Multimodal Logistics Network Design over Planning Horizon through a Hybrid Meta-Heuristic Approach*

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Abstract
Logistics has been acknowledged increasingly as a key issue of supply chain management to improve business efficiency under global competition and diversified customer demands. This study aims at improving a quality of strategic decision making associated with dynamic natures in logistics network optimization. Especially, noticing an importance to concern with a multimodal logistics under multiterms, we have extended a previous approach termed hybrid tabu search (HybTS). The attempt intends to deploy a strategic planning more concretely so that the strategic plan can link to an operational decision making. The idea refers to a smart extension of the HybTS to solve a dynamic mixed integer programming problem. It is a two-level iterative method composed of a sophisticated tabu search for the location problem at the upper level and a graph algorithm for the route selection at the lower level. To keep efficiency while coping with the resulting extremely large-scale problem, we invented a systematic procedure to transform the original linear program at the lower-level into a minimum cost flow problem solvable by the graph algorithm. Through numerical experiments, we verified the proposed method outperformed the commercial software. The results indicate the proposed approach can make the conventional strategic decision much more practical and is promising for real world applications.

Key words: Logistics Network Optimization, Hybrid Tabu Search, Multimodal Transport, Multi-Term Planning, Graph Algorithm

1. Introduction

Logistic optimization has been acknowledged increasingly as a key issue of supply chain management to improve business efficiency under global competition and diversified customer demands. Though many studies regarding this topic have been made in the area of operations research associated with combinatorial optimization (for examples, Ref. (1) and (2)), they payed major attentions to benchmarking of the proposed method in terms of simply formulated problems. To cope with complex and complicated real world problems in manufacturing systems, therefore, we need to make different efforts. Starting with the original problem(3), to increase the practicability, we concerned various logistic optimization problems associated with the system coupled with supply-side chain(4), consideration on the stair-wised discount transport cost and multi-commodity cases(5), the flexible system against demand variations(6) and the parallel algorithm(7).

However, all of these concerns were confined to strategic or static decision making. During the planning horizons, there usually occur various deviations assumed constant in this strategic or static model. Taking into accounts such dynamic circumstance, we can make
a more reliable and operational decision-making regarding logistic network design. In this study, therefore, we have extended our previous approach termed hybrid tabu search (HybTS) to cope with dynamic circumstances. Accordingly, we can incorporate a production planning such as an inventory management and a multimodal transport into the optimization of logistics network design.

Below, presenting a general formulation and its solution method, we will show the validity of the proposed method through numerical experiments. There, after a few preliminary considerations, we compare the results with those obtained from the commercial software.

2. Hybrid Tabu Search for Logistics Network Design

2.1. Preliminary Statements

Many studies in the area of operations research make point to develop new algorithms and compete their abilities through simple benchmarking, and/or to reveal theoretical truth about how fast, how exactly and how large problem to be solvable by the developed algorithm (for examples, Ref. (8) – Ref. (10)). However, easy applications following these outcomes often cause a dramatic increase in problem size in real world problems, and accordingly such a difficulty that makes almost impossible to solve the resulting problem by any currently available software.

Under such understanding, to cope with the specific problem in complex and complicated real world situation, we concerned various logistics optimization problems for strategic or static decision making. As mentioned already, we concerned certain conditions such like realistic discount of transportation cost, flexibility against demand deviations, multi-commodity delivery and so on.

Eventually, mathematical formulations for these studies refers to a mixed-integer programming (MIP) problem. The HybTS used there decomposes the original problem into upper-level and lower-level sub-problems, and applies a suitable method for each sub-problem. Figure 1 illustrates schematically this solution procedure. There, the upper level sub-problem decides the locations of DC by the sophisticated tabu search based on the result of the lower-level sub-problem.

On the other hand, the lower level sub-problem decides the network routes under the prescribed DC locations at the upper level. After the locations is fixed, the problem refers to a linear program (LP) possible to be transformed into a minimum cost flow (MCF) problem. In practice, this transformation is carried out by adding virtual nodes and edges to the physical configuration as illustrated in Fig.2. Then we can apply the graph algorithm to solve the resulting MCF problem for which especially fast solution algorithm such as CS2(11) is known.

Now, by returning to the upper-level, neighbor location is generated following the algorithm of the sophisticated tabu search whose local search operation is summarized in Table 1. To enhance the efficiency of algorithm, selection probability of each operation is decided based on the following ideas. It is likely that the search types like “Add” and/or “Subtract”
might be used rather often at the earlier stage to explore possibly the whole search space while "Swap" is more suitably applied at the final stage when the number of open DCs approaches almost optimal.

<table>
<thead>
<tr>
<th>Search type</th>
<th>Selection probability</th>
<th>Neighborhood operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add</td>
<td>( p_{\text{add}} )</td>
<td>Let closed DC open.</td>
</tr>
<tr>
<td>Subtract</td>
<td>( p_{\text{subtract}} )</td>
<td>Let open DC close.</td>
</tr>
<tr>
<td>Swap</td>
<td>( p_{\text{swap}} )</td>
<td>Let closed DC open and open DC close.</td>
</tr>
</tbody>
</table>

Letting it be a basis to decide the selection probability in the table, we further present an idea to do it more effectively using the long-term memory or a history of so far search processes. That is, the probability of each operation will be increased by a certain rate if it has brought about the update of solution. In contrast, these values are reset when the tentative solution is not updated by the prescribed consecutive duration and/or a feasible solution has not been obtained.

These procedures will be repeated until a certain convergence criterion has been satisfied.

2.2. Multimodal Model over Planning Horizon

By making use of the available stock of DC to descendant terms (inventory control), we can bring about significant effects on the strategic decision making of logistic network design. However, as supposed easily, problem size will expands greatly along with the increase in the number of planning horizon. To cope with such multiterm problem, we extended the foregoing static or single-term development in a practical manner\(^1\)\(^2\).

Taking into accounts, for such the dynamic circumstance, we can make a more reliable and comprehensive decision if we consider the effects of multimodal transport. It is commonly known as a transport operation carried out using different modes of transport, \textit{e.g.}, truck, train, ship, \textit{etc.} This is because we can make better use of transportation vehicles depending on the situation, say, either fast (short lead time) but expensive and small or slow (long lead time) but cheap and large as illustrated in Fig. 3. However, introduction of such interest into the model will expands drastically the difficulty to solve the resulting problem as shown in the latter.

To describe such idea concretely, we formulate mathematically the problem under consideration as follows.

\[
(p.1) \min \limits_{x,f,s} f(x,f,s) = \sum \limits_{t \in T} \sum \limits_{t_a \in T} \sum \limits_{j \in J} (E_{t_a}^{f_{ij}} + C_{t}^{j} + H_{t}^{j}) f_{t_a}^{f_{ij}} + \sum \limits_{t \in T} \sum \limits_{t_a \in T} \sum \limits_{j \in J} (E_{t_a}^{f_{ij}} + H_{t}^{j}) f_{t_a}^{f_{ij}} + \sum \limits_{t \in T} \sum \limits_{t_a \in T} \sum \limits_{j \in J} \sum \limits_{k \in K} E_{t_a}^{f_{ij}} f_{t_a}^{f_{ij}} \cdot f_{t_a}^{f_{jk}}
\]
\[ + \sum_{i \in I} \sum_{t \in T} \sum_{j \in J} (E_{ij}^{\ell t} + C_i j f_{ij}^{\ell t}) + \sum_{j \in J} K_j \cdot s_j + \sum_{j \in J} F_j \cdot x_j \]

subject to

\[ \sum_{i \in I} \sum_{t \in T} f_{ij}^{\ell t} + \sum_{j \in J} f_{ij}^{\ell t} = D_k^t, \quad \forall k \in K, \forall t \in T \quad (2) \]
\[ \sum_{i \in I} \sum_{t \in T} f_{ij}^{\ell t} + \sum_{j \in J} f_{ij}^{\ell t} + s_j^t \leq U_j^t \cdot x_j, \quad \forall j \in J, \forall t \in T \quad (3) \]
\[ \sum_{i \in I} \sum_{t \in T} f_{ij}^{\ell t} + \sum_{j \in J} f_{ij}^{\ell t} \leq \bar{P}_i^t, \quad \forall i \in I, \forall t \in T \quad (4) \]
\[ \sum_{i \in I} \sum_{t \in T} f_{ij}^{\ell t} + \sum_{j \in J} f_{ij}^{\ell t} \geq P_i^t, \quad \forall i \in I, \forall t \in T \quad (5) \]
\[ \sum_{i \in I} \sum_{t \in T} f_{ij}^{\ell t} + \sum_{j \in J} f_{ij}^{\ell t} + s_j^t = \sum_{i \in I} \sum_{t \in T} f_{ij}^{\ell t} + \sum_{j \in J} f_{ij}^{\ell t} + s_j^{t+1}, \quad \forall j \in J, \forall t \in T \quad (6) \]

\[ f, s \geq 0, f, s \in \mathbb{R} \quad (7) \]
\[ x \in \{0, 1\} \quad (8) \]

Below, we summarize the notations used to describe this problem.

Variable
\[ x_j \] : it takes one if DC is open at place \( j \), otherwise 0  
\[ f_{ij}^{\ell t} \] : quantity that leaves facility \( i \) in term \( t \) and arrive at facility \( j \) in term \( t \)  
\[ s_j^t \] : stock remaining in the next term at DC \( j \) in term \( t \)

Index set
\[ I \] : Index set indicating plant  
\[ J \] : Index set indicating DC  
\[ K \] : Index set indicating customer  
\[ T \] : Index set indicating planning horizon

Parameter
\[ P_i^t \] : lower bound for production at plant \( i \) in term \( t \)  
\[ \bar{P}_i^t \] : upper bound for production at plant \( i \) in term \( t \)  
\[ C_i^t \] : unit production cost at plant \( i \) in term \( t \)  
\[ F_j \] : fixed-charge for opting DC \( j \)  
\[ U_j^t \] : maximum capacity of DC \( j \) in term \( t \)  
\[ H_j^t \] : unit operational cost of DC \( j \) in term \( t \)  
\[ K_j^t \] : unit inventory cost of DC \( j \) in term \( t \)  
\[ D_k^t \] : demand of customer \( k \) in term \( t \)  
\[ E_{ij} \] : unit transport charge for the transportation that leaves plant \( i \) in term \( t \) and arrive at DC \( j \) in term \( t \)  
\[ E_{ij}^{\ell t} \] : unit transport charge for the transportation that leaves DC \( j \) in term \( t \) and arrive at DC \( j' \) in term \( t \)
Now, let us explain the above optimization problem concretely. The first, forth, sixth and seventh terms of the objective function Eq.(1) correspond to the total transportation costs between plant and DC, DC and DC, DC and customer, and plant and customer, respectively. The second and eighth terms are the total production costs at plant, and the third and fifth terms denote the total costs spent for the operations between plant and DC, and DC and DC, respectively. Moreover, the ninth term represents the total holding cost at DC while the tenth term total fixed-charge for opening DC.

On the other hand, the first constraint Eq.(2) requires to meet the demand of every customer every term. The capacity constraint at each DC is given by Eq.(3) every term. Moreover, Eq.(4) and Eq.(5) are the upper and lower bounds respectively on the production ability of each plant every term. Finally, Eq.(6) describes the material balance at each DC every term. Additionally, non-negative conditions on the material flows and binary condition on the open/close selection are given by Eq.(7) and Eq.(8), respectively.

In this model, the transportation modes are distinguished by the difference of transportation lead time. It is described by the superscripts as $t_{s_t}$, which means that the product leaves in term $t_s$ and arrive in term $t_a$. Then we assume the different lead time implies the different transportation mode. The larger the lead time is, the cheaper the transportation cost is assumed, and vice versa.

In numerical solution of optimization problems, we often impose certain additional conditions that might enhance the solution speed by limiting the search space properly. The followings are the augmented constraints and variables for this purpose (tight bound constraints).

$$\sum_{j \in J} s^s_j + \sum_{j \in J} s^s_j = 0$$

$$f^{ts}_{i,j} \leq M \cdot A^{ts}_{i,j}, \quad \forall i \in I, \ j \in J, \ \forall t_s \in T, \ \forall t_a \in T \quad (10)$$

$$f^{ts}_{j,j'} \leq M \cdot A^{ts}_{j,j'}, \quad \forall j \in J, \ j' \in J, \ \forall t_s \in T, \ \forall t_a \in T \quad (11)$$

$$f^{ts}_{j,k} \leq M \cdot A^{ts}_{j,k}, \quad \forall j \in J, \ k \in K, \ \forall t_s \in T, \ \forall t_a \in T \quad (12)$$

$$f^{ts}_{k,k} \leq M \cdot A^{ts}_{k,k}, \quad \forall i \in I, \ k \in K, \ \forall t_s \in T, \ \forall t_a \in T \quad (13)$$

In the above, Eq.(9) restricts the initial and final stocks to be zero. Equations (10) thru (13) exclude the infeasible transportations explicitly where $A^{ts}_{i,j}$ denotes the variable that takes one if there exists such a transportation that leaves place $i$ in term $t_s$ and arrives at place $j$ in term $t_a$, and otherwise zero. Moreover, $M$ is a very large number.

In my knowledge, any studies have not been reported regarding the practical logistics network design problem associated with the multiterm and multimodal conditions\textsuperscript{(13)}.

3. The Hybrid Tabu Search for Multiterm and Multimodal Transport

Adding some ideas to the original one, we show it is still possible to apply effectively the HybTS for the present case as outlined below. Under the multiterm condition, the lower level sub-problem of the HybTS needs to decide the network routes for every term. It refers to a huge and bulk LP whose coefficient matrix becomes almost block diagonal per each term.

To keep high performance of the HybTS, solving the lower level sub-problem fast is essential. That is, we still need to transform the LP into the MCF problem and apply the graph algorithm to solve the resulting problem. Hence, noticing a special topological structure and adding some dealing for the multiterm and multimodal transport, we have invented a systematic procedure to derive a compact minimum cost flow problem as follows.

Step 1: For every term, place the nodes that stand for plant, DC (doubled), and customer. Next, virtual nodes termed source, center $\Sigma$ and sink are prepared. Then connect the nodes...
between source and center $\Sigma$ (ID 1), source and plant (ID 2), $\Sigma$ and plant (ID 3), duplicated DC nodes (ID 5), and customer and sink (ID 9) in turn. This results in the initial three term graph as depicted in Fig.4 (a), for example.

Step 2: Letting $z$ be the total amount of customer demand over planning horizon, i.e., $z = \sum_{t \in T} \sum_{k \in K} D_{t}^{k}$, flow this amount in the source, and flow out from the sink.

Step 3: To constrain the amounts of flow, set the capacities on the edges identified by ID 1, 2, 3, 5 and 9, respectively as shown in “capacity column” in Table 2. Apparently, there never
Table 2  Labeling on the edge

<table>
<thead>
<tr>
<th>Edge ID</th>
<th>Cost</th>
<th>Capacity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0</td>
<td></td>
<td>source-Σ</td>
</tr>
<tr>
<td>#2</td>
<td></td>
<td></td>
<td>source-plant i (term t)</td>
</tr>
<tr>
<td>#3</td>
<td></td>
<td></td>
<td>plant i (term t)</td>
</tr>
<tr>
<td>#4</td>
<td></td>
<td></td>
<td>plant i-DC j (leave in term t and arrive in t_a)</td>
</tr>
<tr>
<td>#5</td>
<td></td>
<td></td>
<td>between doubled nodes representing DC (term t)</td>
</tr>
<tr>
<td>#6</td>
<td></td>
<td></td>
<td>DC j-DC j' (leave in term t and arrive in t_a)</td>
</tr>
<tr>
<td>#7</td>
<td></td>
<td></td>
<td>DC j-customer k (leave in term t and arrive in t_a)</td>
</tr>
<tr>
<td>#8</td>
<td></td>
<td></td>
<td>plant i-customer k (leave in term t and arrive in t_a)</td>
</tr>
<tr>
<td>#9</td>
<td>0</td>
<td></td>
<td>customer k-sink (term t)</td>
</tr>
<tr>
<td>#10</td>
<td></td>
<td></td>
<td>stock at DC j (term t)</td>
</tr>
</tbody>
</table>

induce any costs on edge ID 1 and 9 for the connections.

Step 4: To allow the stock at DC, add the edges from down-DC node to up-DC node in the next term as shown in Fig.4 (b). See the “ID 10” row of the table for the labeling.

Step 5: According to the transportation mode (transportation lead time), connect the edges between plant and DC (ID 4), DC and DC (ID 6), DC and customer (ID 7) and plant and customer (ID 8) every term (See Fig.4 (c)).

Step 6: Finally, place the appropriate label on each edge (See Fig.4 (d)).

From all of these, we have obtained the final graph as shown in Fig.4 (d) that makes it possible to still apply the graph algorithm as before. Consequently, in the lower-level sub-problem of the HybTS, we can solve the extensively expanded problem extremely fast compared with the linear programs.

4. Numerical Experiments

4.1. Evaluation of Multiterm Model

Instead of the dynamic model, a strategic or conceptual decision is often made based on the averaged values that will fluctuate in reality over the planning horizon such as demand. This is equivalent to say that we intend to obtain the result from the static or single-term problem as a preliminary decision. To verify the advantage of considering the dynamic nature that enables us to make use of the stock of inventory, we compared the results between the (averaged) single-term model and the multiterm model using small size benchmark problems. There, the multiterm model is derived(12) by neglecting the difference between t_i and t_a and by simplifying them just as i in the foregoing formulation stated in problem (p.1) and Table 2.

Table 3  Effect of dynamic model

<table>
<thead>
<tr>
<th>Properties of problem</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant</td>
<td>DC</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

In Table 3, we summarize the results taken place under the conditions of demand deviations. Thereat, we know that the dynamic model can derive decisions with less total costs (the value of average model is represented as the rate to the value of the multiterm model to be one hundred). Particularly, it is remarkable that there appeared the case where the average model could not obtain the feasible solution against the demand deviations while the multiterm model could cope with every situation by virtue of the production planning and the inventory control. Particularly, let us notice the situation illustrated in Fig.5 where the adequate production planning and the stocks at the DC are utilized to meet the customer demands varying beyond the upper bound of production ability for the second problem in Table 3.

In Table 4, we summarize the computation environment for the numerical experiments. In the following, results of the HybTS are averaged over five trials, and the problem size is set...
Figure 6 compare the CPU times along with the number of planning horizon between the HybTS and CPLEX 9.0. The maximum problem size in this case becomes as large as what is shown in the figure. Thereat, we can observe the increase in CPU time is almost linear for the HybTS while it is exponential for the CPLEX. Moreover, we confirm that the HybTS can derive the same results as those by CPLEX for every problem until one hundred term problem that is a limit for the CPLEX. This fact supports a prospect that the HybTS may keep high accuracy even for the larger scale problems solved here.

4.2. Evaluation of Multiterm and Multimodal Model

In Table 5, we compare the problem size between only multiterm and the multiterm and multimodal model in terms of the system parameters. Rapid increase in real variables can expand the difficulty to obtain the result unexpectedly. To cope with this dimensionality problem, we note the role of the tight bounds that are described thru Eqs.(9) to (13). By introducing these constraints, we can expect to reduce the solution time when we try to solve the problem based on the mathematical programming like CPLEX. In fact, as shown in Fig.7, we know the computation load without the tight bound constraints tends to increase rapidly with the problem size. Relying on the results, we will adopt this formulation to solve the problems in the following numerical experiments.

To examine a rising computational load with the increase in numbers of modal transport
### Table 5 Number of decision variables and constraints

<table>
<thead>
<tr>
<th>Model</th>
<th>Decision variable</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiterm</td>
<td>$</td>
<td>I</td>
</tr>
<tr>
<td>Multimodal</td>
<td>$</td>
<td>I</td>
</tr>
</tbody>
</table>

* $I$: Plant, $J$: DC center, $K$: Customer, $T$: Term

![Fig. 7 Effect of tight bound formulation](image)

**With:** tight bound constraints are employed  
**Without:** tight bound constraints are not employed

Along the problem size (number of planning horizon), we carried out another preliminary numerical experiment. Figure 8 shows a comparison of CPU times between uni- and two-modal transports. (The problems are solved till 25 terms using CPLEX 9.0 when $|I| = 2$, $|J| = 25$ and $|K| = 30$). We know the computation load grows rapidly when the two-modal model is considered even for such small size problems. It implies the multimodal model will confront the difficulties associated with the dimensionality for real world applications.

From so far discussions, it is interesting to examine effectiveness of the HybTS in terms of the problem size. Figure 9 shows the CPU times compared between the HybTS and the CPLEX along with the number of planning horizon in dual-modal case. Thereat, we can observe the increase is moderate and almost linear for the HybTS, but its increase is slightly rapid compared with the foregoing multiterm case due to the great increase in number of real decision variables as shown in Table 5 and also in Fig. 9. Meanwhile it is more sharply exponential for the CPLEX (See Fig. 6).

In addition, we confirm the approximation rate or the accuracy of the objective function value of the HybTS as long as we can do it. As shown in Table 6, for every problem, the HybTS can derive the same results as those by CPLEX, i.e., no gap with very short CPU times. After all, relying on these numerical experiments, we can ascertain that the HybTS is promising for real world applications. Prospectively, we can accelerate such effect more by applying the parallel computing environment that is shown suitable for the present two-level
hybrid meta-heuristic algorithm\(^7\).

**5. Conclusion**

This paper concerned a multimodal and multiterm (dynamic) logistics optimization problem. To cope with such huge and complicated problem, we have extended a method termed hybrid tabu search (HybTS) that is developed previously for a static and basic model by the authors. In practice, we have invented a systematic procedure to transform the mathematical model into the graph model and finally complete it to the multiterm and multimodal models that can hold high performance like the basic model.

Numerical experiments revealed that production planning or inventory control and multimodal transport could bring about an economical effect and robustness against demand deviations during the planning horizon. The validity of the HybTS as a solution method was also shown through comparison with the commercial software both in accuracy and solution speed.

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Appendix

Outline of tabu search

Tabu search (TS)\(^{(14),(15)}\) is a metaheuristic algorithm on a basis of local search technique. TS repeats the local search iteratively to move from a current solution \(x\) to a possible and best solution \(x'\) in the neighbor of \(x\), \(N(x)\). To avoid the cycling of the solution, TS uses a short-term memory structure termed tabu list that prohibits transition to any solutions for a while even if this will improve the current solution. The basic iteration process of TS is outlined for minimization problem as follows:

Step 1: Generate an initial solution \(x\) and let \(x^* := x\), where \(x^*\) denotes the current best solution. Set \(k := 0\) and let the tabu list \(T(k)\) be empty.

Step 2: If \(N(x) \setminus T(k)\) becomes empty, stop. Otherwise, set \(k := k + 1\) and select \(x'\) such that \(x' = \arg\min f(x)\) for \(\forall x \in N(x) \setminus T(k)\).

Step 3: If \(x'\) outperforms the current solution \(x^*\), i.e., \(f(x') \leq f(x^*)\), let \(x^* := x'\).

Step 4: If a chosen number of iterations has elapsed either in total or since \(x^*\) was last improved, stop. Otherwise, update \(T(k)\), and go back to Step 2.