Development of Scheduling System Considering Statistical Distributions of Processing Times for Human Operators*

Koji IWAMURA**, Takuya YOSHIOKA**, Yoshitaka TANIMIZU** and Nobuhiro SUGIMURA**
**Graduate School of Engineering, Osaka Prefecture University
1-1 Gakuen-cho, Naka-ku, Sakai-shi, Osaka, Japan
E-mail: iwamura@me.osakafu-u.ac.jp

Abstract
A new scheduling method is proposed, in this paper, to consider the statistical distributions of the processing times which are represented by normal distributions for the manufacturing systems where the human operators carry out the machining operations by using manual machine tools. The proposed scheduling method consists of two steps. In the first step, the machining schedules are determined for the individual machine tools based on average processing times by using utility value based method. In the second step, the candidate human operators are assigned to the individual machine tools taking into consideration of the statistical distributions of the processing times. A new scheduling system is also developed to generate suitable schedules based on the proposed scheduling method by using the processing time data executed by the combination of human operators, manual machine tools and machining operations. Some case studies have been carried out to verify the effectiveness of the proposed method.

Key words: Scheduling System, Human Operator, Statistical Distribution, Normal Distribution, Manufacturing System

1. Introduction
The scheduling problem is often treated as a combinational optimization problem based on the definite processing times. In real production systems, however, the processing times have statistical distributions. Especially, it is not easy to consider variations of the processing times which are different from one operator to another, in the scheduling problems of the manufacturing systems with human operators.

Some researches have been carried out to deal with the scheduling method considering the variations of processing times. The variations of processing times are represented by fuzzy numbers in the fuzzy scheduling problems. An effective method has been proposed to solve the fuzzy scheduling problems by applying triangular or trapezoidal fuzzy processing times(1). Morikawa et al. (2) proposed a scheduling method which minimizes the average and variance of total flow-time under the assumption that the processing times of jobs are obtained by uniform distributions for the parallel machine scheduling problem. Single machine scheduling problems have been studied for stochastic processing times which are represented by gamma distributions, negative binomial distributions, normal distributions, and that consist of a deterministic component and a random disturbance(3).

However, the total scheduling system has not yet been discussed to estimate the statistical distributions of the processing times executed by the human operators in the
existing manufacturing systems, and to generate the suitable schedule based on the statistical distributions of the processing times. A new scheduling method is proposed, in this paper, to consider the statistical distributions of the processing times which are represented by normal distributions for the manufacturing systems where the human operators carry out the machining operations by using manual machine tools. A new scheduling system is also developed to generate suitable schedules based on the processing time data executed by the combination of human operators, manual machine tools and machining operations.

2. Conventional Scheduling Method

2.1 Scheduling methods based on stochastic processing times represented by normal distributions

Following researches have been carried out to propose scheduling methods based on the stochastic processing times which are represented by normal distributions.

Aoyama et al. (4) estimate statistical distributions of make spans by carrying out Monte Carlo simulations under the assumption that the processing times are obtained by normal distributions. The average and standard deviation of make spans are calculated based on the results of Monte Carlo simulations in order to estimate the probability of keeping due date.

Kise et al. (5) consider an n-job one machine scheduling problem, in which the processing time of each job is a random variable subject to a normal distribution and the objective function is to maximize the weighted number of individual jobs which keep due date. A theoretical algorithm is proposed to solve the problem in $O(n^2)$ time.

Experimental analyses have been carried out to compare the make spans of the schedules obtained by the estimated processing times and that obtained by the actual processing times fluctuated randomly around the estimated values (6).

Many researches have been carried out, as mentioned above, to deal with the scheduling method based on the stochastic processing times which are represented by normal distributions, and to verify the effectiveness of the proposed scheduling methods. Therefore, it is assumed, in this paper, that the processing times are represented by normal distributions for the cases where the human operators carry out the machining operation by using manual machine tools.

2.2 Real-time scheduling method based on utility values

In the previous research (7), a real-time scheduling method based on the utility values has been proposed to determine a suitable combination of the jobs and the machine tools which carries out the machining operations in the next time period based on average processing times.

At a time $t$, all the ‘idling’ jobs and machine tools determine their machining schedules in the next time period, as shown in the Fig. 1. The individual jobs and machine tools firstly select all the candidates for the machining operations in the next time period, and determine the utility values for the individual selected candidates, based on their own objective functions.

The utility values have the range from 0 to 1, which indicates the ratio of the improvement of the objective function values for the cases where the jobs and machine tools select the candidates for the next machining operations. It is assumed that the individual jobs and machine tools have one of the objective functions shown in Table 1 for evaluating the utility values.

All the jobs and the machine tools send the selected candidates and their utility values of the candidates to the coordinator, in the second step. The coordinator determines a suitable combination of the jobs and the machine tools which carry out the machining operations in the next time period, based on the utility values. The decision criteria of the coordinator is
to maximize the total sum of the utility values of all the jobs and machine tools.

The combinations of the jobs and machine tools are determined by repeating the process mentioned above. The machining schedules are determined by using utility value based method, in this research.

3. Scheduling Method Considering Statistical Distributions of Processing Times

It is assumed that the processing times executed by the human operators are represented by random variables according to normal distributions $N(\mu_{ijkm}, \sigma_{ijkm}^2)$, in this paper. Where, $\mu_{ijkm}$ and $\sigma_{ijkm}$ represent the average and the standard deviation for the cases where a human operator $i$ carries out a $m$-th machining operation of a job $k$ by using a machine tool $j$ ($i=1,2,\ldots,\eta$, $m=1,2,\ldots,\omega$, $k=1,2,\ldots,\kappa$, $j=1,2,\ldots,\phi$). The scheduling method consisted of following two steps is proposed in order to consider the statistical distributions of processing times executed by human operators.

STEP1 Machining schedules are determined for the individual machine tools.

STEP2 Candidate human operators are assigned to the individual machine tools.

3.1 Determination of machining schedules

STEP1 is carried out by using utility value based method described in § 2.2. The individual machine tools and jobs evaluate the utility values to select the most suitable combinations for the machining operations based on their objective functions such as flow-time, machining accuracy, efficiency and machining cost. It is set, in this research, that the objective functions of the machine tools are efficiency and that of the jobs are flow-time, aimed at improving the production efficiency.

The machine tools and the jobs evaluate the utility values based on the average processing time $\mu_{ijkm}$ given by the following equation.
$\mu_{ijkm} = \frac{1}{\eta} \sum_{i=1}^{\eta} \mu_{ijkm}$ \hspace{1cm} (1)

$\mu_{ijkm}$ gives the average processing time of the $m$-th machining operation of the job $k$ by $j$-th machine tools, without considering the difference of human operators.

### 3.2 Determination of assignment of human operators

Candidate human operators are assigned to the individual machine tools in the STEP2. It is assumed that the individual human operators are assigned to the same machine tools until all the machining operations are finished in the manufacturing systems.

After assigning the human operators, it is required to calculate the averages and standard deviations of starting times and finishing times of the individual machining operations in order to estimate the statistical distribution of the make span which is the objective function of the system.

Let us consider the case to calculate the average and standard deviation of starting time for a Target-Operation 1 in Fig. 2. As shown in the figure, a Preceding Operation-1 and a Preceding Operation-2 are the machining operation which precedes the Target Operation-1 based on the sequence of machining operations for the machine tools, and that precedes the Target Operation-1 based on the machining sequences of the jobs, respectively. The Target Operation-1 can be carried out after finishing both the Preceding Operation-1 and Preceding Operation-2.

The operations and its processing times, starting times and finishing times are given by the following parameters.

- $O_{ijkm}$ : $m$-th machining operation of job $k$ which is carried out by human operator $i$ by using machine tool $j$.
- $PT_{ijkm}$ : Processing time of operation $O_{ijkm}$. Where, $\mu_{ijkm}$ and $\sigma_{ijkm}$ represent the average and standard deviation of $PT_{ijkm}$.
- $ST_{ijkm}$ : Starting time of operation $O_{ijkm}$. Where, $\mu_{ijkm}$ and $\sigma_{ijkm}$ represent the average and standard deviation of $ST_{ijkm}$.
- $FT_{ijkm}$ : Finishing time of operation $O_{ijkm}$. Where, $\mu_{ijkm}$ and $\sigma_{ijkm}$ represent the average and standard deviation of $FT_{ijkm}$.

The operation $O_{ijkm}$ can be started at the time which is the maximum value of the finishing time of both preceding operations $O_{ij+1}$ and $O_{i-1jkm-1}$. Therefore, the distribution of the maximum value is estimated by applying the equation proposed by Clark(8), which approximately evaluates the distribution.
Following equations give the average $\mu_{ijkm}$ and standard deviation $\sigma_{ijkm}$ of starting time for operation $O_{ijkm}$ calculated by equation proposed by Clark based on the averages $\mu_{f_{km}^{**}}$, $\mu_{f_{ij}^{**}}$ and standard deviations $\sigma_{f_{km}^{**}}$, $\sigma_{f_{ij}^{**}}$ of finishing times for preceding operations $O_{ij}^{**}$, $O_{km}^{**}$.

\[
\mu_{ijkm} = \mu_{f_{km}^{**}} \Phi(\alpha) + \mu_{f_{ij}^{**}} \Phi(-\alpha) + a \phi(\alpha) \quad (2)
\]

\[
\sigma_{ijkm} = \{(\mu_{f_{km}^{**}})^2 + (\sigma_{f_{km}^{**}})^2\} \Phi(\alpha) + \{(\mu_{f_{ij}^{**}})^2 + (\sigma_{f_{ij}^{**}})^2\} \Phi(-\alpha) + 2\sigma_{f_{ij}^{**}} \phi(\alpha) - (\mu_{ijkm})^2 \quad (3)
\]

where,
\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt
\]
\[
\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}
\]
\[
a = (\sigma_{f_{km}^{**}})^2 + (\sigma_{f_{ij}^{**}})^2 \quad (3a)
\]
\[
\alpha = (\mu_{f_{km}^{**}} - \mu_{f_{ij}^{**}}) / a \quad (3b)
\]

The average $\mu_{ijkm}$ and the standard deviation $\sigma_{ijkm}$ of finishing time for operation $O_{ijkm}$ are calculated by following equations based on the averages and the standard deviations of the starting time and the processing time according to the reproductive property of normal distribution.

\[
\mu_{f_{ijkm}} = \mu_{s_{ijkm}} + \mu_{ijkm} \quad (4)
\]

\[
\sigma_{f_{ijkm}} = \{(\sigma_{s_{ijkm}})^2 + (\sigma_{ijkm})^2\}^{1/2} \quad (5)
\]

The distributions of starting times and finishing times for all the machining operations are obtained by repeating the two processes mentioned above.

Figure 3 shows an example of the generated schedules considering statistical distributions of processing times in the form of gantt chart obtained by STEP2. The triangular shapes around the starting times and the finishing times of the individual operations in the Fig. 3(a) mean the averages ($\mu$) and the three times of the standard deviations ($+3\sigma$) of the starting and finishing times, as shown in the Fig. 3(b). Both the values of $+3\sigma$ and $-3\sigma$ are obtained by Eqs. (3) and (5), however only $+3\sigma$ is presented in
Fig. 3(b), since attention is paid mainly to the delay of schedule in management and control of the manufacturing systems.

Following equation is defined to estimate the maximum value of the make span.

\[ MS_{3\sigma} = \max_{i=1 \cdots q, j=1 \cdots q, k=1 \cdots \kappa, m=1 \cdots \nu} (\mu_{ijkm} + 3\sigma_{ijkm}) \]  

(6)

\( MS_{3\sigma} \) means the finishing time of all the machining operations with probability 99.73% based on the property of normal distribution. The optimal assignment of the human operators which minimizes \( MS_{3\sigma} \) is determined from all the candidate assignments.

### 4. Scheduling System Using Processing Time Database

Figure 4 shows the proposed system to generate the schedules based on the scheduling method considering the distributions of the processing times. The scheduling system consists of a planning system, a manufacturing system, and a processing time database.

The planning system generates the schedules based on the proposed scheduling method considering the statistical distributions of the processing times. The human operators carry out the machining processes according to the schedules determined by the planning system, in the manufacturing system. The manufacturing system collects and stores the processing time data \( PT_{ijkn} \) executed by the individual human operators to the processing time database by using the POP (Point of production) systems. \( PT_{ijkn} \) is the \( n \)-th processing time for the cases where the human operator \( i \) carries out \( m \)-th machining operation of job \( k \) by using machine tool \( j \) \((n=1, 2, \ldots, \nu)\).

The processing time database has a function to calculate the following statistical distributions of the processing time which are referred to by the planning system.

(a) Average processing time \( \mu_{ijkm} \) and standard deviation \( \sigma_{ijkm} \) of the individual human operators calculated by following equations.

\[ \mu_{ijkm} = \frac{1}{\nu} \sum_{n=1}^{\nu} PT_{ijkmn} \]  

(7)

\[ \sigma_{ijkm} = \left\{ \frac{1}{\nu} \sum_{n=1}^{\nu} (PT_{ijkmn} - \mu_{ijkm})^2 \right\}^{1/2} \]  

(8)
(b) Average processing time for all the human operators $\mu_{ijk}$ which are calculated by the Eq. (1).

The proposed scheduling system generates the suitable schedule based on the statistical distributions of processing times obtained by repeating the planning and collection cycles consisting of planning of the schedule, execution of the schedule, and collection of the executed processing times.

5. Case Study

5.1 Simulation of human operators

Some case studies have been carried out to verify the effectiveness of the proposed method. It is required to execute the machining processes by human operators, and to collect the processing times by using the POP system in the proposed scheduling system. However, the processing times executed by human operators are generated by using simulator, in the case study, in order to simplify the experimental analysis. The normal distribution is inputted to the simulator to represent the statistical distributions of processing times executed by human operators.

It is assumed that the human operators carry out the machining operations based on their skills, and the processing times for carrying out the machining operations depend on their skills. The following equations give the assumptions for the effects of the skills of human operators to the averages and the standard deviations of the processing times.

\[ x_1 \mu_{(1)ijk} = x_2 \mu_{(2)ijk} = \mu_{(3)ijk} \]  \hspace{1cm} (9)
\[ \sigma_{(1)ijk} = y_1 \mu_{(1)ijk} \]  \hspace{1cm} (10)
\[ \sigma_{(2)ijk} = y_2 \mu_{(2)ijk} \]  \hspace{1cm} (11)
\[ \sigma_{(3)ijk} = y_3 \mu_{(3)ijk} \]  \hspace{1cm} (12)

where, $\mu_{(1)ijk}$, $\mu_{(2)ijk}$, $\mu_{(3)ijk}$, $\sigma_{(1)ijk}$, $\sigma_{(2)ijk}$, and $\sigma_{(3)ijk}$ are the averages and the standard deviations of processing times in the cases where the skill of human operator is first grade, second grade, and third grade, respectively. $x_1$, $x_2$, $y_1$, $y_2$, and $y_3$ are the weighting factors representing the difference of skills.

The simulator determines the processing time $t_{ijk}$ for the case where the human operator $i$ carries out the $m$-th machining operation of job $k$ by using machine tool $j$ based on the random variables according to the normal distributions. The finishing times $f_{ijk}$ of individual jobs can be obtained from the processing times $t_{ijk}$ of individual jobs and the scheduling results. The make span $MS_{sim}$ given by following equation is obtained based on the finishing times $f_{ijk}$ of individual jobs through the simulation.

\[ MS_{sim} = \max_{i=1,\ldots,\eta; j=1,\ldots,\nu; k=1,\ldots,\kappa; m=1,\ldots,\alpha} f_{ijk} \]  \hspace{1cm} (13)

5.2 Experimental analysis

Some case studies have been carried out by using proposed scheduling system. The conditions of case studies are followings.

(1) The manufacturing system consists of 6 machine tools which are classified into 2 types. Same type of machine tools can carry out the same machining operations.

(2) Twenty jobs are considered in the case studies, which are machined by 2 types of machine tools.
(3) Two human operators for each skill level are considered. Therefore, total number of human operators is 6.

(4) The factors representing the skill levels are set to be $x_1 = 12/10$, $x_2 = 12/11$, $y_1 = 0.05$, $y_2 = 0.1$, and $y_3 = 0.15$, respectively.

(5) The average processing times for the human operators who have 1st grade are set to be 30 minutes to 100 minutes.

Table 2 shows the statistical distributions of processing times for human operators who have 1st grade.

<table>
<thead>
<tr>
<th>Human operator 1 and 2</th>
<th>Machine tool 1</th>
<th>Machine tool 2</th>
<th>…</th>
<th>Machine tool 5</th>
<th>Machine tool 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job 1</td>
<td>$N(50,3^2)$</td>
<td>$N(30,2^2)$</td>
<td>…</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Process 2</td>
<td>-</td>
<td>-</td>
<td>…</td>
<td>$N(60,3^2)$</td>
<td>$N(40,2^2)$</td>
</tr>
<tr>
<td>Job 2</td>
<td>$N(50,3^2)$</td>
<td>$N(30,2^2)$</td>
<td>…</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Process 2</td>
<td>-</td>
<td>-</td>
<td>…</td>
<td>$N(60,3^2)$</td>
<td>$N(40,2^2)$</td>
</tr>
<tr>
<td>Job 3</td>
<td>$N(50,3^2)$</td>
<td>$N(30,2^2)$</td>
<td>…</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Process 2</td>
<td>-</td>
<td>-</td>
<td>…</td>
<td>$N(60,3^2)$</td>
<td>$N(40,2^2)$</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>Job 20</td>
<td>$N(50,3^2)$</td>
<td>$N(30,2^2)$</td>
<td>…</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Process 2</td>
<td>-</td>
<td>-</td>
<td>…</td>
<td>$N(60,3^2)$</td>
<td>$N(40,2^2)$</td>
</tr>
</tbody>
</table>

Fig. 5 Scheduling result

The make span $MS_{sim}$ obtained through the simulation increases rapidly in the first phase, and converge to suitable values in Fig. 5. It is because that unsuitable machining schedule is firstly generated based on the initial values in the processing time database,
where the initial value 10 and 1 are given to the averages $\mu_{ijkm}$ and standard deviations $\sigma_{ijkm}$ respectively. After repeating the simulations, both the $\mu_{ijkm}$ and $\sigma_{ijkm}$ converge to reasonable values, and a suitable schedule is generated.

The differences between the estimated maximum values of make spans $MS_{3\sigma}$ and make spans $MS_{sim}$ obtained by the simulation are relatively large until the repeat count reaches about 200. It shows that suitable schedules and estimation of make spans are not obtained, because the information of processing times executed by human operator is not enough in the processing time database.

The upper limits of make spans $MS_{sim}$ obtained by the simulation fit the estimated maximum value of make span $MS_{3\sigma}$ after repeat count of scheduling reaches about 200. It shows that the scheduling system generates suitable schedules for the cases where the enough information of processing times have been stored to the processing time database, and obtained schedules are effective to estimate the maximum value of the make span.

If suitable statistical distributions of the processing times are given to the database in the initial stage, the system can generate suitable schedule and statistical distribution of make span within less repeat count.

Additional case studies have been carried out to consider the case where the number of jobs is changed during the repetition of production. Following conditions are considered for the additional case studies.

(a) Twenty jobs are repeatedly produced until repeat count reaches to 250. The number of jobs is reduced from 20 to 6 after repeat count reaches to 250.

(b) Six jobs are repeatedly produced until repeat count reaches to 250. The number of jobs is increased from 6 to 20 after repeat count reaches to 250.

Figure 6 shows the example of scheduling results. The make span $MS_{sim}$ obtained through the simulation increases rapidly in the first phase, and converge to suitable values in Fig. 6. This reason is the same situation in Fig. 5.

As show in the figure (a), the upper limits of make spans $MS_{sim}$ obtained by the simulation fit the estimated maximum value of make span $MS_{3\sigma}$ after the number of jobs is reduced. It is because that enough information of processing times have already been stored to the processing time database.

However, the differences between the estimated maximum values of make spans $MS_{3\sigma}$ and make spans $MS_{sim}$ obtained by the simulation become large after the number of jobs is increased, as shown in the figure (b). It is because that the information of processing times executed by human operator is not enough in the processing time database. Therefore, scheduling system generates suitable schedules and estimates the maximum value of the make span after the enough information of processing times have been stored to the processing time database.

As shown in the case study results, proposed scheduling system is effective to generate suitable schedules for the manufacturing systems where the product mix of jobs is changed and/or some of jobs are changed to new ones in the case where the information of processing times has already been stored to the processing time database.

6. Conclusions

New scheduling method is proposed, in this paper, to consider the statistical distribution of processing times executed by human operators. The following remarks are concluded.

(1) New scheduling method is proposed to determine the machining schedules and the assignments of the human operators to the individual machine tools taking into consideration of the statistical distributions of the processing times which are represented by normal distributions.
The scheduling system is developed to carry out the proposed scheduling method by using processing time database.

Some case studies have been carried out to verify the effectiveness of the proposed method. It was shown, through case studies, that the scheduling system generates suitable schedules and estimates the maximum value of the make span for the cases where the enough information of processing times have been collected and stored to the processing time database.

It is required to verify the effectiveness of the proposed scheduling system for the cases where the processing times are represented by probability distribution other than normal distribution e.g. “beta distribution” as a future issue.

References


