Heat Flux Estimation at Heat Sources of Machine Tools by Solving Inverse Problems*

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Abstract
An increase in frictional heat due to the increase in rotational speed and feed rate causes thermal deformation of machine tools, resulting in a decrease in machining accuracy. To establish a method for reducing thermally induced machining errors of machine tools, a method has been proposed to estimate the heat flux acting upon a machine tool body by solving inverse problems. Experiments were carried out with the cross slide of a machine tool. Using the heat flux estimated at the heat source, the temperature rise of the body in a steady state could be obtained with sufficient accuracy. Effects of both the number of evaluating and measuring points of the temperature on the estimation accuracy were also examined experimentally. More accurate estimation of the heat flux requires that the number and position of both temperature measurement points and candidates for heat sources are selected considering how the machine tool is actually operated.

Key words: Machine Tool, Machining Accuracy, Thermal Deformation, Heat Flux, Temperature, Inverse Problem, FEM Simulation

1. Introduction
Both higher efficiency and higher precision are essential requirements for machining with machine tools. An increase in frictional heat due to the increase in rotational speed and feed rate causes thermal deformation of machine tools, resulting in a decrease in machining accuracy. In the conventional system, to compensate for the deformation, tool tip displacements are estimated by substituting the temperatures at several points of a machine tool into the experimentally obtained equations of the relationship between the displacement and the temperature. NC equipments mounted on a machine tool numerically compensate for the displacement predicted by the equation. However, the present multifunctional trend of machine tools causes the deformation to be complicated. The complexity of the machine tool structure makes it more difficult to guarantee machining accuracy by using a simple system based on the temperature to compensate for the deformation. Machine tools have many heat sources such as the main spindle motor and linear guides. A heat source causes the temperature of machine tools to rise, and thermal deformation is induced according to the temperature distribution and structure of the machine tool. This indicates that to compensate for the thermal deformation in operation using an NC equipment, it is very important to estimate the heat flux at the heat source as accurately as possible.

A recent study on the thermal deformation of machine tools demonstrated that the thermal deformation could be estimated by measuring the temperature of the machine tool structure and the surrounding region *(1)-(7)*. Although the heat source of a simplified structure can be detected by numerical simulations *(8)-(12)*, it is difficult to find the heat source of
complicated structures such as machine tools.

In this paper, in order to establish a method to reduce thermally induced machining errors of machine tools, an algorithm is proposed to estimate the heat flux acting upon the machine tool body by solving inverse problems, and the effectiveness of the algorithm is examined experimentally using the cross slide of a commercially available machine tool.

2. Algorithm for Estimating Heat Flux and Experimental Examinations

2.1 Algorithm for Estimating Heat Flux of Heat Source

Figure 1 shows an algorithm to estimate the heat flux acting upon the machine tool body. The heat flux, which is assumed uniform, is estimated as follows. First, temperature sensors installed at several points in the machine tool body measure the temperature rise. The inverse problem solution estimates the heat flux at the heat source, which can be identified by comparing the temperature rises obtained by finite element analyses with those measured experimentally.

![Fig. 1  Algorithm for estimating heat flux acting upon machine tool body.](image)

2.2 Cross Slide Model

Figure 2 shows a schematic diagram of a cross slide model used in the experiments to verify the effectiveness of the algorithm shown in Fig.1. In practical operation, the model, which is a unit of a commercially available machine tool, is installed on the bed and is moved along the slide guide ways. In the experiments, the model was set alone on supports with hard rubber sheets held between the lower slide guide ways in Fig.2 and the supports. Table 1 shows the physical properties of the model. A silicon rubber heater of 600 mm length and 50 mm width was installed to supply the uniform heat inflow of 300 W on slant slide guide way B in Fig.3. To prevent heat dissipation into the air from the heater, heat insulation plates made of Bakelite were placed on the heater and clamped on the slant slide guide ways. To promote heat transfer from the heater to the slant slide guide way, a suitable grease was selected and was applied to the guide way.
Table 1  Physical properties used for thermal analyses of cross slide model.

<table>
<thead>
<tr>
<th>Material</th>
<th>Cast iron</th>
</tr>
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<tbody>
<tr>
<td>Density kg/m$^3$</td>
<td>7000</td>
</tr>
<tr>
<td>Thermal conductivity W/(m·K)</td>
<td>50</td>
</tr>
<tr>
<td>Specific heat kJ/(kg · K)</td>
<td>0.55</td>
</tr>
<tr>
<td>Heat transfer coefficient at surface W/(m$^2$ · K)</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 2  Cross slide model and temperature rise evaluation points ($t_1$–$t_{15}$).

Fig. 3  Candidates for heat source.
2.3 Estimation of Heat Flux at Heat Source

The temperature rises of the cross slide model were measured with temperature sensors set at the 15 points \( t_1, \cdots, t_{15} \) shown in Fig.2. The temperature rises were recorded in a microcomputer at a sampling interval of 1 min. The experiments were carried out at a room temperature of 294 K.

Figure 4 shows the experimental results of the temperature rises of the cross slide model. In the experiments, the heater was only installed on the slant slide guide way \( B \) in Fig.3. The temperatures rose rapidly after the start of heating and reached a quasi-steady state at 480 min. The maximum temperature rise of 28.4 K was obtained at point \( t_{15} \) shown in Fig.2.

Heat inflows from the heat sources shown in Fig.3 were estimated with the quasi-steady state temperature rises shown in Fig.4 in combination with thermal analyses by the finite element method and the optimization technique. To examine the effectiveness of the algorithm mentioned in section 2.1, in the estimation, as shown in Fig.3, though except for candidate \( B \) for heat source area, candidates \( A, C-I \) had no heaters, all candidates for heat source area were selected considering the practical operation of the cross slide. Candidate \( B \) had a length of 600 mm and a width of 50 mm, which were the same as those of the heater installed on \( B \). The physical properties shown in Table 1 were used for thermal analyses of the cross slide model. Optimization was carried out in combination with linear programming, sequential quadratic programming, downhill simplex method, and genetic algorithm. The searches for the heat sources were advanced in the direction where the objective function \( \delta \) defined by the following equation was minimized:

\[
\delta = \sum_{i=1}^{n} (T_{ex}(i) - T_{an}(i))^2 \quad (i = 1, \cdots, n) \tag{1}
\]

where \( T_{ex}(i) \) is the temperature rise after 480 min heating at point \( t_i \) shown in Fig.2, \( n \) is the number of the temperature rise evaluation points, and \( T_{an}(i) \) is the corresponding one obtained by the FEM analysis. \( \delta \) was expressed as the sum of squares of \( (T_{ex}(i)-T_{an}(i)) \).
Table 2 shows the search range of the heat flux applied to the candidate for the heat source area. Considering the heat dissipation from each heat source, the lower limit of the heat flux was taken as a negative value.

### 2.4 Results of Heat Flux Estimation

The heat inflows that minimize the objective function $\delta$ defined in Eq.(1) were determined from the results of repetitive temperature rise analyses of the cross slide model. Figure 5 shows an example of the relation between the objective function $\delta$ and the number of repetitions. The total of repetitions carried out for optimization was 506. $\delta$ had the minimum of 1.970 K$^2$ at the number of 401.

Figure 6 corresponding to Fig. 5 shows the heat inflow for the optimal solution at each of the nine heat sources. The heat inflow was obtained by multiplying the heat flux in the optimal solution by the area of the candidate for the heat source. The largest heat inflow of 250.6 W was obtained at candidate $B$, where the heater of 300 W was installed. It was about 16% lower than the applied inflow of 300 W in the experiment. In candidates $G$, $H$, and $I$, where no heaters were installed, non-negligible heat inflows were estimated. The total of the estimated inflows was 343 W, which was about 15% higher than the applied heat inflow of 300 W. The effects of the number of candidates for the heat source on the estimation accuracy of the heat inflow will be discussed in section 3.2.
Figure 7 shows the correlation between the measured temperature rises and the analytical ones corresponding to the optimal solution shown in Fig. 6. They were in good agreement. This indicated that the heat inflows shown in Fig. 6 had a negligible effect on temperature rise estimation.

3. Discussion

The thermal deformation of the machine tool body is directly affected by the temperature rise and temperature rise distribution of the body. As clarified in section 2.4, unlike the heat inflow at the heat source, the temperature rise in practical use could be estimated with sufficient accuracy. In this section, the effects of the number of temperature rise evaluation points and the number of the candidates for heat source on the accuracy of heat inflow estimation are examined.

3.1 Effects of Number of Temperature Rise Evaluation Points

Table 3 shows the analytical conditions used to examine the effects of the number of temperature rise evaluation points in Fig. 2 on the estimation accuracy of heat inflow at the heat source. In Table 3, the number of points increase from one (Task p1) to 15 (Task p8).

<table>
<thead>
<tr>
<th>Task</th>
<th>Temperature rise evaluation points</th>
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<tbody>
<tr>
<td></td>
<td>$t_1$</td>
</tr>
<tr>
<td>p1</td>
<td>○</td>
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<tr>
<td>p2</td>
<td>○</td>
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<td>p3</td>
<td>○</td>
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<td>p4</td>
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<td>p5</td>
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<td>p6</td>
<td>○</td>
</tr>
<tr>
<td>p7</td>
<td>○</td>
</tr>
<tr>
<td>p8</td>
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</table>
Figure 8 shows the results corresponding to Table 3. The estimated heat inflows in Tasks p1 and p2 at candidate B, where the heater was installed, were respectively 64.7 W and 61.1 W. Both of the values were much smaller than the applied heat inflow of 300 W in the experiment. In Task p2, the estimated heat inflow at candidate B was about 80 % lower than the applied heat inflow in the experiment. Two temperature rise evaluation points were not adequate to evaluate the heat inflow with sufficient accuracy for candidate B. In Task p8 with 15 temperature rise evaluation points, the estimated heat inflow at candidate B was 250.6 W. It was the closest value to the applied heat inflow in the experiment among all tasks in Fig. 8. The more the temperature rise evaluation points were increased, the more the estimation accuracies for candidates D, E, and F where no heat inflows were applied in the experiment were improved.

3.2 Effects of Number of Candidates for Heat Source

Table 4 shows the analytical conditions used to examine the effects of the number of candidates for the heat source in Fig.3 on the estimation accuracy of the heat inflow at the heat source. The combination of the candidate for the heat source was selected considering the applied heat inflow in the experiment.

Figure 9 shows the effects of the number of candidates for the heat source on the estimation accuracy of the heat inflow. The number of temperature rise evaluation points was 15, which had the most accurate estimation of the heat inflow in Fig.8. The estimation accuracies of the heat inflow in Tasks s2 and s3 were higher than the accuracy in Task s1 with more candidates. In Tasks s2 and s3, the estimated heat inflows at candidate B were approximately equal to the applied heat inflow in the experiment (300 W). This demonstrates that in estimating the heat inflow at the heat source of the machine tool body, the number and position of candidates for heat source must be determined by taking into account the state of the machine tool operation. Candidates may be avoided where less heat inflow is forecasted.
4. Conclusion

Experiments were carried out with the cross slide of a commercially available machine tool to verify the effectiveness of a method for estimating the heat flux acting on the machine tool body by solving inverse problems. The results demonstrated that by using the heat flux estimated at the heat source, the temperature rise of the body in a steady state could be obtained with sufficient accuracy for practical use. To estimate the heat flux as accurately as possible, it is important to suitably select the number and position of both temperature measurement points and candidates for heat source considering the state of machine tool operation.

References


