Investigations on the Formability of Incremental Punching Process*

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Abstract

It was reported that the incremental sheet metal forming (ISMF) produces larger deformations as compared to the conventional forming processes. We developed a new ISMF method based on incremental punching. In this study, a theoretical method based on M-K analysis is presented to determine the forming limit curve (FLC) in this forming process and it has been integrated to our in-house FEM program. Also, a ductile fracture model is used for comparing of the forming limit of the process. A numerical example is then given and verified by experiments. The results indicate that the proposed method provides an available approach to determine the FLC of incremental punching process.

Key words: Forming Limit Curve (FLC), Incremental Punching, M-K Analysis, FEM

1. Introduction

Incremental Sheet Metal Forming (ISMF) is a dieless sheet forming process that has emerged over the last decade as an economic alternative for rapid manufacturing and small batch production. It has become one of the leading R&D topics in the industry. In ISMF process, the final component shape is determined by the relative movement of the punch with respect to the blank. This process is usually carried out on CNC machines and robot arms⁽¹⁾⁽²⁾⁽³⁾. The process has two variants - (1) with support, and (2) without support. In the former, the blank is supported with a die that increases the local deformations and probability to produce parts with complex shape in better accuracy⁽⁴⁾.

Luo *et al.*⁽⁵⁾ develop a new ISMF system that is incremental punching machine. Its mainly components include – slider, workpiece, blank holder, high frequency punch and forming tool, and X-Y table, which are shown in Fig. 1. In incremental punching process, a simple tool punches on the sheet metal that produces local plastic deformations which is accumulated to a large deformation. The tool path is similar to single point incremental forming (SPIF), but the forming tool is rather different that using a punching tool instead of a rotating tool.

Formability is one of mainly concerned issues to investigate the capability of the proposed forming system. In ISMF, there are two variants to define the formability - (1) maximum forming angle, and (2) forming limit curves. These researches are introduced below. The former method is largely depended on experiments and the maximum forming

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angle is less than 75° . Gao *et al.* ⁽⁶⁾ conducted experiments to evaluate the forming angle of aluminum alloy. The maximum forming angle is 66° . It was also reported by Takano *et al.* ⁽⁷⁾ that the forming angle of aluminum is $65^\circ \sim 70^\circ$ in CNC ISMF process. Cerro *et al.* ⁽⁸⁾ conducted experiment and a 75° wall angle square-based pyramid was formed from aluminum sheet for the study of different forming strategies. This method is difficult to be used in theoretical analysis and numerical simulation. Hence, some researcher studied the later method. The formability in incremental forming of sheet metal is studied by Kim *et al.* by using commercial FEM and experiments ⁽⁹⁾. It was pointed out that the forming limit curve is quite different from that in conventional forming. Silva *et al.* ⁽¹⁰⁾ formed analytical model into a theoretical framework to investigate the formability limits of SPIF in terms of ductile damage mechanics. The fracture forming limit in SPIF can be approximately expressible in the form $\varepsilon_1 + \varepsilon_2 = q$, where $\varepsilon_t = q$ is the thickness strain at the onset of fracture in plane. This result is in close agreement with the typical loci of failure strains in conventional sheet-forming processes, where the slope of the fracture forming line is often about -1 ⁽⁹⁾⁽¹⁰⁾. Eyckens *et al.* ⁽¹¹⁾ presented the formability predictions using an M-K type of forming limit model. The method is well predicted the fracture, however, it should account for anisotropic yielding and anisotropic hardening at strain path changes, as well as an accurate description of the complex deformation path in ISMF.

Fig. 1 The incremental punching machine

An example of a strain path of the sheet metal formed by ISMF process is shown in Fig. 2. It represents the evolution of the major principal strain and minor principal strain of an element located in the region of the wall of a cone on the top layer of brick elements. This strain path evolution is due to the deformed region in each step is small, progressively increasing the level of plastic deformation.

Fig. 2 Example of cyclic strain history ⁽¹¹⁾

It's well known that FLC is determined by the necking theory that should satisfy the following assumptions: (1) the strain path is proportional, (2) the deformation is dominantly caused by membrane forces (plane stress state). However, the ISMF is rather complicated that the local material will experience deformations and the localized deformations is violated the above mentioned assumptions. This is quite different from the conventional stamping. Based on the numerical simulation program of incremental punching in our previous research ⁽⁵⁾, the forming limit diagram is therefore needed to predict the potential fracture of workpiece during the forming process. The objective of this paper is to study the formability of the forming process and integrate it into the proposed numerical simulation program. The rest of the paper is organized as follows. Section 2 presents methodology based on M-K analysis to determine FLC. Section 3 describes the implementation procedure. Section 4 introduces the ductile fracture model. Section 5 gives one numerical example as well as the experiment result. Finally, Section 6 contains conclusions.

2. Methodology

2.1 Yield Criterion

The yield criterion is usually used in the FEM simulation of sheet metal forming processes. It describes the relationship between the stress components in the region where the sheet metal is undergoing plastic yielding. If the local deformation is pure tension, the yield point is easily expressed by using the tensile stress-strain curve of material. In most of cases, the sheet metal experiences multi-axial stress state. It cannot be easily modeled by the tensile stress-strain curve. The maximum shear stress criterion (Tresca) and strain energy criterion (von-Mises) are usually used as the yield criteria. However, the properties of sheet metal are anisotropic due to the complex production process that is from crystallization to the rolling process. Hill ⁽¹²⁾ proposed the relation for the plastic yielding criterion for the deformation of planar isotropy sheet metals.

$$f = \sigma_x^2 + \sigma_y^2 + R(\sigma_x + \sigma_y)^2 - (1 + R)X^2 = 0 \quad (1)$$

where R is the Lankford factor, X is the equivalent yield stress.

Hill's ⁽¹³⁾ non-quadratic normal anisotropic plastic potential is a generalization of the normal anisotropic potential given in Eq. (1). In this case, the yield function is with normal anisotropic coefficient, R , and additional material constant a , which is usually larger than 2.

$$f = |\sigma_x + \sigma_y|^a + (1 + R)|\sigma_x - \sigma_y|^a - 2(1 + R)X^a = 0 \quad (2)$$

2.2 Modified M-K Analysis

The localized necking and diffuse necking are common failure methods in sheet metal forming. Maricniak and Kuczynski ⁽¹⁴⁾ proposed an analysis method (M-K analysis) to calculate the right side of FLD by assuming that there is a preexisting defect that lies perpendicular to the major stress axis. For calculation purpose, this defect can be approximated as a thinner (region B) than the rest of the sheet (region A), as shown in Fig. 3. The calculations are based on imposing strain increments $\Delta\epsilon_{B11}$ on the material inside of the defect and finding the corresponding value of $\Delta\epsilon_{B11}$. This involves an iterative procedure until the strain in region B is 10 times of region A. It can be determined that this

point of loading will result the fracture happens on the weak region. Hutchinson and Neale⁽¹⁵⁾ extended M-K theory using a J_2 deformation theory. Therefore, the left and right hand sides of the forming limit diagram can be calculated by M-K analysis. However, the traditional M-K method is used for calculate the conventional stamping, in which the strain ratio is a constant. In incremental forming process, the sheet metal will experience many times of loading and unloading in forming a typical part. Hence, the strain ratio will not be a constant during the whole process. This has already been shown in Fig. 2.

Fig. 3 Schematic representation of M- K analysis

During the entire process, the groove force balance must be respected. Therefore, the force balance condition requires:

$$F_{1A} = F_{1B} \quad (3)$$

where, F_{1A} and F_{1B} stand for the force on one direction inside and outside the groove region respectively. Therefore,

$$\sigma_{11A} t_A = \sigma_{11B} t_B \quad (4)$$

Assumed the initial thickness of Region A and Region B are t_{A0} and t_{B0} respectively, we can characterize the initial imperfection by $f = k_B t_{B0} / k_A t_{A0}$, then $t = t_0 \exp \varepsilon_{33}$, and substitute this value into the Eq. (4). We can obtain

$$\sigma_{11A} = f \sigma_{11B} \quad (5)$$

Defining the slope of principle stress $\xi = \frac{\bar{\sigma}}{\sigma_{11}}$, Eq. (5) can be rewritten as

$$\frac{\bar{\sigma}_A}{\xi_A} = \frac{f \bar{\sigma}_B}{\xi_B} \quad (6)$$

The ratio of the principal stress and strain are defined as

$$\alpha = \frac{\sigma_{22}}{\sigma_{11}} \quad (7)$$

$$\rho = \frac{d\varepsilon_{22}}{d\varepsilon_{11}} \quad (8)$$

According to the yield function in Eq. (2), the slope of the principle stress is

$$\xi = \frac{\bar{\sigma}}{\sigma_{11}} = \left[\frac{(\alpha + 1)^a + (1 + R)(1 - \alpha)^a}{2(R + 1)} \right]^{1/a} \quad (9)$$

Based on the yield criteria in Eq. (2), the effective stress and plastic work rate yield functions are

$$\bar{\sigma} = \left[\frac{|\sigma_{11} + \sigma_{22}|^a + (1 + R)|\sigma_{11} - \sigma_{22}|^a}{2(1 + R)} \right]^{1/a} \quad (10)$$

and

$$\dot{\bar{\epsilon}} = \frac{[2(1 + R)]^{1/a}}{2} \left(|\dot{\epsilon}_{11} + \dot{\epsilon}_{22}|^{a/(a-1)} + \left(\frac{|\dot{\epsilon}_{11} - \dot{\epsilon}_{22}|^a}{(1 + 2R)} \right)^{1/(a-1)} \right)^{(a-1)/a} \quad (11)$$

Assume the deformation is proportional during the deformation step, and then the effective strain is

$$\bar{\epsilon} = \int \dot{\bar{\epsilon}} dt \quad (12)$$

The relationship between the strain ratio and stress ratio are

$$\rho = \frac{(1 + \alpha)^{a-1} - (1 + 2R)(1 - \alpha)^{a-1}}{(1 + \alpha)^{a-1} + (1 + 2R)(1 - \alpha)^{a-1}} \quad (13)$$

and its inverse

$$\alpha = \frac{[(1 + 2R)(1 + \rho)]^{1/(a-1)} - (1 - \rho)^{1/(a-1)}}{[(1 + 2R)(1 + \rho)]^{1/(a-1)} + (1 - \rho)^{1/(a-1)}} \quad (14)$$

The slope of ratio of strain rate and major strain is

$$\lambda = \frac{[2(1 + R)]^{1/a}}{2} \left((1 + \rho)^{a/(a-1)} + \left(\frac{(1 - \rho)^a}{(1 + 2R)} \right)^{1/(a-1)} \right)^{(a-1)/a} \quad (15)$$

Assuming the hardening effects of the material are represented as

$$\bar{\sigma} = K \bar{\epsilon}^n \dot{\bar{\epsilon}}^m \quad (16)$$

where, K is a material constant $\bar{\sigma}$ and $\bar{\epsilon}$ are the effective stress and strain. The strain rate sensibility parameter is given by m , while n is the strain coefficient.

Since the cross section areas of region A is normal while it thinner in region B, $\epsilon_1^A = \epsilon_1$ and $\epsilon_1^B > \epsilon_1$. And the elastic strain will recovered in both of A and B when the punch released from the sheet metal.

$$\epsilon_{ij}^{e(t)} = \frac{1}{E} (1 + \nu) \sigma_{ij}^{(t)} - \nu \delta_{ij} \sigma \quad (17)$$

where E is the Young's modulus, ν is the Poisson's ratio, ' e ' and ' (t) ' denotes the elastic recovered after the t times punch released. Therefore, the plastic deformation reached by both the nominal and weak areas can be calculated as:

$$\bar{\epsilon}^{p(t)} = \bar{\epsilon}^{(t)} - \bar{\epsilon}^{e(t)} \quad (18)$$

where superscript ' p ' denotes the plastic deformation during ' t ' stage. The strain hardening power law used for next stage is modified as:

$$\bar{\sigma}^{(t+1)} = K(\bar{\varepsilon} + \bar{\varepsilon}^{p(t)})^n \dot{\bar{\varepsilon}}^m \quad (19)$$

where $\dot{\bar{\varepsilon}}$ is defined as Eq. (11), $\bar{\varepsilon}^{p(t)}$ and $\bar{\varepsilon}$ are the effective plastic strains defined in Eq. (12) for the prestraining stage and currents stage respectively.

If the prestrain results in a strain state $(\varepsilon_{11}^{p(t)}, \varepsilon_{22}^{p(t)})$, where the index t denotes t time strain, and the current stage results in final strain state $(\varepsilon_{11}^{p(t+1)}, \varepsilon_{22}^{p(t+1)})$, then the principal stresses at the end of current stage are given by⁽¹⁶⁾:

$$\sigma_{11} = \frac{\bar{\sigma}(\varepsilon_{11}^{p(t)}, \varepsilon_{22}^{p(t)}) + \bar{\varepsilon}(\varepsilon_{11}^{p(t+1)} - \varepsilon_{11}^{p(t)}, \varepsilon_{22}^{p(t+1)} - \varepsilon_{22}^{p(t)})}{\xi(\alpha(\varepsilon_{22}^{p(t+1)} - \varepsilon_{22}^{p(t)}) / (\varepsilon_{11}^{p(t+1)} - \varepsilon_{11}^{p(t)}))} \quad (20)$$

and

$$\sigma_{22} = \alpha\left(\frac{\varepsilon_{22}^{p(t+1)} - \varepsilon_{22}^{p(t)}}{\varepsilon_{11}^{p(t+1)} - \varepsilon_{11}^{p(t)}}\right) \sigma_{11} \quad (21)$$

where, $\bar{\sigma}(\bar{\varepsilon})$, $\bar{\varepsilon}(\varepsilon_{11}, \varepsilon_{22})$ and $\xi(\alpha)$ are defined as Eq. (16), Eq. (12) and Eq. (9) respectively, and $\alpha(\rho)$ is the inverse function of Eq. (13) and Eq. (14).

$$\begin{aligned} & \frac{1}{\lambda_A} K_A t_A \frac{\bar{\sigma}(\varepsilon_{11A}^{p(t)}, \varepsilon_{22A}^{p(t)}) + \bar{\varepsilon}(\varepsilon_{11A}^{p(t+1)} - \varepsilon_{11A}^{p(t)}, \varepsilon_{22A}^{p(t+1)} - \varepsilon_{22A}^{p(t)})}{\xi(\alpha(\varepsilon_{22A}^{p(t+1)} - \varepsilon_{22A}^{p(t)}) / (\varepsilon_{11A}^{p(t+1)} - \varepsilon_{11A}^{p(t)}))} \\ &= \frac{f}{\lambda_B} K_B t_B \frac{\bar{\sigma}(\varepsilon_{11B}^{p(t)}, \varepsilon_{22B}^{p(t)}) + \bar{\varepsilon}(\varepsilon_{11B}^{p(t+1)} - \varepsilon_{11B}^{p(t)}, \varepsilon_{22B}^{p(t+1)} - \varepsilon_{22B}^{p(t)})}{\xi(\alpha(\varepsilon_{22B}^{p(t+1)} - \varepsilon_{22B}^{p(t)}) / (\varepsilon_{11B}^{p(t+1)} - \varepsilon_{11B}^{p(t)}))} \end{aligned} \quad (22)$$

Hence, with a given stain path in region A, we can compute the stresses in strong region A. Also, the stresses $(\sigma_{11A}^{t+1}, \sigma_{11B}^{t+1})$ can be obtained by using the balance equation. Using the compatibility requirement $d\varepsilon_{22A}^{t+1} = d\varepsilon_{22B}^{t+1}$, we can get $\varepsilon_{22B}^{t+1} = \varepsilon_{22A}^{t+1}$. The major strain in region B can be obtained by:

$$\varepsilon_{11B}^{t+1} = \varepsilon_{11B}^t + \frac{\bar{\varepsilon}(\bar{\sigma}(\sigma_{11B}^{t+1}, \sigma_{22B}^{t+1})) - \bar{\varepsilon}(\varepsilon_{11B}^t, \varepsilon_{22B}^t)}{\lambda(\rho(\sigma_{11B}^{t+1} / \sigma_{22B}^{t+1}))} \quad (23)$$

Though, it's too complicated to compute the FLC using analytical method. It can be solved in numerical method by using Eq. (22) and Eq. (23) which will be presented in next section.

3. Implementation Procedure

In order to demonstrate the application of the proposed model, an in-house program is developed to compute the FLC by this model. The computing procedure can be summarized in Fig. 4. First, we can define strains in the first step is (0, 0). Next step is to estimate the final status using the computing procedure of fracture. If it is success to finish the strain path, then it will go to the FLC procedure with the stresses and strains in final step. It should increase the value of the minor strain from -0.3 to 0.3. Also, in each loop, the incremented strain ($\Delta\varepsilon_{11a}$) outside of the groove is 0.001 in each step. The presented algorithm was implemented in Matlab[®].

Fig. 4 The computing procedure of FLC

Fig. 5 show a given strain path (solid line), and with the assumption of $f = 0.999$, the strain in region B can be obtained (dot line). If the strain path only goes to the point (0.4,-0.02), the FLC can be obtained as shown in Fig. 6.

Fig. 5 - A typical strain path in incremental punching and its fracture

Assume that the material experienced 4 steps deformations and with a higher pre-strain, the pre-strain can be obtained by using Eq. 22 and Eq. 23. The FLC of multi-step forming is achieved as shown in Fig. 6.

Fig. 6 - FLC with complex pre-strain path

From the figure, it can be seen that the forming limit of a typical point on the sheet metal which is undergoing multi-step of pre-strain can be achieved by the proposed method. The equivalent strain with pre-strain can be higher before fracture happens, that means higher forming limit curve than conventional stamping will be achieved. This method can be integrated into our in-house program ⁽⁵⁾ for predicting the risk of fracture incremental punching process.

4. Ductile Fracture Model

According to previous research ⁽⁹⁾⁽¹⁰⁾⁽¹⁵⁾, the forming limits of ISMF is possible to conclude that plastic deformation takes place by uniform thinning (without necking) until fracture. The fracture forming limit in ISMF causes by ductile damage mechanics based on void growth models. The fracture forming limit can be approximately expressible in the form $\varepsilon_1 + \varepsilon_2 = -q$, where $\varepsilon_t = -q$ is the thickness strain at the onset of fracture in plane. As q is a constant, the risk of fracture is proportional thickness reduction ratio. Silva *et al.* ⁽¹⁷⁾ deduced the constant by using Tresca yield criterion and ductile fracture mechanics (please refer to Ref. (17) for the details of the proof). It was found that the slope of ductile forming limit is high depending on diameter of the forming tool and the thickness of the sheet metal, that is:

$$q = -\frac{5(r_{tool}/t) - 2}{3(r_{tool}/t) + 6} \quad (24)$$

where r_{tool} is the diameter of the forming tool, t is the thickness of the sheet metal.

A special point $(\varepsilon_{11}, \varepsilon_{22})$ on the fracture limit can be obtained by using biaxial stretching and the incompressibility condition. The model can be used to compare with the modified M-K method.

5. An Example

SPCC steel sheet was used in this work. Its mechanical properties are summarized in Table 1. The power-law material model, as shown in Eq. (16), is introduced to describe the stress-strain relationship.

Table 1 Mechanical properties of SPCC

σ_y	K	n	m	Poisson ratio ν	Lankford value R
206 MPa	576 MPa	0.23	0.008	0.3	1.87

The thickness of initial sheet metal is 1mm and its size is 310mm×310 mm. The forming tool is with the hemispherical end of 15mm diameter punched on the sheet metal at a speed of 300rpm. In this experiment, a facial mask is designed. The tool path is generated based on the design of part in MasterCAM® using contour method. The feed step changes from 1mm to 3 mm based on the curvature of current contour. However, the layer thickness is uniform with 1 mm. The punch locations map is shown in Fig. 7 and the blank holder is with the inner size of 220×220 mm.

Fig. 7 Punch locations map

5.1 Simulation results

We developed a mechanics model that combined the minimum energy principle and inverse FEM to predict the final shape of the part, as well as the strain and stress distributions of the part⁽⁵⁾. Note that the springback has not been considered in this model. The predictions of the final shape are effectively and the error is with in 2mm. This is probably because of springback of the sheet metal after the punching released. In this experiment, we adopt the mechanics model and its program to simulate the forming process. The thickness strain distribution of the experiment is predicted as shown in Fig. 8. Its maximum thickness strain is 0.82. It's conventional to compute the risk of fracture by using the proposed model. However, it takes more than 30 minutes to obtain the risk of crack which is shown in Fig. 9.

Fig. 8 Predicted thickness distribution

Fig. 9 Predicted risk of forming part

5.2 Experiment Result

In order to evaluate the formability of the SPCC sheet in incremental punching, two tests were carried to investigate the strains on the sheet metal. The former test is with a small blank holder that cracks will probably happen. For measurement of the strain distributions on the deformed sheet metal, circular grids are marked on sheet before being clamped on the blank holder. To effectively measure the local deformations, the diameter of the circle is 3 mm. The circular grids were deformed as ellipsoids in the region where the sheet metal deformed in the forming process. The strains are measured from the formed part to evaluate the forming limit diagram (FLD). In sheet metal forming, the sheet can be described as shell element, and logarithmic strains are used. Hence, the major strains ε_1 and minor strains ε_2 are:

$$\varepsilon_1 = \ln \frac{l_1}{l_0} \quad \text{and} \quad \varepsilon_2 = \ln \frac{l_2}{l_0} \quad (25)$$

where, l_1 is the major axis of the ellipse, l_2 is the minor axis of the ellipse, and l_0 is the diameter of the initial circle.

Table 2 Measured strains from the experimental part

#	ε_1	ε_2	Thickness	#	ε_1	ε_2	Thickness
A 1	0.73	0.087	0.44	C9	0.068	0.02	0.92
A 2	0.78	0.003	0.46	C10	0.05	0.006	0.93
A 3	0.81	0.017	0.44	C11	0.25	0.049	0.74
A 4	0.63	-0.05	0.56	C12	0.22	0	0.81
B5	0.46	-0.03	0.65	D13	0.48	0.03	0.60
B6	0.44	-0.05	0.68	D14	0.48	-0.01	0.62
B7	0.46	0.02	0.62	D15	0.39	0.02	0.67
B8	0.46	0.03	0.61	D16	0.36	0	0.70

Fig. 10 shows the experiment part, and four interested regions are selected for

further analysis. In region A, the angle of the wall is 80° , and hence, the sheet metal cracked. No crack happened in others region. The strains distributions in the four interested regions were measured and summarized in Table 2. From the results, we can see that the effective strains are extremely large in region B and they are basically reasonable in the other regions.

Fig. 10 Photo of the experimental part

Comparison of experiment results and two forming limit in the Forming Limit Diagram (FLD) is shown in Fig.11. We can see that the strains of region A exceed the forming limit in both of forming limit. Strains in region D exceed the necking limit but not for the ductile fracture limit. Also, no crack happens in this region in the experiment. Hence, from the results, we can see that the failure in incremental punching is largely depended on the ductile limit. It can be seen that the complex strain path increases the forming limit of the sheet metal, and therefore higher equivalent strain than conventional stamping can be achieved.

As above mentioned, the effect of friction in the incremental punching is negligible in horizontal plane due to the contact happens in a short time. The zigzag crack propagation path in region A is probably caused by the local stretching of the punches.

Fig. 11 Comparison of simulation and experiment results in FLD

6. Conclusions

This paper presents the forming limit with pre-strain by modified M-K method. It can be seen that this method can be used for predicting the formability of the negative incremental punching process. Compare to the forming limit of conventional sheet metal forming process, following conclusions can be drawn:

- 1). The forming limit is higher than conventional FLC with pre-strain that means the forming process will have higher formability than conventional sheet metal forming methods.
- 2). Similar to the CNC ISMF process, the failure of this process is largely depended on the ductile limit. It indicates that the formability of the process will increase for small forming tool and thick blank.
- 3). The formability of incremental punching is also related to strain rate and strain path, and therefore the forming processing parameters, feeding speed, layer thickness, punch velocity e.g., should be related to the failure of the process. The future work will take effort on studying the effects of the process parameters.

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