Web Tension and Velocity Control of Two-Span Roll-to-Roll System for Printed Electronics*

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Abstract
Due to the increasing demand of high accuracy in printed electronics industry at a micrometer-level, it is necessary to have a precise control scheme for web velocity and tension in the presence of disturbances. In this paper, a generalized mathematical model of non-linear control system is proposed and a systematic procedure is presented to design a backstepping controller taking the modified backstepping approach. With application of the proposed theory, a precise control algorithm is developed for a nonlinear two-span roll-to-roll web control system based on the backstepping method. The design parameters are chosen optimally by using the modified genetic algorithm. The reliability of the proposed algorithm is validated through simulations in Matlab/Simulink and real experiments

Key words: Backstepping, Non-Linear Control, Roll-to-Roll Web System, Web Tension Control

1. Introduction
In the last decades, there have been many applications which employed the roll-to-roll (R2R) web technology for mass production such as web printing, papers machine, film processing, and textiles fabrics and so on to make cheaper production in shorter time. Especially, RFID (radio frequency identification) and printed electronics use the principle of R2R manufacturing to create devices at high speed and lower cost, and they have a big impact on the printed electronics and publishing industries. Several developments have pushed the burgeoning printed electronics industry and up to now the R2R system technology is seen as the key to producing flexible electronic components, such as organic thin film transistors and other applications. In order to improve the product quality with low production cost, many control algorithms have been proposed but most of them have been developed for linear systems. With the rapid development of mathematical tools and digital computers, nonlinear control systems are getting more attention. Depending on the mathematical model and experimental researches, some algorithms for a web tension and velocity control for an R2R web system have been proposed. K. H. Shin11 and K. C. Lin8,5 have achieved low cost and high quality through the implementation of observer techniques, the synthesis of an observer-based controller in place of tension transducer, and the estimation of friction, and rotational inertias of the rewinder and the unwinder. An important aspect in R2R web control system design is to fully understand the physical and mathematical models9,13,16,17 and to come up with disturbance rejection algorithms2,6,8. In reality, one usually uses linearized models to design the control systems but due to the linearization, some crucial nonlinearity is ignored and as a result there is a discrepancy

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between the actual system performance and the simulated results. In the recent years, many researchers have taken special attention in designing nonlinear controllers by using backstepping techniques that have the advantages of avoiding the cancellation of benign dynamic nonlinearity, and not forcing the designed system to appear linear. A web tension control algorithm for a nonlinear single-span R2R web control system is proposed. In the present paper, a general mathematical model is developed and applied to design a backstepping controller (BSC) for this model such that a precise control algorithm is created for a non-linear two-span R2R web control system. The design parameters in the BSC are chosen optimally using the modified genetic algorithm (MGA). The reliability of the closed-loop system is demonstrated throughout simulation results in Matlab/Simulink and experimental results implemented on a real two-span R2R web system.

2. Backstepping controller design for a two-span R2R web system

Firstly, some mathematical definitions and theorems are presented. Secondly, a systematic procedure for obtaining the BSC for a non-linear system is proposed. Finally, an application of the BSC design to a two-span R2R web system is presented.

2.1 Mathematical basis

Consider the following affine system

\[ \dot{x} = f(x) + g(x)u \]
\[ y = h(x) \]

where

\[ x \in \mathbb{R}^n, f(x) \text{and } g(x) \text{are } n \text{- dimensional vector-valued functions} \]
\[ u \in \mathbb{R}^m, y \in \mathbb{R}^m, h(x) \text{is } m \text{- dimensional vector-valued function} \]

**Definition 1**

The non-linear system (1) and (2) is called input-output decoupled if, after a possible relabeling of the inputs, the following two properties hold.

- For each \( i, 1 \leq i \leq m \), the output \( y_i \) is invariant under the inputs \( u_j, j \neq i \)
- For each \( i, 1 \leq i \leq m \), the output \( y_i \) is not invariant under the inputs \( u_i \)

**Definition 2 (Lie Derivative)**

Let \( h: \mathbb{R}^n \rightarrow \mathbb{R} \) be a smooth scalar function, and \( f: \mathbb{R}^n \rightarrow \mathbb{R}^n \) a smooth vector field on \( \mathbb{R}^n \). Then the Lie derivative of \( h \) with respect to \( f \) is a scalar function defined by

\[ L_f h = \frac{\partial h}{\partial x} f \]

**Definition 3**

The system (1) and (2) is said to have a vector relative degree \( \{ r_1, \ldots, r_m \} \) at a point \( x_0 \) if

- For each \( i, 1 \leq i \leq m \),

\[ \left( L_{g_1} L_f^{r_i-1} h_1(x), \ldots, L_{g_m} L_f^{r_i-1} h_l(x) \right) = (0, \ldots, 0) \]

For all \( k < r_i - 1 \), and for all \( x \) in a neighborhood of \( x_0 \)

- The mxm matrix

\[ \Lambda(x) = \begin{pmatrix}
L_{g_1} L_f^{r_1-1} h_1(x) & \ldots & L_{g_m} L_f^{r_1-1} h_l(x) \\
L_{g_1} L_f^{r_2-1} h_2(x) & \ldots & L_{g_m} L_f^{r_2-1} h_l(x) \\
\vdots & \ddots & \vdots \\
L_{g_m} L_f^{r_m-1} h_m(x) & \ldots & L_{g_m} L_f^{r_m-1} h_l(x)
\end{pmatrix} \]

is non-singular at \( x = x_0 \)

If the vector relative degree is well defined for the multivariable nonlinear system (1) and (2), then there exists a feedback law which achieves decoupling of the input and output dynamics.
Theorem 1

Consider the multivariable nonlinear system (1) and (2) with m inputs and m outputs. Suppose for each i, \(1 \leq i \leq m\)

\[
(L_{g_1} L_f^k h_i(x), \ldots, L_{g_m} L_f^k h_i(x)) = (0, \ldots, 0)
\]

for all \(k < r_i - 1\), and for all \(x\) in a neighborhood of \(x_0\)

\[
(L_{g_1} L_f^{r_i-1} h_i(x_0), \ldots, L_{g_m} L_f^{r_i-1} h_i(x_0)) \neq (0, \ldots, 0)
\]

Then the decoupling control problem is solvable by the state feedback law if and only if the matrix \(A(x_0)\) is non-singular, i.e. if the system has a vector relative degree \(\{r_1, \ldots, r_m\}\) at \(x_0\).

Theorem 2 (Lyapunov Stability Theorem)

Let \(V : \mathbb{R}^n \to \mathbb{R}\) be a \(C^1\) radially unbounded function and \(x = 0\) be an equilibrium point of \(\dot{x} = f(x)\) such that

- \(V(0) = 0\)
- \(V(x) > 0\) when \(x \neq 0\)
- \(\dot{V}(x) < 0\) when \(x \neq 0\)

then the system is globally asymptotically stable at \(x = 0\).

2.2 Backstepping controller design

The input-output decoupling problem for the non-linear system (1) and (2) can be obtained by a feedback control law with the conditions mentioned in the above theorem. The question arises on how to determine such a control law. In this paper, a systematic method is proposed to transform the non-linear system into the equivalent linear system by a change of state coordinates and the state feedback using the backstepping technique. The backstepping design methodology was originally introduced in adaptive control theory to systematically construct a feedback control law. Various backstepping design techniques have been introduced such as integrator backstepping, backstepping for strict-feedback systems, adaptive backstepping, robust backstepping, lead backstepping, and so on. In this paper, a modified backstepping approach is employed for the non-linear MIMO system in form (1) and (2). The modified backstepping approach for MIMO systems involves recursive processes which break a design problem on the full system down to a sequence of sub-problems on lower order systems. Considering each lower order system and paying attention to the interaction between two subsystems makes it modular and easy to design a BSC. The resultant final BSC for the full system meets the performance specifications and achieves globally asymptotical stability.

Consider an affine system in the following form

\[
\begin{align*}
\dot{x}_1 &= f_1(x_1, x_2, x_3) \\ \dot{x}_2 &= f_2(x_1, x_2, x_3, x_4) + g_1(x_1, x_2, x_3, x_4)u_1 \\ \dot{x}_3 &= f_3(x_1, x_2, x_3, x_4) \\ \dot{x}_4 &= f_4(x_1, x_2, x_3, x_4) + g_2(x_1, x_2, x_3, x_4)u_2 
\end{align*}
\]

with the output equations:

\[
\begin{align*}
y_1 &= h_1(x) \\ y_2 &= h_2(x)
\end{align*}
\]
$x \in \mathbb{R}^4, f(x)$ and $g(x)$ are 4-dimensional vector-valued functions. $u \in \mathbb{R}^2, y \in \mathbb{R}^2, h(x)$ is 2-dimensional vector-valued function.

It is assumed that the conditions in Theorem 1 are satisfied and the $x_2, x_4$ variables in Eqs. (4) and (6) are solvable explicitly in terms of the other variables. Thus, input-output decoupling problem is solvable by a state feedback law and the state feedback law is designed using the backstepping approach.

The objective is to design a control law for the non-linear control system (4)-(9) such that $x_1 \to x_{1\text{ref}}$ and $x_3 \to x_{3\text{ref}}$ asymptotically where $x_{1\text{ref}}$ and $x_{3\text{ref}}$ are constant and globally asymptotical stability is achieved with no overshoots in the system. The following are the steps for formulating the BSC for the system (4)-(9).

**Step 1:** $x_2$ is regarded as a control input in Eq. (4) that is considered as the first subsystem. Thus, $x_2$ is chosen to make the first subsystem globally asymptotically stable. The choice is called a virtual control law, i.e.

By putting

$$\zeta_1 = x_1 - x_{1\text{ref}}$$

By differentiating both sides in time and combining with Eq. (4), we have:

$$\dot{\zeta}_1 = x_1' = f_1(\zeta_1 + x_{1\text{ref}}, x_2, x_3)$$

For the Eq. (11), a CLF $V(\zeta_1)$ can be chosen such that when the virtual control law is applied, its time derivative becomes negative definite, i.e.

$$V_1 = \frac{1}{2}\zeta_1^2$$

By taking derivative in time of Eq. (12) and combining with Eq. (11) results in;

$$V_1 = \zeta_1\dot{\zeta}_1 = \zeta_1 f_1(\zeta_1 + x_{1\text{ref}}, x_2, x_3)$$

By satisfying the asymptotically stable condition in a sense of Lyapunov in Theorem 2 for Eq. (13), a virtual control law $\alpha_1$ can be chosen as follows;

$$-c_1\zeta_1 = f_1(\zeta_1 + x_{1\text{ref}}, x_2, x_3)$$

$$\Rightarrow \alpha_1 = \lambda_1(c_1, \zeta_1, x_3) \equiv x_2$$

where $c_1$ is the positive gain.

By doing so, we have:

$$\dot{V}_1 = \zeta_1\dot{\zeta}_1 = -c_1\zeta_1^2 < 0 \forall \zeta_1 \neq 0$$

**Step 2:** By choosing the state feedback (14) and a change of coordinate (15)

$$\zeta_2 = x_2 - \alpha_1$$

The second subsystem can be rewritten as follows:

$$\dot{\zeta}_1 = -c_1\zeta_1$$

$$\dot{\zeta}_2 = f_2(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, x_3) + g_1(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, x_3)u_1 - \alpha_1$$

A CLF $V_2(\zeta_1, \zeta_2)$ can be chosen such that it makes the subsystem (16) asymptotically stable with the virtual control law, i.e.

$$V_2 = V_1 + \frac{1}{2}\zeta_2^2$$

Taking the derivative of the Eq. 17 in time and combining with Eqs. 16 result in;

$$\dot{V}_2 = -c_1\zeta_1^2 + \zeta_2(f_2(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, x_3, x_4) + g_1(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, x_3, x_4)u_1 - \alpha_1)$$

To meet the asymptotically stable condition in the sense of Lyapunov in Theorem 2 for Eq. (18), a control law $u_1$ can be chosen such that

$$-c_2\zeta_2 = f_2(\zeta_1, \zeta_2, x_3, x_4) + g_1(\zeta_1, \zeta_2, x_3, x_4)u_1 - \alpha_1$$

$$\Rightarrow u_1 = \bar{\lambda}_2(c_2, \zeta_1, x_1, x_2, x_3, x_4)$$

where $c_2$ is the positive gain.

By doing so, we have:

$$\dot{V}_2 = -c_1\zeta_1^2 - c_2\zeta_2^2 < 0 \forall \zeta_1, \zeta_2 \neq 0$$
Step 3: by choosing the state feedbacks (14) and (19) and a change of state transformations (10) and (15), the third subsystem can be rewritten as follows:

\[
\begin{align*}
\dot{\zeta}_1 &= -c_1\zeta_1 \\
\dot{\zeta}_2 &= -c_2\zeta_2 \\
\dot{x}_3 &= f_3(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, x_3, x_4)
\end{align*}
\]  
(20)

By putting

\[\zeta_3 = x_3 - x_{3\text{ref}}\]  
(21)

The third subsystem can be rewritten as follows:

\[
\begin{align*}
\dot{\zeta}_1 &= -c_1\zeta_1  \\
\dot{\zeta}_2 &= -c_2\zeta_2  \\
\dot{\zeta}_3 &= f_3(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, \zeta_3 + x_{3\text{ref}}, x_4)
\end{align*}
\]  
(22)

Now, \(x_4\) is regarded as a control input in the subsystem (22). So, \(x_4\) can be chosen to make the subsystem (22) globally asymptotically stable. A CLF \(V_3(\zeta_1, \zeta_2, \zeta_3)\) can be chosen such that it makes the subsystem (22) asymptotically stable with the virtual control law, i.e.

\[V_3 = V_2 + \frac{1}{2}\zeta_3^2\]  
(23)

By taking the derivative of the Eq. (23) in time and combining with Eq. (22) result in;

\[\dot{V}_3 = -c_4\zeta_1^2 - c_5\zeta_2^2 - c_3f_3(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, \zeta_3 + x_{3\text{ref}}, x_4)\]  
(24)

To satisfy the asymptotically stable condition in the sense of Lyapunov in Theorem 2 for Eq. (24), a virtual control law \(\alpha_2\) can be chosen such that

\[-c_3\zeta_3 = f_3(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, \zeta_3 + x_{3\text{ref}}, x_4)
\]

\[\alpha_2 = \lambda_3(c_1, c_2, c_3, \zeta_1, \zeta_2, \zeta_3) = x_4\]  
(25)

where \(c_3\) is the positive gain

By doing so, we have:

\[V_3 = -c_4\zeta_1^2 - c_5\zeta_2^2 - c_3\zeta_3^2 < 0 \forall \zeta_1, \zeta_2, \zeta_3 \neq 0\]

Step 4: by choosing the state feedbacks (14) (19) and (25) and a change of state transformations (10) (15) and (21), the complete system can be rewritten as follows:

\[
\begin{align*}
\dot{\zeta}_1 &= -c_1\zeta_1  \\
\dot{\zeta}_2 &= -c_2\zeta_2 \\
\dot{\zeta}_3 &= -c_3\zeta_3 \\
\dot{\zeta}_4 &= f_1(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, \zeta_3 + x_{3\text{ref}}, x_4) \\
&+ g_2(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, \zeta_3 + x_{3\text{ref}}, x_4)u_2
\end{align*}
\]  
(26)

By putting

\[\zeta_4 = x_4 - \alpha_2\]  
(27)

the complete system is rewrite as follows:

\[
\begin{align*}
\dot{\zeta}_1 &= -c_1\zeta_1  \\
\dot{\zeta}_2 &= -c_2\zeta_2 \\
\dot{\zeta}_3 &= -c_3\zeta_3 \\
\dot{\zeta}_4 &= f_4(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, \zeta_3 + x_{3\text{ref}}, \zeta_4 + \alpha_2) \\
&+ g_2(\zeta_1 + x_{1\text{ref}}, \zeta_2 + \alpha_1, \zeta_3 + x_{3\text{ref}}, \zeta_4 + \alpha_2)u_2 - \alpha_2
\end{align*}
\]  
(28)

A CLF \(V_4(\zeta_1, \zeta_2, \zeta_3, \zeta_4)\) can be chosen such that it makes the subsystem (28) asymptotically stable with the real control law, i.e.

\[V_4 = V_3 + \frac{1}{2}\zeta_4^2\]  
(29)

By taking the derivative of Eq. (29) in time and combining with Eq. (28) Result in;

\[\dot{V}_4 = -c_4\zeta_1^2 - c_5\zeta_2^2 - c_3\zeta_3^2 + \zeta_4(f_4(\zeta_1 + \zeta_2, \zeta_3, \zeta_4) + g_2(\zeta_1 + \zeta_2, \zeta_3, \zeta_4)u_2 - \alpha_2)\]  
(30)

To satisfy the asymptotically stable condition in a sense of Lyapunov in Theorem 2 for Eq.
(30), a control law $u_2$ can be chosen such that

$$-c_4 f_4(\zeta_1, \zeta_2, \zeta_3, \zeta_4) + g_2(\zeta_1, \zeta_2, \zeta_3, \zeta_4)u_2 - \ddot{\alpha}_2$$

$$u_2 = \dot{\lambda}_4(c_1, c_2, c_3, c_4, x_1, x_2, x_3, x_4)$$

(31)

where $c_4$ is the positive gain

By doing so, we have:

$$V_4 = -c_1 \dot{\zeta}_1^2 - c_2 \dot{\zeta}_2^2 - c_3 \dot{\zeta}_3^2 - c_4 \dot{\zeta}_4^2 < 0 \forall \zeta_1, \zeta_2, \zeta_3, \zeta_4 \neq 0$$

Thus, by choosing the state feedbacks (14) (19) (25) and (31) and a change of state transformations (10) (15) (21) and (27), the non-linear system (4) – (7) is transformed into the linear system as follows

$$\begin{aligned}
\dot{\zeta}_1 &= -c_1 \zeta_1 \\
\dot{\zeta}_2 &= -c_2 \zeta_2 \\
\dot{\zeta}_3 &= -c_3 \zeta_3 \\
\dot{\zeta}_4 &= -c_4 \zeta_4
\end{aligned}$$

(32)

By examining the system (32), it is clear that the system is stable and converges to zero with positive gains and the response of the system has no overshoots. The desired settling time and rising time of the system are obtained by tuning the gains. Thus, the stability and performance specifications on the system (4)-(7) are achieved with the BSC.

In summary, the BSC for the system (4) – (7) is given as follows

$$u_1 = \dot{\lambda}_3(c_1, c_2, x_1, x_2, x_3, x_4)$$

$$u_2 = \dot{\lambda}_4(c_1, c_2, c_3, c_4, x_1, x_2, x_3, x_4)$$

(33)

where $c_1, c_2, c_3$ and $c_4$ are positive gains

### 2.3 Application of BSC design to a two-span R2R web system

Figure 1 shows a two-span R2R web control system consisting of an unwinder, a rewinder, an infeeder, a dancer system, two load cells, rollers, and a web lateral control system. The idle rollers guide the moving web around the load cell in a fixed angle. In order to control the web tension at span 1 and span 2, motors at the unwinder and the rewinder are used to produce control torques $\tau_u$ and $\tau_r$ respectively to keep web tensions at the desired values and the infeeder is used for web velocity control. The dancer system put on the unwinder side is to take up the slack during the start-up and the shutdown. On the other hand, two load cells are used to feedback the web tension during the operating process and a web guide mechanism is used to control any web lateral error.

![Fig. 1 Two-span R2R web control system](image-url)
It is assumed that no web slippage occurs, the web has no permanent deformation due to applied tension, and the load cell and dancer dynamics is ignored.

By using Newton’s law and the principle of mass conservation with aforementioned assumptions, the non-linear dynamic equations of two-span R2R web control system can be written as follows:

\[
\begin{align*}
\dot{\omega}_1 &= k_7 T_2 + k_8 T_1 + k_9 \omega_1 + k_{10} \tau_1 \\
T_1 &= k_1 \omega_1 T_1 + k_2 \omega_1 + k_3 \omega_u \\
\dot{\omega}_u &= k_4 T_1 + k_5 \omega_u + k_6 \tau_u \\
\dot{T}_2 &= k_{11} \omega_1 T_1 + k_{12} \omega_2 T_2 + k_{13} \omega_r + k_{14} \omega_1 \\
\dot{\omega}_r &= k_{15} T_2 + k_{16} \omega_r + k_{17} \tau_r
\end{align*}
\]

We rewrite equations (34)-(38) in vector form:

\[
\begin{bmatrix}
\dot{\omega}_1 \\
\dot{T}_1 \\
\dot{\omega}_u \\
\dot{T}_2 \\
\dot{\omega}_r
\end{bmatrix}
= \begin{bmatrix}
k_7 T_2 + k_8 T_1 + k_9 \omega_1 \\
k_1 \omega_1 T_1 + k_2 \omega_1 + k_3 \omega_u \\
k_4 T_1 + k_5 \omega_u \\
k_{11} \omega_1 T_1 + k_{12} \omega_2 T_2 + k_{13} \omega_r + k_{14} \omega_1 \\
k_{15} T_2 + k_{16} \omega_r
\end{bmatrix}
+ \begin{bmatrix}
k_{10} \tau_1 \\
0 \\
0 \\
k_{17} \tau_r
\end{bmatrix}
\]

(39)

with output

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix}
= \begin{bmatrix}
\omega_1 \\
T_1 \\
\omega_2
\end{bmatrix}
\]

(40)

where

\[
\begin{align*}
k_1 &= -R_1 / L_1, k_2 &= EAR_1 / L_1, k_3 = -EAR_u / L_1, k_4 = R_u / J_u, k_5 = -B_u / J_u, k_6 = 1 / J_u \\
k_7 &= R_1 / J_1, k_8 = -R_1 / J_1, k_9 = -B_1 / J_1, k_{10} = 1 / J_1, k_{11} = R_1 / L_2, k_{12} = -R_r / L_2 \\
k_{13} &= EAR_r / L_2, k_{14} = -EAR_1 / L_2, k_{15} = -R_r / J_r, k_{16} = -B_r / J_r, k_{17} = 1 / J_r
\end{align*}
\]

\[T_1 = \text{web tensions of span 1}, T_2 = \text{web tensions of span 2}, \omega_u = \text{angular velocity of unwinder}, \omega_2 = \text{angular velocity of infeeder}, \omega_r = \text{angular velocity of rewinder}, J_u = \text{inertia moment of unwinder and motor at unwinder}, J_1 = \text{inertia moment of roller 1 and motor at roller 1}, J_r = \text{inertia moment of winder and motor at winder}, R_u = \text{radius of unwinder}, R_r = \text{radius of infeeder roller 1}, R_1 = \text{radius of rewinder}, L_1 = \text{the length of span 1}, L_2 = \text{the length of span 2}, E = \text{Young’s modulus of web materials}, A = \text{Area of cross-section}, \tau_u = \text{control torque generated by the motor at unwinder}, \tau_r = \text{control torque generated by the motor at rewinder}, \rho = \text{the density of the web}, h = \text{the thickness of web}
\]

The first step of the BSC design is to check if the conditions in Theorem 1 are satisfied, i.e. the control system can be decoupled by the BSC and then a systematic procedure is given for obtaining the BSC for the two-span R2R web control system.
From the Eqs. (39) and (40), it is easy to show that the Eqs. (39) and (40) are in the form (1) and (2) with the following

\[
\begin{align*}
 f_1 &= k_7 T_2 + k_8 T_1 + k_9 \omega_1 \\
 f_2 &= k_1 \omega_1 T_1 + k_2 \omega_1 + k_3 \omega_u \\
 f_3 &= k_4 T_1 + k_5 \omega_u \\
 f_4 &= k_{11} \omega_1 T_1 + k_{12} \omega_1 T_2 + k_{13} \omega_r + k_{14} \omega_1 \\
 f_5 &= k_{15} T_2 + k_{16} \omega_r
\end{align*}
\]  \tag{41}

\[
\begin{align*}
 g_1 &= k_{10} \\
 g_2 &= 0 \\
 g_3 &= k_6 \\
 g_4 &= 0 \\
 g_5 &= k_{17}
\end{align*}
\]  \tag{42}

and

\[
\begin{align*}
 h_1 &= \omega_1 \\
 h_2 &= T_1 \\
 h_3 &= T_2
\end{align*}
\]  \tag{43}

In our application, we have \( k = 0 \), \( m = 3 \) and \( r_1 = r_2 = r_3 = 2 \), thus

Because \( L_{g_1} J_{g_1}^1 h_1(x) = L_{g_2} J_{g_2}^1 h_2(x) = L_{g_3} J_{g_3}^1 h_3(x) = L_{g_4} J_{g_4} h_4(x) = L_{g_5} J_{g_5}^1 h_5 = 0 \) due to \( g_2 = 0 \), \( g_4 = 0 \) we have,

\[
\begin{align*}
 L_{g_1} J_{g_1}^1 h_1(x) &= L_{g_2} J_{g_2}^1 h_2(x) = L_{g_3} J_{g_3}^1 h_3(x) \\
 &= L_{g_5} J_{g_5}^1 h_5 = 0
\end{align*}
\]

At \( x = [\omega_1 \ T_1 \ \omega_u \ T_2 \ \omega_r] = [0 \ 0 \ 0 \ 0 \ 0] \)

Thus, we can conclude that the input-output decoupling problem is solvable by the state feedback law. The next step is to show the detail of BSC design for the two-span R2R web control system.

The given problem is to design the controller that is required to keep web tension and web velocity at prescribed reference values, to satisfy the performance specifications with no overshoot and settling time of 0.2 second and to obtain the high precision and stability. By using the above developed theory, the complete system (34) - (38) is divided into five subsystems. The first one consists of Eq. (34), the second one consists of the (34) and (35), the third one consists of the (34), (35) and (36), the fourth one consists of the (34), (35) (36) and (37) and the last one consists of the whole system. After applying the modified backstepping method with each subsystem as shown above, the resulting controller called the backstepping controller is proven to achieve globally asymptotical stability using a CLF and Theorem 2. The following is the procedure to get the BSC with desired tensions at span 1 and 2 of \( T_{1ref}, T_{2ref} \) respectively and desired angular velocity of \( \omega_1 \).

**Step 1.** By considering the first subsystem and putting:

\[
\zeta_1 = \omega_1 - \omega_{1ref}
\]  \tag{44}

Taking the derivative both sides in time and combining with Eq. (34), we have:

\[
\dot{\zeta}_1 = \dot{\omega}_1 = k_7 T_2 + k_8 T_1 + k_9 (\zeta_1 + \omega_{1ref}) + k_{10} \tau_1
\]  \tag{45}

For Eq. (45), a CLF \( V(\zeta_1) \) can be chosen such that when the control law is applied, its time derivative becomes negative definite or mathematically

\[
V_1 = \frac{1}{2} \dot{\zeta}_1^2
\]  \tag{46}

By taking the derivative in time of Eq. (46) and combining with Eq. (45) results in;

\[
\dot{V}_1 = \zeta_1 \dot{\zeta}_1 = \zeta_1 (k_7 T_2 + k_8 T_1 + k_9 (\omega_1 + \omega_{1ref}) + k_{10} \tau_1)
\]  \tag{47}

To meet the asymptotically stable condition in the sense of Lyapunov in Theorem 2 for Eq. (47), the controller \( \tau_1 \) can be chosen as follows;

\[
-\zeta_1 \dot{\zeta}_1 = k_7 T_2 + k_8 T_1 + k_9 \omega_1 + k_{10} \tau_1
\]

\[
\Rightarrow \tau_1 = \frac{1}{k_{10}} (-c_1 (\omega_1 - \omega_{1ref}) - k_7 T_2 - k_8 T_1 - k_9 \omega_2)
\]  \tag{48}

Where \( c_1 \) is the positive gain.
By doing so, we have:
\[ \dot{\zeta}_1 = \xi_1 \dot{\xi}_1 = -c_1 \xi_1 < 0 \] \forall \zeta_1 \neq 0

**Step 2:** by choosing the state feedbacks (48) and a change of state transformations (44), the second subsystem can be rewritten as follows:
\[
\begin{aligned}
\dot{\xi}_1 &= -c_1 \xi_1 \\
T_1 &= k_1(\xi_1 + \omega_{1\text{ref}})T_1 + k_2(\xi_1 + \omega_{1\text{ref}}) + k_3 \omega_u \\
\end{aligned}
\] (49)

By putting
\[ \zeta_2 = T_1 - T_{1\text{ref}} \] (50)
the second subsystem can be rewritten as follows:
\[
\begin{aligned}
\dot{\xi}_1 &= -c_1 \xi_1 \\
\dot{\xi}_2 &= k_1(\xi_1 + \omega_{1\text{ref}})(\xi_2 + T_{1\text{ref}}) + k_2(\xi_1 + \omega_{1\text{ref}}) + k_3 \omega_u \\
\end{aligned}
\] (51)

Now, \( \omega_u \) is regarded as a control input to the subsystem (51). So, \( \omega_u \) can be chosen to make the subsystem (51) globally asymptotically stable. A CLF \( V_2(\xi_1, \xi_2) \) can be chosen such that it makes the subsystem (51) asymptotically stable with a virtual control law, i.e.
\[ V_2 = V_1 + \frac{1}{2} \xi_2^2 \] (52)

By taking the derivative of Eq. (52) in time and combining with Eq. (51) results in:
\[ \dot{V}_2 = -c_1 \xi_2^2 - \xi_2(k_1(\xi_1 + \omega_{1\text{ref}})(\xi_2 + T_{1\text{ref}}) + k_2(\xi_1 + \omega_{1\text{ref}}) + k_3 \omega_u) \] (53)

By satisfying the asymptotically stable condition in the sense of Lyapunov in Theorem 2 of Eq. (53), a virtual control law \( \alpha_2 \) can be chosen as follows;

By choosing:
\[ -c_2 \xi_2 = k_1(\xi_1 + \omega_{1\text{ref}})(\xi_2 + T_{1\text{ref}}) + k_2(\xi_1 + \omega_{1\text{ref}}) + k_3 \omega_u \]
\[ \Rightarrow \alpha_2 = \frac{1}{k_3} \left(-c_2(T_1 - T_{1\text{ref}}) - k_3 \omega_1 T_1 - k_2 \omega_2 \right) \equiv \omega_u \] (54)

Where \( c_2 \) is the positive gain

By doing so, we have:
\[ \dot{V}_2 = -c_1 \xi_1^2 - c_2 \xi_2^2 < 0 \] \forall \zeta_1, \xi_2 \neq 0

By choosing the state feedbacks (48) and (54) and a change of state transformations (44) and (50), the third subsystem can be rewritten as follows:
\[
\begin{aligned}
\dot{\xi}_1 &= -c_1 \xi_1 \\
\dot{\xi}_2 &= -c_2 \xi_2 \\
\dot{\omega}_u &= k_3(\xi_2 + \omega_{1\text{ref}}) + k_3 \omega_u + k_6 \tau_u \\
\end{aligned}
\] (55)

**Step 3:** By putting
\[ \zeta_3 = \omega_u - \alpha_2 \] (55)

The third subsystem can be rewritten as follows:
\[
\begin{aligned}
\dot{\xi}_1 &= -c_1 \xi_1 \\
\dot{\xi}_2 &= -c_2 \xi_2 \\
\dot{\xi}_3 &= k_4(\xi_2 + T_{1\text{ref}}) + k_5(\xi_3 + \alpha_2) - \dot{\alpha}_2 + k_6 \tau_u \\
\end{aligned}
\] (56)

where \( \dot{\alpha}_2 \) is determined by (34), (48) and (54) or
\[
\dot{\alpha}_2 = \frac{1}{k_3} \left(-(c_2 + k_4 \omega_1)(k_1 \omega_1 T_1 + k_2 \omega_1 + k_3 \omega_u) + (k_1 T_1 + k_2)(k_7 T_2 + k_8 T_3 + k_9) \right)
\]

So, \( \tau_u \) can be chosen to make the subsystem (56) globally asymptotically stable. A CLF \( V_3(\xi_1, \xi_2, \xi_3) \) can be chosen such that it makes the subsystem (56) asymptotically stable with a virtual control law, i.e.
\[ V_3 = V_2 + \frac{1}{2} \xi_2^2 \]  

(57)

Taking the derivative of Eq. (57) in time and combining with Eq. (56) result in;
\[ \dot{V}_3 = -c_1 \xi_1^2 - c_2 \xi_2^2 + c_3 (k_4 \xi_2 + T_{1 \text{ref}}) + k_4 (\xi_3 + \alpha_2) - \alpha_2 + k_6 \tau_u \]  

(58)

To satisfy the asymptotically stable condition in the sense of Lyapunov in Theorem 2 for Eq. (58), a control law \( \tau_u \) can be chosen as follows;
\[ -c_3 \dot{\xi}_3 = k_4 (\xi_2 + T_{1 \text{ref}}) + k_5 (\xi_3 + \alpha_2) - \alpha_2 + k_6 \tau_u \]
\[ \Rightarrow \tau_u = \frac{1}{k_6} (-c_3 (\alpha_u - \alpha_2) - k_4 T_1 - k_5 \alpha_u + \alpha_2) \]  

(59)

where \( c_2 \) is the positive gain

By doing so, we have:
\[ \dot{V}_3 = -c_1 \xi_1^2 - c_2 \xi_2^2 - c_3 \xi_3^2 < 0 \ \forall \xi_1, \xi_2, \xi_3 \neq 0 \]

By choosing the state feedbacks (48), (54) and (59) and a change of state transformations (44), (50) and (55), the fourth subsystem can be rewritten as follows:
\[ \begin{aligned}
\dot{\xi}_1 &= -c_1 \xi_1 \\
\dot{\xi}_2 &= -c_2 \xi_2 \\
\dot{\xi}_3 &= -c_3 \xi_3 \\
\end{aligned} \]

(60)

where \( \dot{\xi}_4 = T_2 - T_{2 \text{ref}} \)

(61)

Now, \( \omega_r \) is regarded as a control input in (62). So, \( \omega_r \) can be chosen to make the subsystem (62) globally asymptotically stable. A CLF \( V_4(\xi_1, \xi_2, \xi_3, \xi_4) \) can be chosen such that it makes the subsystem (62) asymptotically stable with a virtual control law, i.e.
\[ V_4 = V_3 + \frac{1}{2} \xi_4^2 \]  

(63)

Taking the derivative of Eq. (63) in time and combining with Eq. (62) result in;
\[ \dot{V}_4 = -c_4 \xi_4^2 - c_2 \xi_2^2 - c_3 \xi_3^2 + \xi_4 (k_11 (\xi_1 + \omega_{1 \text{ref}}) (\xi_2 + T_{1 \text{ref}}) + k_{12} \omega_r (\xi_4 + T_{2 \text{ref}}) + k_{13} \omega_r + k_{14} (\xi_1 + \omega_{1 \text{ref}}) \]

(64)

To satisfy the asymptotically stable condition in the sense of Lyapunov in Theorem 2 for Eq. (64), a virtual control law \( \alpha_3 \) can be chosen as follows;
\[ -c_4 \xi_4 = k_{11} (\xi_1 + \omega_{1 \text{ref}}) (\xi_2 + T_{1 \text{ref}}) + k_{12} \omega_r (\xi_4 + T_{2 \text{ref}}) + k_{13} \omega_r + k_{14} (\xi_1 + \omega_{1 \text{ref}}) \]
\[ \Rightarrow \alpha_3 = \frac{1}{k_{12} T_2 + k_{13}} (-c_4 (T_2 - T_{2 \text{ref}}) - k_{12} \omega_r T_1 - k_{14} \omega_{1 \text{ref}}) \equiv \omega_r \]  

(65)

where \( c_4 \) is the positive gain

By doing so, we have:
\[ \dot{V}_4 = -c_4 \xi_4^2 - c_2 \xi_2^2 - c_3 \xi_3^2 - c_4 \xi_4^2 < 0 \ \forall \xi_1, \xi_2, \xi_3, \xi_4 \neq 0 \]

By choosing the state feedbacks (48), (54), (59) and (65) and a change of state transformations (44), (50), (55), and (61) the complete system can be rewritten as follows:
\[
\begin{align*}
\dot{\xi}_1 &= -c_1 \xi_1 \\
\dot{\xi}_2 &= -c_2 \xi_2 \\
\dot{\xi}_3 &= -c_3 \xi_3 \\
\dot{\xi}_4 &= -c_4 \xi_4 \\
\dot{\omega}_r &= k_{15}(\xi_4 + T_{2\text{ref}}) + k_{16}\omega_r + k_{17}\tau_r
\end{align*}
\] (66)

**Step 5:** by putting
\[
\xi_5 = \omega_r - \alpha_3
\] (67)
the complete system can be rewritten as follows:
\[
\begin{align*}
\dot{\xi}_1 &= -c_1 \xi_1 \\
\dot{\xi}_2 &= -c_2 \xi_2 \\
\dot{\xi}_3 &= -c_3 \xi_3 \\
\dot{\xi}_4 &= -c_4 \xi_4 \\
\dot{\xi}_5 &= k_{15}(\xi_4 + T_{2\text{ref}}) + k_{16}(\xi_5 + \alpha_3) - \dot{\alpha}_3 + k_{17}\tau_r
\end{align*}
\] (68)
where \( \dot{\alpha}_3 \) is determined by (34), (37), (48) and (65) or
\[
\dot{\alpha}_3 = \frac{1}{(k_{12}T_2 + k_{13})^2} \left\{ \begin{array}{l}
-\left[ -c_4(k_{11}\omega_1T_1 + k_{12}\omega_rT_2 + k_{13}\omega_r + k_{14}\omega_1) - k_{11}\omega_1(k_{12}T_2 + k_{13}\omega_r + k_{14}\omega_1) \\
-\left( -c_4(k_{12}T_2 + T_{2\text{ref}}) - k_{11}\omega_1T_1 - k_{14}\omega_1 \\
(k_{11}\omega_1T_1 + k_{12}\omega_rT_2 + k_{13}\omega_r + k_{14}\omega_1)k_{12}
\end{array} \right]
\right \}
\]
Thus, \( \tau_r \) can be chosen to make the subsystem (68) globally asymptotically stable. A CLF \( V_5(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5) \) can be chosen such that it makes complete system asymptotically stable with the control law;
\[
V_5 = V_4 + \frac{1}{2}c_5^2
\] (69)
Taking the derivate of Eq. (69) in time and combining with Eq. (68) result in;
\[
\dot{V}_5 = -c_1\xi_1^2 - c_2\xi_2^2 - c_3\xi_3^2 - c_4\xi_4^2 + \xi_5(k_{15}(\xi_4 + T_{2\text{ref}}) + k_{16}(\xi_5 + \alpha_3) - \dot{\alpha}_3 + k_{17}\tau_r)
\] (70)
To satisfy the asymptotically stable condition in a sense of Lyapunov in Theorem 2 for Eq. (70), a control law \( \tau_r \) can be chosen as follows;
\[
-\dot{c}_5\xi_5 = k_{15}(\xi_4 + T_{2\text{ref}}) + k_{16}(\xi_5 + \alpha_3) - \dot{\alpha}_3 + k_{17}\tau_r
\Rightarrow \tau_r = \frac{1}{k_{17}}(-\dot{c}_5(\omega_r - \alpha_3) - k_{15}T_2 - k_{16}\omega_r + \dot{\alpha}_3)
\] (71)
where \( c_5 \) is the positive gain
Thus, there exists a CLF of the Eq. (69) and the state feedbacks (48), (54), (59), (65) and (71) and a change of state transformations (44), (50), (55), (61) and (67), such that the complete system can be rewritten as follows:
\[
\begin{align*}
\dot{\xi}_1 &= -c_1 \xi_1 \\
\dot{\xi}_2 &= -c_2 \xi_2 \\
\dot{\xi}_3 &= -c_3 \xi_3 \\
\dot{\xi}_4 &= -c_4 \xi_4 \\
\dot{\xi}_5 &= -c_5 \xi_5
\end{align*}
\] (72)
By the backstepping approach shown above, the backstepping controller of the (34) – (38) system is given as follows
\[
\begin{align*}
\tau_1 &= \frac{1}{k_{10}}(-c_1(\omega_1 - \omega_{1\text{ref}}) - k_{7}T_2 - k_{9}T_4 - k_{9}\omega_1) \\
\tau_4 &= \frac{1}{k_{4}}(-c_4(\omega_r - \alpha_3) - k_{4}T_1 - k_{5}\omega_r + \dot{\alpha}_3) \\
\tau_r &= \frac{1}{k_{17}}(-c_5(\omega_r - \alpha_3) - k_{15}T_2 - k_{16}\omega_r + \dot{\alpha}_3)
\end{align*}
\] (73)
where \( c_1, c_2, c_3, c_4, and c_5 \) are positive gains and determined by the modified genetic
algorithms (MGA), and \( a_2, a_3, \dot{a}_2, \text{and} \dot{a}_3 \) are determined in the equations (34), (37), (48), (54) and (65).

The positive gains \( c_4, c_2, c_3, c_4, \text{and} c_5 \) in the (73) can be determined optimally by using the MGA by objective function (74).

\[
J = \beta_1 \sum_{i=1}^{N} (\Delta_T^2) + \beta_2 \sum_{i=1}^{N} (\Delta \alpha^2) + \beta_3 \sum_{i=1}^{N} (\Delta \beta^2) + \beta_4 \sum_{i=1}^{N} (\Delta \phi^2)
\]

(74)

where \( N \) is an integer number of iterations in control simulations, \( \beta_1, \beta_2, \beta_3, \beta_4 \) are scale factors, \( \Delta T = T_i - T_{i_{ref}} \) is the errors between the operating and reference tensions of span 1 and 2. The detailed explanation of the genetic algorithm and the MGA are developed by the authors in Refs. (18) and (19). Here is some brief explanation about the development of MGA proposed in the paper. The following is with block of modified genetic algorithm.

1. Initiate the strategy parameters
2. Create and initialize the initial gains
3. For each gain
   a. Integrate the motion equations
   b. Evaluate the objective function \( J \) in the (74)
4. End
5. While stopping condition(s) not true do
   a. For \( i = 1, n \) do
      i. Choose \( i \geq 2 \), new gains at random
      ii. Create offspring through application of crossover operator
      iii. Mutate offspring strategy parameters
      iv. Integrate the motion equations
      v. Evaluate the objective function (74) of new gains
     a. If value of objective function is less than epsilon
        i. Best gains
     a. End
   a. End
6. Select the new population
7. \( t = t + 1 \)
8. End

The use of real-coded GAs with search operator find more suited than binary GAs in finding feasible gains from feasible parent gains. In this paper, the use of real-coded GAs with simulated binary crossover (SBX) and a parameter-based mutation operator is implemented. The mutation probability, mutation parameter, crossover rate and crossover probability is selected depending on the speed of convergence of algorithm.

3. Precise control algorithm design using the backstepping controller

Figure 4 shows the block diagram of the two-span R2R web control system algorithm using the BSC. In this algorithm, the FPGA module based on PC shown in Fig. 3 depends on the input data to generate the control signals of torques to keep web velocity and tension at prescribed reference values due to the presence of inertia change and viscous friction. These values are determined by the BSC (73).
Load cells in channel 1 are used for web tension feedbacks of span 1 and 2, ultrasonic sensors are used to determine the radius change in time and encoders are used for velocity feedback of motors 1, 2, and 3. The FPGA module of Input/Outputs shown in Fig. 3 depends on input signals to generate the output signals. Using the Labview programming language, control software is given out for the two-span R2R web system as shown in Fig. 8. With the rapid development of digital computers, the algorithm will be developed with the function that can detect errors and automatically recover the errors of the control system due to disturbances and changing parameters.

Fig. 4. Block diagram of the R2R web tension and velocity control system

4. Simulation results

4.1 Simulation Condition and Parameters
The simulation parameters of the two-span R2R web control system are shown in Table I. The simulation condition is set up with the zero initial conditions, the desired web tension of $T_{2ref} = 20 \ (N)$ and $T_{2ref} = 15 \ (N)$ and the desired angular velocity of the infeeder of 0.5 (rad/s). In order to observe the effectiveness of the proposed algorithm of the BSC, one compensates the initial friction and inertia change of the rewinding and unwinding rolls in time.

| TABLE I |
| SIMULATION PARAMETERS OF WEB CONTROL SYSTEM |
| Parameters | values | units |
| Initial radius of the unwinder ($R_u$) | 0.1455 | (m) |
| Initial radius of the winder ($R_r$) | 0.0483 | (m) |
| Radius of the roller 1 ($R_1$) | 0.02535 | (m) |
| Total moment of inertia of motor at unwinder ($J_u$) | 0.0000707 | (kg-m) |
| Total moment of inertia of motor at winder ($J_r$) | 0.00001 | (kg-m-s$^2$) |
| Total moment of inertia of motor at Winder ($J_w$) | 0.0000707 | (kg-m-s$^2$) |
| Total length of span 1 ($L_1$) | 1.490 | (m) |
| Total length of span 2 ($L_2$) | 1.335 | (m) |
| Coefficient of vicious friction at unwinder ($B_u$) | 0.00002533 | (kg-m-s/rad) |
| Coefficient of vicious friction at winder ($B_r$) | 0.00002533 | (kg-m-s/rad) |
| Coefficient of vicious friction at roll 1 ($B_1$) | 0.00002533 | (kg-m-s/rad) |
| The thickness of web (h) | 0.0001 | (m) |
| The width of web (w) | 0.12 | (m) |
| Young’s module (E) | $2.710^8 \ (N/m^2)$ |
| Area of cross-section (A) | 0.000013 | $m^2$ |
4.2 Simulation results

The BSC algorithm in (73) is implemented in Matlab/Simulink. The optimal gains obtained by the MGA in Refs (18) and (19) are shown below:

\[ c_1 = 9.12, c_2 = 19.00, c_3 = 14.23, c_4 = 11.2, c_5 = 12.3 \]

The simulation results are presented in Fig. 5 and Fig. 6:

Fig. 5. The angular velocity change in time of infeeder

Fig. 6. The tension change in time of span 1 and 2

Figures 5 and 6 show the angular velocity change of the infeeder and the web tension change in spans 1 and 2 in time, respectively. From the simulation results, the following comments can be made:

- The time response of R2R web system of the BSC with optimal gains determined by the MGA has no overshoot and yields settling time of 0.2 second.
- The proposed algorithm based on the BSC achieved the precision, high stability and met the performance specifications.

5. Experimental results

5.1 Experimental setup

Figure 7 shows the diagram of the two-span R2R web control system for experimental study. In order to operate the web, the unwinder, infeeder unit and rewinder motors HC-KFS43 and Driver (MR-J2S-40A) with torque control mode (0–8 V) are used. The two load cells are used to measure the web tensions at span 1 and span 2 of the system and an ultrasonic sensor MIC+25/IU/TC is used to determine the change in the radii in time. The system with NI FPGA board (PCI 7811R reconfigurable I/O) is integrated with input/output equipments to receive and send out the input and output signals. Depending on the input, the control program shown in Fig. 8 with the proposed algorithm generates the torques to keep web velocity and tension at prescribed values.
Using the diagram in Figure 7, the aforementioned mathematical model and the Labview language of Labview FPGA module, the web tension and velocity control program for R2R web system is shown in Figure 8. In this program, the reference web tension and velocity and tension can be changed arbitrarily by users. The control program displays the change of web tension and change of angular velocity of rewinder and unwinder as shown in Figure 8. The change of gains in the BSC is also available to help the users understand and tune the gains to get the reference response of system in the presence of disturbance.

5.2 Experimental results

The desired web tensions of $T_{1ref} = 20 \, (N)$, $T_{2ref} = 15 \, (N)$ and the desired angular velocity change are 0.5 (rad/s). The experimental results are given in Figure 9 and Figure 10.
Figure 9 shows the change of angular velocity of the infeeder. It can be observed that the system response with low velocity shows no overshoot. The minus sign represents the opposite direction of the motor.

Figure 10 shows the web tension change in span 1 and span 2 respectively. The red line shows the time response of web tension in span 2 and the black line shows the time response of the web tension in span 1. The big variation of web tension in span 1 is due to the static charge existence in the unwinder. The static charge is removed by high voltage. Thus, the variation of web tension in span 2 is small. It is clear that the web tensions approach the reference values in short time with no overshoots.

From the above simulation results, the following comments can be made:

- The system response for the proposed algorithm has no overshoot and the fast response and the variations of web tension in time are due to the static charge. It can be eliminated when a specified device is installed.
- By comparing with the simulation results, it is evident that the system response in time with the proposed algorithm is working well with high reliability

6. Conclusions

In this paper, a mathematical basis is developed and applied to design a BSC and then a new precise control algorithm is proposed for a non-linear two-span R2R web tension and velocity control system by using the BSC with optimal gains determined by the MGA. The efficiency of this method is validated by the simulation results in Matlab/Simulink and the
experimental results. The following conclusions are drawn:
1. The proposed algorithm is employed for simulation in Matlab/Simulink. The simulation results with optimal gains give a better response in comparison with existing algorithms.
2. The simulation and experimental results show that the proposed algorithm is reliable and highly accurate.
3. The proposed algorithm based on the BSC achieves stability with positive gains.
Also, from the obtained results, it follows that the proposed algorithm using the BSC meets the desired performance specifications of the high stability in the presence of inertia change of the unwinder and rewinder and viscous friction. With the rapid development of sensors and electronic devices, the proposed control algorithm of BSC results in a control system with high precision and is useful for applications with high digital computational system.

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