Workspace and Orientation Analysis of a Parallel Structure for Robotic Fingers*

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Abstract

This paper investigates the workspace and the orientation capability of robotic fingers with a five-bar linkage and its variance defined by the parameter of a base link. Associating with the kinematics model and the link parameter analysis of a general structure in the form of coincidence of two ground pivots, the workspace and the orientation mapping of the robotic finger are presented and singularity conditions under different link parameters are discussed. This is then extended to variant mechanisms with separate ground pivots forming a structure of a parallel mechanism for robotic fingers, leading to further singularity analysis and workspace discussion. The orientation capability is then depicted in the workspace of this variance. A robotic finger is design to show the industrial application of the Analysis.

Key words: Orientation Mapping, Workspace, Singularity, Robotic Finger, Parallel Mechanism

1. Introduction

It has been widely required in industry to develop a new generation of robotic manipulators with the dexterity to meet the changed market. However, the manipulators currently used are still far from the desired level of dexterity and flexibility. Robotic fingers of various types have subsequently been developed and applied to new developed robotic systems as modules.

With respect to the structure selection, closed-chain linkages like four-bar linkages⁽¹⁾⁻⁽³⁾ and five-bar linkages⁽⁴⁾⁻⁽⁷⁾ have been implemented as a structure for a robotic finger because of their reliability at an industry-preferred speed. For instance, Yi, Oh, and Hong⁽⁸⁾⁽⁹⁾ proposed a five-bar finger mechanism by studying methodologies for stiffness and motion frequency modulations via redundant actuation.

In most aspects of kinematics of robotic fingers, the flexibility is one of the key analyses and it gives rise to the workspace study of a close-chain mechanism. Yang and Lai⁽¹⁰⁾ in 1985 first presented a method of service angles on the sphere to analyze the workspace of a mechanism. At the same time Jo and Haug⁽¹¹⁾ obtained boundaries of mechanism workspaces and traced a regular one-dimensional solution from non-linear equations. Furthermore, Yang and Haug⁽¹²⁾⁽¹³⁾ presented a method that models a planar mechanism in a three-dimensional workspace. Through this method, reduced-dimension representation can be derived by fixing either a possible point or a certain orientation of an end-effector. A new kinematic representation with respect to the algebra of rotations was depicted to formulate kinematics equations of manipulators by Xi and Fenton⁽¹⁴⁾ in 1997. As an example of the other possible methods, Ceccarelli⁽¹⁵⁾ introduced a numerically efficient method that can be applied in terms of an algebraic description of the workspace geometry. This formulation has been useful to deduce an interesting algorithm for workspace analysis

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by using an evaluation of the workspace boundary. More recently, Tanev ⁽¹⁶⁾ proposed an algorithm for the determination of the workspace of a hybrid type manipulation system based on the obtained closed form solution of the forward and inverse kinematics problems in 2000.

The concern with the dexterity of robotic fingers in the workspace has been raised, particularly the orientation with respect to end-effector position changes of a robotic finger. In some applications, robotic fingers are required to keep a certain orientation during manipulation that characterizes their kinematic dexterity. Kumar and Waldron ⁽¹⁷⁾⁽¹⁸⁾ were the first to depict the concept of a dexterous workspace where end-effectors can rotate about any axes in this workspace. Dai and Wang ⁽¹⁹⁾ and Cui and Dai ⁽²⁰⁾ investigated the manipulative dexterity of robotic hands and fingers. Luo and Dai ⁽²¹⁾ explored the characteristics of a three-fixed-pivoted multi-phalanx robotics finger. Ghafoor *et al* ⁽²²⁾ proposed stiffness modeling of a finger contact. Minor and Mukherjee ⁽²³⁾ represented a surgical manipulator by defining its dexterous workspace within an orientation range of 270 degrees. Dai, Holland and Kerr ⁽²⁴⁾ developed an orientation representation by introducing a finite twist to describe a manipulator's motion. With this representation, joint variables can be represented as lines on the mapping manifold. Dai and Shah ⁽²⁵⁾⁽²⁶⁾ then presented a new approach for analyzing and synthesizing the orientation capability of planar serial manipulators by relating the end-effector to the base joint of a manipulator in the form of a virtual adjustable link. This method converted the open-chain kinematic problem into that of an equivalent close-chain mechanism, and characterized the relationship between the orientation capability of a serial manipulator and the rotatability of the equivalent mechanism.

This paper presents an orientation analysis with respect to end-effector position changes of a 5-bar two-degree-of-freedom robotic finger. Although some literatures ⁽²⁷⁾⁻⁽³²⁾ have examined the kinematics, the singularity and the workspace of a five-bar mechanism, this paper is the first to analyze the orientation capability of such linkages as a robotic finger which is the key issue of the linkage's dexterity. The orientation analysis is carried out by studying the motion of the end-effector from the geometry parameters of the finger mechanism. The boundary of the workspace is presented from the singularity positions. The mapping of the fingertip expressed by the angle between a fingertip link and a base link is studied. A base-link variance of the mechanism is discussed. The workspace and the orientation capability is analysed and compared with a general 5-bar linkage. At the end of the paper, a robotic finger for carton packaging is developed. It shows the application of the workspace and orientation analysis for the mechanism design.

2. Geometry and Link Parameter Analysis

The orientation analysis is based on a five-bar linkage, represented in Fig. 1(a) in the form of its topology structure. This linkage has two ground-pivoted links which form a virtual link between two fixed pivots. It is identified as a general structure of a five-bar-linkage robotic finger. When two ground pivots are separated physically giving a distance l_v , the mechanism is converted to a parallel manipulator, as shown in Fig. 1(b). Geometry parameters can be obtained from Fig. 2 as following

Fig. 1 Topology structure of a five bar linkage of a robotic finger

Fig.2 Structure of the 5-bar linkage

The five-bar linkage in Fig. 2 is a closed chain linkage and the two ground pivoted links providing inputs are denoted OA and OC. Angles between two input links OA, OC and X-axis are given as ϕ and θ that provide two input parameters. The fingertip is made from the extension of the transmission bar AB and the position P of the linkage is identified and described by

$$p = \overline{OA} + \overline{AB} + \overline{BP} = \begin{pmatrix} l_1 \cos(\phi) + (l_4 + l_p) \cos(\psi) \\ l_1 \sin(\phi) + (l_4 + l_p) \sin(\psi) \end{pmatrix} \quad (1)$$

While l_1, l_2, l_3, l_4, l_p are lengths of links OA, OC, BC, AB and BP. The angle ψ between the fingertip and X-axis is defined as the orientation angle for the linkage. The geometric relationship can be derived from Fig. 3. A construction line has been added between A and C for the purpose of studying the orientation angle ψ about input angles θ and ϕ . This construction line can thus be expressed as follows:

$$\begin{aligned} \overline{AC}^2 &= \overline{OA}^2 + \overline{OC}^2 - 2\overline{OA} \cdot \overline{OC} \cdot \cos(\phi - \theta) \\ &= l_1^2 + l_2^2 - 2l_1l_2 \cos(\phi - \theta) \end{aligned} \quad (2)$$

Fig. 3 Symbolic representation of the 5-bar linkage

The angle ζ is defined by the angle between two links OA and AP. It can be described by the following equation:

$$\zeta = \arccos\left(\frac{l_2 \cos(\phi - \theta) - l_1}{\sqrt{l_1^2 + l_2^2 - 2l_1l_2 \cos(\phi - \theta)}}\right) + \arccos\left(\frac{l_4^2 - l_3^2 + l_1^2 + l_2^2 - 2l_1l_2 \cos(\phi - \theta)}{2l_4 \sqrt{l_1^2 + l_2^2 - 2l_1l_2 \cos(\phi - \theta)}}\right) \quad (3)$$

While, the angle ζ is divided by added line AC into two parts, which the first angle can be calculated using trigonometric functions from triangle $\angle OAC$ and triangle $\angle ABC$.

Furthermore, the angle ζ is the summation of the orientation angle ψ and the input angle ϕ 's supplement angle. Thus the orientation angle ψ can be given as:

$$\psi = \zeta - 180^\circ + \varphi \quad (4)$$

By substituting the transmission angle ζ from Eq. (3) into Eq. (4), the orientation angle ψ about input angles θ and φ can be given as:

$$\psi(\varphi, \theta) = \arccos\left(\frac{l_2 \cos(\varphi - \theta) - l_1}{\sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) + \arccos\left(\frac{l_4^2 - l_3^2 + l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}{2l_4 \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) - 180^\circ + \varphi \quad (5)$$

Then, the moving distance of the fingertip can be expressed in terms of input angles θ and φ :

$$d = p_x = l_1 \cos(\varphi) + (l_4 + l_p) \cos(\psi(\varphi, \theta)) \quad (6)$$

From Eq. (1) and Eq. (5) the complete equation of the fingertip position in Cartesian space can be expressed as:

$$p = \begin{pmatrix} l_1 \cos(\varphi) + (l_4 + l_p) \cos\left(\arccos\left(\frac{l_2 \cos(\varphi - \theta) - l_1}{\sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) + \arccos\left(\frac{l_4^2 - l_3^2 + l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}{2l_4 \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) - 180^\circ + \varphi\right) \\ l_1 \sin(\varphi) + (l_4 + l_p) \sin\left(\arccos\left(\frac{l_2 \cos(\varphi - \theta) - l_1}{\sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) + \arccos\left(\frac{l_4^2 - l_3^2 + l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}{2l_4 \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) - 180^\circ + \varphi\right) \end{pmatrix} \quad (7)$$

Hence, these kinematic equations can be used to produce workspace diagrams of the linkage. Moreover these kinematics equations can also be applied to the analysis of the workspace, the singularity and the orientation mapping of the linkage.

The analysis of the fixed-orientation-position and link parameters leads to the study of the orientation mapping of the linkage. The orientation mapping can be obtained from a set of ranges where the end-effector of a robotic finger can reach with certain orientation. The orientation angle ψ in different configuration of the fingertip is a key study for the flexibility of the manipulator. In the case when $\psi=0$ the orientation angle ψ should keep zero and there is a relationship as follows. Equation (5) will now reduce to:

$$\arccos\left(\frac{l_2 \cos(\varphi - \theta) - l_1}{\sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) + \arccos\left(\frac{l_4^2 - l_3^2 + l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}{2l_4 \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varphi - \theta)}}\right) = 180^\circ - \varphi \quad (8)$$

The singularity contains an extreme position that the linkage can approach in the work space. The singularity analysis is crucial to the study of the linkage's workspace and the orientation capability. In following singularities the finger tip reaches the boundary of the workspace.

A general case of singularity occurs when links are collinear, as illustrated in Fig.4, where the link lengths are given as $l_1=400\text{mm}$, $l_2=450\text{mm}$, $l_3=300\text{mm}$, $l_4=350\text{mm}$. This singularity occurs at the boundary of the workspace that the fingertip can reach. The analysis of the singularities and the workspaces will be divided into three cases, which are determined by the relative lengths of transmission bars.

Fig. 4 The workspace arc analysis of the fingertip of the linkage

The analysis of the range of the movement can be obtained by mapping the distance l and the distance d between two singularities. However the singularity depends upon the parameters of the linkage and link lengths that satisfy different singularity conditions.

Case 1 occurs when $l_1+l_4>l_2+l_3$ and $l_3<l_4$ that p'' is the singularity and satisfies the condition:

$$\theta'' = \arccos\left(\frac{l_4^2 + (l_2 + l_3)^2 - l_1^2}{2l_4(l_2 + l_3)}\right) - \psi \quad (9)$$

and

$$\varphi'' = \arccos\left(\frac{(l_2 + l_3)^2 + l_1^2 - l_4^2}{2l_1(l_2 + l_3)}\right) \quad (10)$$

The other singularity is p' when

$$\theta' = \arccos\left(\frac{l_4^2 + l_2^2 - l_1^2}{2l_4l_2}\right) + \psi \quad (11)$$

and

$$\varphi' = \arccos\left(\frac{l_1^2 + l_2^2 - l_4^2}{2l_1l_2}\right) \quad (12)$$

then

$$l = p''_x - p'_x = l_1 \cos(\varphi'' - \varphi') \quad (13)$$

and

$$h = p''_y - p'_y = l_1 \sin(\varphi'' - \varphi') \quad (14)$$

The workspace arc based on this configuration can be given as the arc between φ' and φ'' on the circle that the center is on $(l_p \cos \psi, l_p \sin \psi)$ and the radius is l_1 . The arc starts from $\varphi' + \psi$. And the angle of the arc is

$$\begin{aligned} \delta &= \varphi' - \varphi'' \\ &= \arccos\left(\frac{l_1^2 + l_2^2 - l_4^2}{2l_1l_2}\right) - \arccos\left(\frac{(l_2 + l_3)^2 + l_1^2 - l_4^2}{2l_1(l_2 + l_3)}\right) \end{aligned} \quad (15)$$

Case 2 occurs when the configuration is $l_1=400\text{mm}$, $l_2=350\text{mm}$, $l_3=400\text{mm}$, $l_4=350\text{mm}$, $l_p=800\text{mm}$ and $l_1=l_3$, $l_2=l_4$. The arrangement of the curve is a circle with radius l_1 and origin $(l_p, 0)$. The angle θ keep zero, where in this case $l_1=400\text{mm}$, $l_p=800\text{mm}$.

Fig. 5 The workspace acre analysis when $l_1+l_4=l_2+l_3$ and $l_1=l_3$, $l_2=l_4$

Case 3 occurs when the configuration is $l_1+l_4<l_2+l_3$. In this case, $l_4-l_1 \leq l_2-l_3$, when $l_2 \leq l_4$ the angle θ keeps changing under X-axis. The linkage is in a singularity configuration and the workspace arc is shown in Fig. 6 where:

$$\varphi' = \pi - \arccos\left(\frac{l_4^2 + l_1^2 - (l_2 - l_3)^2}{2l_1l_4}\right) \quad (16)$$

In this case $l_1=400\text{mm}$, $l_2=550\text{mm}$, $l_3=325\text{mm}$, $l_4=240\text{mm}$, $l_p=800\text{mm}$. from Eq. (16). Then the arc is a circle with radius l_1 and origin $(l_p, 0)$.

Fig. 6 The workspace acre analysis when $l_1+l_4 < l_2+l_3$

From the analysis, the conclusion can be drawn that different configurations of the linkage can lead different manipulation workspaces when the orientation angle keeps horizontal. In this case the workspace is larger in the condition when $l_1+l_4 > l_2+l_3$ and $l_3 < l_4$ than when and $l_1=l_3$, $l_2=l_4$. While, for the third condition, $l_1+l_4 < l_2+l_3$, the workspace would be less than the first condition due to the physical limitation of the linkage.

3. Orientation Map

The orientation map when the orientation angle is zero has been discussed above and workspace arcs are given for divert linkage configurations. By expanding the orientation map to every orientation angle between -90° to 90° , a reachable workspace of an end-effector based on the extreme and singularity positions of the robotic finger can be given. The same three cases as before will be examined.

Case 1 will be defined as when the configuration is $l_1+l_4=l_2+l_3$ and $l_1=l_3$, $l_2=l_4$. Workspace for changed orientation angles can be obtained from Fig. 7, as follows:

Fig. 7 Workspace and orientation mapping when $l_1+l_4=l_2+l_3$ and $l_1=l_3$, $l_2=l_4$

The arrangement of the workspace of the end-effector in orientation angles from -90° to 90° is within the area between the half circle with radius $l_P+l_1=1200\text{mm}$ and the half circle with $l_P-l_1=400\text{mm}$ where their origin is $(0,0)$, $l_1=400\text{mm}$, and $l_P=800\text{mm}$. The top edge of the workspace is the left half of a circle with radius l_1 and origin $(0, l_P-l_1)$, when the orientation angle $\psi=90^\circ$. The bottom edge of the workspace is the right half of a circle with radius l_1 and origin $(0, -(l_P-l_1))$, when the orientation angle $\psi=-90^\circ$.

Case 2 is the workspace for changed orientation angles when the configuration is $l_1+l_4 < l_2+l_3$.

Fig. 8 Workspace and orientation mapping when $l_1 + l_4 < l_2 + l_3$

From the extreme positions the end-effector can reach, it follows that

$$\varphi = \pi - \arccos \frac{(l_4 + l_3)^2 + l_1^2 - l_2^2}{2(l_4 + l_3)l_1} - \psi \quad (17)$$

In this singularity position the extreme position can be gotten from a virtual linkage between fingertip l_p and origin O as follows:

$$\cos \zeta = \frac{(l_4 + l_3)^2 + l_1^2 - l_2^2}{2l_1(l_4 + l_3)} \quad (18)$$

where ζ is the internal angle $\angle PAO$, then from this angle ζ the virtual link OP can be obtained as:

$$OP = \sqrt{(l_1 + l_p)^2 + l_1^2 - \frac{(l_1 + l_p)[(l_4 + l_3)^2 + l_1^2 - l_2^2]}{(l_4 + l_3)}} \quad (19)$$

The arrangement of the reachable workspace of the end-effector in orientation angles from -90° to 90° is a ring within the area between half circle with radius $l_p + l_1 = 1200\text{mm}$ and half circle with $OP = 635\text{mm}$ from Eq. (18) where their origin is (0,0), in this case $l_1 = 400\text{mm}$, $l_2 = 300\text{mm}$, $l_3 = 480\text{mm}$, $l_4 = 240\text{mm}$, $l_p = 800\text{mm}$. The top edge of the workspace is an arc on the left half of a circle with radius l_1 and origin $(0, l_p - l_1)$, when the orientation angle $\psi = 90^\circ$. The bottom edge of the workspace is on the arc of the right half of circle with radius l_1 and origin $(0, -(l_p - l_1))$, when the orientation angle $\psi = -90^\circ$. Both arc angles can be obtained from Eq. (17).

Case 3 is the workspace analysis for changed orientation angles when the configuration is

$$l_1 + l_4 > l_2 + l_3 \quad \text{and} \quad l_3 \leq l_4.$$

The arrangement of the reachable workspace (see Fig. 9) of the end-effector in orientation angles from -90 to 90 degree is a ring within the area between half circle with radius OP' and half circle with OP' where its origin is (0,0), in this case $l_1 = 400\text{mm}$, $l_2 = 300\text{mm}$, $l_3 = 350\text{mm}$, $l_4 = 380\text{mm}$, $l_p = 800\text{mm}$. The top edge of the workspace is an arc on the left half of a circle with radius l_1 and origin $(0, (l_p - l_1))$, when the orientation angle $\psi = 90^\circ$. The bottom edge of the workspace is on the arc right half of a circle with radius l_1 and origin $(0, -(l_p - l_1))$, when the orientation angle is $\psi = -90^\circ$. Both arc angles can be obtained.

Fig. 9 workspace and orientation mapping of the fingertip of the linkage $l_1 + l_4 > l_2 + l_3$

From over cases analysis the dexterity work space the fingertip can be reached is compared as follow Fig. 10. The different configurations of the linkage can lead to different manipulation workspaces. In this comparison the fingertip length l_p and input crank link length l_1 are similar. In case 1, when $l_1 + l_4 = l_2 + l_3$ and $l_1 = l_3$, $l_2 = l_4$, the work space is a maximum ring that is larger than that for the condition when $l_1 + l_4 < l_2 + l_3$, which is a ring within the maximum ring. While for case 3, when $l_1 + l_4 > l_2 + l_3$, the workspace is less than for the first two cases due to the singularity limitation of the linkage.

Fig. 10 workspace comparing of the fingertip of the linkage in different configurations

Hence the orientation map is produced by a representation of the position and the orientation in terms of the fingertip and the orientation angle. The range of the dexterity of the linkage can be given in divert configurations of the linkage and could describe the rotation capability of the fingertip.

4. Workspace Analysis of the Base-Link Variance

The general configuration of the five bars linkage for the structure of the robotic finger has been analyzed. The based link of two pivots is a virtual link with $l_v = 0$ as in Fig. 1(a). The variant case is when the virtual link is not zero with $l_v > 0$, the mechanism is thus changed to a parallel mechanism that is composed by two serial chains as Fig. 11.

Fig. 11 parallel linkage for the robotic finger

A construction line l_d is added between points O_1 and C in Fig. 11, thus the angle β can be settled as a new input angle that relate to input angle θ as

$$\beta = \arccos \frac{l_v^2 + l_d^2 - l_2^2}{2l_d l_v} \quad (20)$$

where the length l_d that can be treated as a new input link as

$$l_d = \sqrt{l_v^2 + l_2^2 - 2l_v l_2 \cos(\pi - \theta)} \quad (21)$$

Hence the position of the fingertip can be obtained.

$$p = \overline{O_1 A} + \overline{AP} = \begin{pmatrix} l_1 \cos(\varphi) + (l_d + l_p) \cos(\psi) \\ l_1 \sin(\varphi) + (l_d + l_p) \sin(\psi) \end{pmatrix} \quad (22)$$

Where input link l_2 is changed to virtual link l_d from the Eq. (5), then a new equation is given as

$$\psi(\varphi, \theta) = \arccos \left(\frac{l_d \cos(\varphi - \theta) - l_1}{\sqrt{l_1^2 + l_d^2 - 2l_1 l_d \cos(\varphi - \theta)}} \right) + \arccos \left(\frac{l_d^2 - l_3^2 + l_1^2 + l_d^2 - 2l_1 l_d \cos(\varphi - \theta)}{2l_1 \sqrt{l_1^2 + l_d^2 - 2l_1 l_d \cos(\varphi - \theta)}} \right) - 180^\circ + \varphi \quad (23)$$

From these equation the workspace can be gotten and compare with zero based-link configuration finger. The singularity positions of the variant mechanism are similar with a parallel mechanism that has been discussed widely.

For the analysis of the workspace, Singularities occur when the end effect point p reaches its limit or at the boundary of the workspace. For the example when the condition is $l_1 + l_4 > l_2 + l_3$, with parameters $l_1 = 400\text{mm}$, $l_2 = 300\text{mm}$, $l_3 = 270\text{mm}$, $l_4 = 350\text{mm}$, $l_p = 800\text{mm}$ and $l_v = 250\text{mm}$, the end-effector curves can be drawn in these singularity configurations.

Fig. 12 Fingertip curves on Singularities of the 5R variant linkages

The coupler curve is shown in Fig. 12(a), denoted as C_a . The points E and G are turning points on the singularity of this new four-bar mechanism when the coupler link O_2C and the output link CB are completely expended to one line. In this case the mechanism is locked and the coupler curve changes its direction. The out edge on the curve C_a is the left part outside boundary of the workspace.

Figure 12(b) is the coupler curve when the coupler link O_2C and the output link CB

are completely expended to one line. The curve C_b is an arc between the points E and G on the circle, centered at the origin with radius $r=l_1+l_p$. This curve consists of the right outermost edge of the workspace.

The curve C_c is a circle, centered at the origin, with radius $r=l_1-l_p$. The inside edge boundary of the workspace consists of the arc between points H and I on the circle where points H and I are the cross points between curve C_c and curve C_e . The workspace of a variant mechanism can thus be given as follows:

Fig. 13 workspace of a variant mechanism when $l_1 + l_4 > l_2 + l_3$

The other case occurs when $l_1+l_4=l_2+l_3$ and $l_1=l_3$, $l_2=l_4$. Such a mechanism is a symmetric structure with the parameter $l_1=400\text{mm}$, $l_2=400\text{mm}$, $l_3=350\text{mm}$, $l_4=350\text{mm}$, $l_p=800\text{mm}$ and $l_v=250\text{mm}$.

Fig. 14 workspace of a variant mechanism when $l_1 + l_4 = l_2 + l_3$ and $l_1 = l_3$, $l_2 = l_4$

From comparing Fig. 13 and Fig. 14, the workspaces are both within the same ring, that is between two circles centered at O_1 , the outer one with radius 1200mm, the inner with radius 400mm. In Fig. 14 the curve C_b is longer than in Fig. 13, so the points E and G have moved left along the circle. The point F has also moved left, hence C_a has expanded. This is a description of how the workspace of case 2 is larger than for case 1.

The third case occurs when the condition is $l_1+l_4<l_2+l_3$ with the parameter $l_1=400\text{mm}$, $l_2=400\text{mm}$, $l_3=4500\text{mm}$, $l_4=350\text{mm}$, $l_p=800\text{mm}$ and $l_v=250\text{mm}$.

Fig. 15 Workspace of an variant mechanism when $l_1 + l_4 < l_2 + l_3$

Comparing this case with other two cases, the workspace is within the same ring but the curves C_a and C_b have moved left further so its workspace is larger than those of the other two cases.

Compared with the workspace of the general linkage, without considering the physical limitations of the fingertip - whose orientation angles can not be negative - the workspace can be generated as being between two rings which are limited by singularity positions as demonstrated in Fig. 16 (a). The workspace of this linkage with the parameter $l_1=400\text{mm}$, $l_2=350\text{mm}$, $l_3=400\text{mm}$, $l_4=350\text{mm}$, $l_p=800\text{mm}$ is a ring between two circles centered at the original point with the radius $r=l_1+l_p$, and radius $r=l_p-l_1$.

(a) General linkage workspace

(b) Variant linkage workspace

Fig. 16 Models on different configurations

For the variant linkage, the workspace is perturbed by the parameter l_d the workspace can be generated by two families of contour lines. They can be obtained by the lay areas of the plotting rings which located in different centers.

For example, the workspace of a linkage with $l_1=l_2=400\text{mm}$, $l_3=l_4=350\text{mm}$, $l_v=250\text{mm}$ and $l_p=800\text{mm}$ that is showed as the hatched region on Fig. 17 is a linkage identified as a symmetric structure.

As the virtual link l_d is keeping variable the workspace is showed different characteristics with the general model. The parameter l_d can be called the perturbation factor that affects the workspace and the orientation capability for these linkages.

5. Orientation Capability of the Base-Link Variance

With the same principle as the general mechanism, the workspace arc of the linkage with fixed orientation angles can be described by the absolute length of the fingertip limit

along the horizontal line l and the distance of the fingertip moving along vertical within the workspace boundary. The example in Fig. 18 shows when the fingertip orientation angle is kept at 90° . The arc is the right half of a circle with radius $r_v = (l_p - l_1)/2$ and center $(0, l_4 + r_v)$. This arc has two cross points with dexterous boundary in Q and R. So the curve between Q and R, denoted as C_r , is the orientation mapping of a fixed orientation angle 90° .

Fig. 17 Orientation mapping for a variant mechanism when $l_1 + l_4 > l_2 + l_3$

Similarly, the curve between S and T, denoted as C_i , is the orientation mapping with fixed orientation angle -90° . By expanding the fixed orientation angle to all angles between 90° and -90° , the orientation mapping can be described as the area within several curves, as shown above in Fig. 17. In this figure curves C_r and C_i are the up and down edges of the mapping. The inside edge is the curve between R and S, on the curve C_c . At same time the curves between points Q and E and curve between points G and T are on the curve C_r . Similarly the outside edge is the curve C_b .

Case 2 occurs when the configuration is $l_1 + l_4 = l_2 + l_3$ and $l_1 = l_3$, $l_2 = l_4$. Such a mechanism is a symmetric structure with the parameters $l_1 = 400\text{mm}$, $l_2 = 400\text{mm}$, $l_3 = 350\text{mm}$, $l_4 = 350\text{mm}$, $l_p = 800\text{mm}$ and $l_v = 250\text{mm}$. The orientation mapping for this case is below,

Fig. 18 Orientation mapping for a variant mechanism with a symmetric structure

As the same length of the input link l_1 and end-effector link length l_p , the orientation mapping is within the same circle with radius 1200mm and its center is at O_1 . In this case the workspace of the mechanism is larger than the first case. Hence the area of the orientation mapping is larger than the first case when the orientation angle is keeping change within between 90° and -90° . This mechanism has more orientation capability than the first case.

Case 3 occurs when the configuration is $l_1 + l_4 < l_2 + l_3$, for example, with the parameters parameter $l_1 = 400\text{mm}$, $l_2 = 400\text{mm}$, $l_3 = 4500\text{mm}$, $l_4 = 350\text{mm}$, $l_p = 800\text{mm}$ and $l_v = 250\text{mm}$. The orientation mapping is shown below in Fig. 19,

Fig. 19 Orientation mapping for a variant mechanism when $l_1 + l_4 < l_2 + l_3$

From Fig. 17 the orientation mapping curves are within the workspace when fixed orientation angles are between 90° and the -90° . The mechanism can reach any point with full orientation capability within these angles. From this analysis, the longer the length $l_2 + l_3$, the more orientation capability the mechanism has.

6. Industrial application

A five-bar mechanism robotic finger is designed as a module for a reconfigurable robotic system for carton packaging of confectionery industry. The 5-bar finger is a closed-loop mechanism as described in Fig. 20 and its structure is same with Fig. 1(a). Only four links are apparent and the fifth link is typically grounded and has zero length. The linkage is driven by two motors which are fixed on the ground arranged with both sides of the finger on two inputs to control the end point of the finger moving freely.

Fig.20 A robotic finger for a reconfigurable packaging system

The reconfigurable robotic system⁽³³⁾⁻⁽³⁵⁾ is required to achieve its modular construction for diverted carton-folding function including poking, tucking and squeezing operations using robotic fingers. The module, a five-bar linkage robotic finger, has been designed through investigate in common motion and manipulation of carton-folding. The position and orientation of robotic fingers vary with shapes of cartons and their folding sequences. The following figure shows the folding of two different type cartons by the robotic fingers.

(a) Folding a skillet-carton

(b) folding a gusset-corner-tray

Fig.21 Folding divert cartons using robotic fingers

The orientation map is produced by a representation of the position and orientation in term of the fingertip and the orientation angle. For example in the first case of folding two size-panels of a skillet carton in Fig. 21(a), two robotic fingers stand at both sides of the carton to squeeze the panels. The finger tips are required to be keeping horizontal when they move forward. The orientation map when the orientation angle is zero had been discussed in Section 2 and workspace arcs had been given for divert linkage configurations. Expanding the orientation map to every orientation angle between -90° to 90° a reachable workspace of an end-effector based on the extreme and singularity positions of the robotic finger can be given.

This analysis on the workspace and orientation capability is useful for the optimum design of a five-bar based robotic finger through comparing its orientation maps. Comparing the general linkage with a zero virtual base link and its variance, the general linkage has a bigger workspace than its variance. In the general linkage when link parameters have the following relation $l_1+l_4=l_2+l_3$ and $l_1=l_3$, $l_2=l_4$ the workspace and the orientation capability reaches their maximum. When the link length l_2+l_3 is longer, the variance has a larger workspace and orientation capability. It shows that the current design of the robotic finger is able to maximally meet the industrial requirements.

7. Conclusions

This paper investigated structures of robotic fingers, particularly five-bar mechanism structures with variance of a base link. The parameters and their effects on the workspace and the orientation capability have been investigated.

The kinematics, the workspace and the orientation mapping of the linkages have been analyzed. The singularity of the linkage and its variance with different base links were analyzed and properties of the linkage with different parameters of links have been discussed. The analysis provided conclusions that singularity is the key factor to affect the workspace and the orientation capability of the mechanism. An industrial robotic finger had been designed for a reconfigurable packaging system for confectionery industry. It showed that the analysis on the workspace and orientation capability is very useful for the optimum design of a five-bar based robotic finger.

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