subsequently three prescribed rigid-body displacements (displacements 1-2, 1-3 and 1-4) for synthesis. The equation sets included in this section are applicable for a numbers of prescribed rigid-body poses beyond four (and subsequently displacements beyond three).

Equation (8) is the objective function (to be minimized) incorporating Eq. (1). The form of this objective function resembles the “best fit” criteria from the method of least squares⁽¹⁵⁾ and can theoretically accommodate an indefinite number of prescribed rigid-body poses. When Eq. (8) is minimized, its constant link length constraints are satisfied. Equation (9) is the minimization condition for the objective function. Equations (10)~(14) are the matrix cells in Eq. (9). These equations will be used to calculate four of the five unknowns for the link defined by parameters $\mathbf{a}_0$, $\mathbf{a}_1$ and R_1 (see Fig. 2) given the parameters for N prescribed rigid-body poses (defined by variables $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$).

$$f(\mathbf{X}) = \sum_{j=1}^N \left(([\mathbf{D}_{1j}] \mathbf{a}_1 - \mathbf{a}_0)^T ([\mathbf{D}_{1j}] \mathbf{a}_1 - \mathbf{a}_0) - R_1^2 \right)^2, \quad \mathbf{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} a_{0x} \\ a_{0y} \\ a_{1x} \\ a_{1y} \\ R_1 \end{pmatrix} \quad (8)$$

$$\nabla f(\mathbf{X}) = \begin{pmatrix} \frac{\partial f(\mathbf{X})}{\partial x_1} \\ \vdots \\ \frac{\partial f(\mathbf{X})}{\partial x_5} \end{pmatrix} = 0 \quad (9)$$

$$\frac{\partial}{\partial a_{0x}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0)^T ([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0) - R_1^2 \right)^2 = 0 \quad (10)$$

$$\frac{\partial}{\partial a_{0y}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0)^T ([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0) - R_1^2 \right)^2 = 0 \quad (11)$$

$$\frac{\partial}{\partial a_{1x}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0)^T ([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0) - R_1^2 \right)^2 = 0 \quad (12)$$

$$\frac{\partial}{\partial a_{1y}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0)^T ([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0) - R_1^2 \right)^2 = 0 \quad (13)$$

$$\frac{\partial}{\partial R_1} \sum_{j=1}^N \left(([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0)^T ([D_{1j}] \mathbf{a}_1 - \mathbf{a}_0) - R_1^2 \right)^2 = 0 \quad (14)$$

Equation (15) is the objective function (to be minimized) incorporating Eqs. (2)~(5). Like Eq. (8), the link constraints in Eq. (15) are satisfied when Eq. (15) is minimized. Equation (16) is the minimization condition for the objective function. Equations (17)~(29) are the matrix cells in Eq. (16). These equations will be used to calculate unknown parameters $\mathbf{b}_0$, $\mathbf{b}_1$, $\mathbf{c}_1$, $\mathbf{d}_1$, $\mathbf{e}_1$, R_2 , R_3 and R_4 (see Fig. 2) given the parameters for N prescribed rigid-body poses (defined by variables $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ and variables $\mathbf{p}^*$, $\mathbf{q}^*$ and $\mathbf{r}^*$). Equations (6), (7) and the rotation matrix $[R_{1j}]$ in Eq. (4) are used accordingly in Eqs. (8)~(29).

$$\begin{aligned} f(\mathbf{X}) = & \sum_{j=1}^N \left(([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0)^T ([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0) - R_2^2 \right)^2 \\ & + \sum_{j=1}^N \left(([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1)^T ([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1) - R_3^2 \right)^2 \\ & + \sum_{j=1}^N \left(([S_{1j}] \mathbf{d}_1 - [R_{1j}](\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0)^T ([S_{1j}] \mathbf{d}_1 - [R_{1j}](\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0) - R_4^2 \right)^2 \\ & + \left[\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1y} \\ 1 & 1 & 1 \end{pmatrix} \right]^2 \end{aligned} \quad (15)$$

where

$$\begin{aligned} \mathbf{X} = & [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10} \ x_{11} \ x_{12} \ x_{13}]^T \\ = & [b_{0x} \ b_{0y} \ b_{1x} \ b_{1y} \ c_{1x} \ c_{1y} \ d_{1x} \ d_{1y} \ e_{1x} \ e_{1y} \ R_2 \ R_3 \ R_4]^T \\ \nabla f(\mathbf{X}) = & \begin{pmatrix} \frac{\partial f(\mathbf{X})}{\partial x_1} \\ \vdots \\ \frac{\partial f(\mathbf{X})}{\partial x_{13}} \end{pmatrix} = 0 \end{aligned} \quad (16)$$

$$\begin{aligned} & \frac{\partial}{\partial b_{0x}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0)^T ([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0) - R_2^2 \right)^2 \\ & + \frac{\partial}{\partial b_{0x}} \sum_{j=1}^N \left(([S_{1j}] \mathbf{d}_1 - [R_{1j}] (\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0)^T ([S_{1j}] \mathbf{d}_1 - [R_{1j}] (\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0) - R_4^2 \right)^2 \\ & + \frac{\partial}{\partial b_{0x}} \left[\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1x} \\ 1 & 1 & 1 \end{pmatrix} \right]^2 = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} & \frac{\partial}{\partial b_{0y}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0)^T ([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0) - R_2^2 \right)^2 + \\ & \frac{\partial}{\partial b_{0y}} \sum_{j=1}^N \left(([S_{1j}] \mathbf{d}_1 - [R_{1j}] (\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0)^T ([S_{1j}] \mathbf{d}_1 - [R_{1j}] (\mathbf{e}_1 - \mathbf{b}_0) - \mathbf{b}_0) - R_4^2 \right)^2 \\ & + \frac{\partial}{\partial b_{0y}} \left[\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1x} \\ 1 & 1 & 1 \end{pmatrix} \right]^2 = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} & \frac{\partial}{\partial b_{1x}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0)^T ([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0) - R_2^2 \right)^2 \\ & + \frac{\partial}{\partial b_{1x}} \sum_{j=1}^N \left(([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1)^T ([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1) - R_3^2 \right)^2 \\ & + \frac{\partial}{\partial b_{1x}} \left[\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1x} \\ 1 & 1 & 1 \end{pmatrix} \right]^2 = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} & \frac{\partial}{\partial b_{1y}} \sum_{j=1}^N \left(([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0)^T ([D_{1j}] \mathbf{b}_1 - \mathbf{b}_0) - R_2^2 \right)^2 \\ & + \frac{\partial}{\partial b_{1y}} \sum_{j=1}^N \left(([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1)^T ([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1) - R_3^2 \right)^2 \\ & + \frac{\partial}{\partial b_{1y}} \left[\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1x} \\ 1 & 1 & 1 \end{pmatrix} \right]^2 = 0 \end{aligned} \quad (20)$$

$$\frac{\partial}{\partial c_{1x}} \left(\sum_{j=1}^N \left(([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1)^T ([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1) - R_3^2 \right)^2 + \left[\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1x} \\ 1 & 1 & 1 \end{pmatrix} \right]^2 \right) = 0 \quad (21)$$

$$\frac{\partial}{\partial c_{1y}} \left(\sum_{j=1}^N \left(([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1)^T ([S_{1j}] \mathbf{c}_1 - [D_{1j}] \mathbf{b}_1) - R_3^2 \right)^2 + \left[\det \begin{pmatrix} b_{0x} & b_{1x} & c_{1x} \\ b_{0y} & b_{1y} & c_{1x} \\ 1 & 1 & 1 \end{pmatrix} \right]^2 \right) = 0 \quad (22)$$

$$\frac{\partial}{\partial d_{1x}} \sum_{j=1}^N \left(([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0)^T ([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0) - R_4^2 \right)^2 = 0 \quad (23)$$

$$\frac{\partial}{\partial d_{1y}} \sum_{j=1}^N \left(([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0)^T ([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0) - R_4^2 \right)^2 = 0 \quad (24)$$

$$\frac{\partial}{\partial e_{1x}} \sum_{j=1}^N \left(([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0)^T ([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0) - R_4^2 \right)^2 = 0 \quad (25)$$

$$\frac{\partial}{\partial e_{1y}} \sum_{j=1}^N \left(([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0)^T ([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0) - R_4^2 \right)^2 = 0 \quad (26)$$

$$\frac{\partial}{\partial R_2} \sum_{j=1}^N \left(([D_{1j}]b_1 - b_0)^T ([D_{1j}]b_1 - b_0) - R_2^2 \right)^2 = 0 \quad (27)$$

$$\frac{\partial}{\partial R_3} \sum_{j=1}^N \left(([S_{1j}]c_1 - [D_{1j}]b_1)^T ([S_{1j}]c_1 - [D_{1j}]b_1) - R_3^2 \right)^2 = 0 \quad (28)$$

$$\frac{\partial}{\partial R_4} \sum_{j=1}^N \left(([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0)^T ([S_{1j}]d_1 - [R_{1j}](e_1 - b_0) - b_0) - R_4^2 \right)^2 = 0 \quad (29)$$

3. Example

Table 1 includes the x and y-coordinates for two groups of seven prescribed rigid-body poses. These poses describe the prescribed finger grasping sequence illustrated in Fig. 5 and they result in twice the maximum number of rigid-body displacements that can be prescribed in the synthesis model in §2.3. The design objective here is to synthesize a finger mechanism to approximate the prescribed grasping sequence.

To assist in specifying judicious initial guesses for Eqs. (10)~(14) and Eqs. (17)~(29), a mechanism solution is first calculated from Eqs. (1)~(5) for a subset of the prescribed poses in Table (poses 1, 2, 4 and 6). Using Eq. (1) with the prescribed values $a_0 = (-1, -1, 1)$, $a_1 = (1, 1, 1)$ and the specified value of $R_1 = 5$, the calculated values are $a_0 = (-1.626374, -1.772792, 1)$, $a_1 = (2.784366, 0.582069, 1)$. Using Eqs. (1)~(5) with the prescribed values $b_0 = (0, 0, 1)$, $b_1 = (3, 0, 1)$, $c_1 = (6, 0, 1)$, $d_1 = (6, 1, 1)$, $e_1 = (1.7, -1, 1)$, $R_2 = 3$, $R_3 = 2$ and $R_4 = 5$, the calculated values are $b_0 = (0.127302, -0.007552, 1)$, $b_1 = (3.254742, -0.029757, 1)$, $c_1 = (5.633234, -0.046525, 1)$, $d_1 = (6.066115, 1.040868, 1)$, $e_1 = (1.624157, -1.025936, 1)$, $R_2 = 3.123883$, $R_3 = 2.375454$ and $R_4 = 4.916735$.

The calculated finger mechanism solution only includes prescribed rigid-body poses 1, 2, 4 and 6. This solution however is the initial guess for a mechanism solution that includes all of the prescribed rigid-body poses. Using Eqs. (10)~(14), the calculated values are $a_0 = (-1.789862, -2.292597, 1)$, $a_1 = (2.713598, 0.692691, 1)$ and $R_1 = 5.404387$. Using Eqs. (17)~(29), the calculated values are $b_0 = (-1.404706, 0.048225, 1)$, $b_1 = (3.043176, 0.017725, 1)$, $c_1 = (5.788283, -0.001046, 1)$, $d_1 = (5.289129, 0.746400, 1)$, $e_1 = (1.896773, -0.971067, 1)$, $R_2 = 4.449757$, $R_3 = 2.745162$ and $R_4 = 3.808070$. The synthesized finger mechanism and approximated grasping poses are illustrated in Fig. 6 and 7 respectively. Table 2 includes the coordinates of the approximated grasping poses. To achieve grasping poses 2 through 7 in Table 2, the proximal phalanx member (ternary link b_0 - b_1 - e_1) is rotated clockwise by 2.511543° , 5.009463° , 7.496273° , 9.974150° , 12.449395° and 14.894576° respectively.

By affixing several mechanical fingers to a rigid palm and thumb, a mechanical hand can

be produced. Figures 8 and 9 illustrate a mechanical hand (comprised of the synthesized mechanical finger) in the extended and grasping poses respectively.

Table 1. Prescribed rigid-body poses

	P	q	r
Pos 1	4.5290, 0.4350	4.5290, 0	3.2673, 0.4350
Pos 2	4.5714, 0.0162	4.5072, -0.4141	3.3235, 0.2024
Pos 3	4.5695, -0.4162	4.4397, -0.8314	3.3653, -0.0395
Pos 4	4.5184, -0.8584	4.3227, -1.2469	3.3916, -0.2908
Pos 5	4.4124, -1.3057	4.1524, -1.6545	3.4006, -0.5517
Pos 6	4.2441, -1.7525	3.9237, -2.0467	3.3906, -0.8232
Pos 7	3.9877, -2.2095	3.6097, -2.4250	3.3627, -1.1134

	p*	q*	r*
Pos 1	6.6793, 0.4350	6.6793, 0	5.7907, 0.4350
Pos 2	6.7127, -0.3515	6.6258, -0.7777	5.8421, -0.1740
Pos 3	6.6326, -1.1747	6.4546, -1.5717	5.8219, -0.8111
Pos 4	6.4168, -2.0225	6.1476, -2.3643	5.7188, -1.4727
Pos 5	6.0359, -2.8762	5.6826, -3.1301	5.5174, -2.1546
Pos 6	5.4468, -3.7044	5.0296, -3.8280	5.1943, -2.8524
Pos 7	4.4786, -4.4799	4.0527, -4.3908	4.6605, -3.6102

Fig. 5 Prescribed finger grasping sequence

Fig. 6 Synthesized finger mechanism

Fig. 7 Prescribed and approximated finger mechanism grasping poses

Table 2. Rigid-body poses achieved by the synthesized finger mechanism.

	p	q	r
Pos 1	4.5290, 0.4350	4.5290, 0	3.2673, 0.4359
Pos 2	4.568824, 0.015308	4.504356, -0.414888	3.321057, 0.202296
Pos 3	4.566165, -0.415811	4.436318, -0.830980	3.361985, -0.039197
Pos 4	4.516190, -0.856473	4.320903, -1.245173	3.388782, -0.290049
Pos 5	4.411779, -1.304463	4.152090, -1.653442	3.399579, -0.551244
Pos 6	4.241070, -1.757837	3.919490, -2.050773	3.391422, -0.825107
Pos 7	3.985380, -2.207549	3.607699, -2.423377	3.359382, -1.112099

	p*	q*	r*
Pos 1	6.6793, 0.4350	6.6793, 0	5.7907, 0.4350
Pos 2	6.708705, 0.345780	6.624153, -0.772484	5.837053, -0.173061
Pos 3	6.629259, 1.165534	6.454573, -1.563918	5.815456, -0.808693
Pos 4	6.416483, -2.013458	6.150181, -2.357418	5.713855, -1.469469
Pos 5	6.037765, -2.871193	5.685857, -3.126900	5.515418, -2.152330
Pos 6	5.447352, -3.704353	5.030348, -3.828176	5.194411, -2.852513
Pos 7	4.485814, -4.475711	4.059257, -4.390422	4.660039, -3.604358

Fig. 8 Synthesized mechanical hand in extended pose

Fig. 9 Synthesized mechanical hand in grasping pose

4. Discussion

The solution data in the Example section were calculated using the mathematical analysis software MATLAB and were expressed to six decimal places. Although the authors employed the three point rigid-body displacement matrix in this work, Eqs. (6) and (7) could be replaced with a displacement matrix that utilizes a single point and angle to describe the planar displacement of a rigid-body⁽¹⁰⁾. The primary advantage in using Eqs. (1)~(7) to calculate a mechanism solution for a subset of the total prescribed rigid-body poses is that the calculated mechanism solution can be used as the initial guess to calculate a mechanism solution using Eqs. (10)~(14) and Eqs. (17)~(29). The latter considers the total number of prescribed rigid-body poses.

5. Conclusion

The motion generation method presented in this work was demonstrated in the synthesis of a multi-loop planar six-bar mechanism comparable to the Belgrade/USC robotic hand⁽¹³⁾⁽¹⁴⁾. In the example, the method was demonstrated to be effective in the synthesis of a six-bar motion generator to approximate two groups of six prescribed rigid-body poses. This motion generation method can accommodate numbers of prescribed rigid-body poses beyond the maximum number of poses allowable using conventional planar six-bar mechanism synthesis models⁽¹⁰⁾.

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