Multi-Objective Analysis Applied to Mixed-Model Assembly Line Sequencing Problem through Elite Induced Evolutionary Method*

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Abstract
To meet higher customer satisfaction and shorter production lead time, assembly lines are shifting to mixed-model assembly lines. Accordingly, sequencing is becoming an increasingly important operation scheduling that directly affects on efficiency of the entire process. In this study, such sequencing problem at the mixed-model assembly line has been formulated as a bi-objective integer programming problem so that decision making through trade-off analysis can bring about significant production improvements. Then we have developed a multi-objective analysis method by hybridizing conventional and recent meta-heuristic methods. After showing its generic idea, the car mixed-model assembly line sequencing problem is concerned as a case study. Certain measures are also introduced to quantitatively evaluate the performances of the method through comparison.

Key words: Multi-Objective Analysis, Mixed-Model Assembly Line, Sequencing Problem, Elite Induction, Strength Pareto Evolutionary Algorithm 2 (SPEA2)

1. Introduction
Contemporary diversified customer demands boost the importance of just-in-time and agile manufacturing in much greater extent. Accordingly, a mixed-model assembly line, which is available for small-lot-multi-kinds production, has been introduced and popularized. Focusing on a car production as a typical example, we concerned with a sequencing problem of the mixed-model assembly line with a paint line as its preceding process. Then, noticing the trade-off between volume of WIP (Work In Process) inventory and line stoppage, we engaged in the multi-objective analysis based on a new method named E-PSA (Elite-induced PSA) (1). It is developed by hybridizing the conventional weighting method and evolutionary method known as PSA (Pareto Simulated Annealing) (2).

As an extension from our previous studies, in this paper, we have developed another elite-induced method named E-SPEA2 (Elite-induced SPEA2) in terms of SPEA2 (Strength Pareto Evolutionary Algorithm 2) (3). Furthermore, we analyze the results not only for the case of multiple elites, but also for the case of single elite induction. Such practice is of special importance since it expands the capability of E-SPEA2 algorithm to the next stage. It enables a decision maker (DM) to manipulate the desired location of the outcome led by the elites. Moreover, to evaluate quantitatively the results from each method, performance
metrics are proposed and employed in a comparative study. Those considerations are discussed and eventually the effectiveness of the proposed method will be examined through a case study applied to the car mixed-model assembly line sequencing problem.

The rest of this paper is organized as follows: problem description is stated in Section 2. In Section 3, we outline the solution methods generally. Section 4 shows experimental results to reveal some properties of the proposed method. Finally, we summarize our conclusions in Section 5.

2. Problem Description

2.1 A Brief Literature Review

In the sequencing problem of mixed-model assembly line, one of the two major goals is to level the workload at each workstation on the line against the different assembly time per product (4). Another one is to keep every usage rate of parts at the assembly line constant (5). These are very important aspects to prevent line stoppage and idle time of workers. Therefore, concerns about these two goals have been widely discussed in the literatures (6)-(8).

Among them, Yano and Rachamadugu (9) considered the problem that aims at minimizing the risk of assembly line stoppage while Sumichrast and Russell (10) discussed a parts usage smoothing problem. According to progress of production manners both from hardware and software, those typical interests have also moved on the modern ones. Sequencing and/or balancing for Parallel and U-type lines were newly studied (11)-(14), and hot production manner and/or optimization method has been often taken for these studies (14)-(16). In addition, an attempt to have an interest in multi-objective optimization was made by Korkmazel and Meral (17) and Tavakkoli-Moghaddam (18). Recently, Rahimi-Vahed et al. (19) developed a multi-objective scatter search algorithm for solving a multi-objective sequencing problem under three objectives, i.e., minimization of total utility work, total production rate variation, and total setup cost.

Moreover, associated with the difference of the production processes of a combined system, Nagamoto and Morito (20) proposed to install two paint lines to simultaneously realize a smooth lot production for the paint line and a level production for the assembly line. As a drawback, this approach expands the investment cost due to the additional installation. On the other hand, Monden (7) introduced an individual sequencing method for the paint line and the assembly line to adjust the sequence disturbed by defective half-products in the paint line. However, this method paid no attention to reduce the WIP inventory.

From this brief review, we know concerns of the previous studies are stayed in a limited extent and are insufficient to resolve certain problems on the sequencing at the mixed-model assembly line. As a prospective step, therefore, it becomes meaningful to engage in the trade-off analysis for the combined process under the substantial goals.

Since it refers to multi-objective analysis for integer programming problems that is hard to work with and have not been successfully applied even by recent multi-objective meta-heuristic methods. It is meaningful, therefore, to develop alternative methods amenable to efficiently solving real world problems.

2.2 Mixed-Model Assembly Line with Painting Line

In this section, we will describe a bi-objective sequencing problem for the mixed-model assembly line including a paint line as shown in Figure 1. The paint line is composed of sub-painting, main painting and check processes. Defective products are sometimes produced and put in the buffer after correction. Due to these uncertain factors that might change the shipping rate, there causes the delay of shipping to the mixed-model
assembly line without WIP inventory.

Generally speaking, the mixed-model assembly line is equipped with \( K \) workstations operated with constant cycle time (CT). Hence necessary completion time becomes ideally \( K \cdot CT \) for every product model. For the integration of notations, the painting process (buffer) is viewed as a workstation \((k=0)\) proceeding to the workstation \((k=1)\).

### 2.3 Two-Objective Sequencing Problem

The sequencing problem under consideration is formulated as (p.1) assuming majorly the following conditions.

1. Paint defects occur randomly under the prescribed probability.
2. Correction times of the defective product vary randomly within the prescribed interval.
3. All workstations are empty at the beginning.

The objective functions \( f_1(z) \) and \( f_2(z) \) correspond to the average volume of WIP inventory and the average line stoppage time, respectively. Here, \( f_1 \) is the difference between shipping and production amounts while \( f_2 \) refers to a stoppage due to part shortage \( P_i \) and work load imbalance \( A_i \) at \( k \) station and in \( t \) period. Moreover, \( z \) denotes a vector whose component is given by a binary decision variables \( z_{it} \) that takes 1 if product model \( i \) is supplied to the assembly line at period \( t \), and otherwise, 0.

On the other hand, the cumulative amount of product model \( i \) till \( t \), \( x_{it} \) is given by

\[
x_{it} = \sum_{j=t}^{T} z_{ij}, \quad (i = 1, \ldots, I).
\]

Moreover, \( y_{it} \) denotes the cumulative amount of product model \( i \) shipped from the paint line in period \( t \).

Among the constraints, Eq.(1) requires that each demand \( d_i \) of product model \( i \) must be satisfied every injection period \( t \) while Eq.(2) requires that only one product model is possible to be shipped to the line every period. Moreover, Eq.(3) shows that total idle time of workstations should be upper bounded (\( W^u \)).

In the above formulation, we modeled some key terms as follows:

1. According to goal chasing method \((7)\), part shortage occurs at workstation \( k \) when the quantity of part \( m \) used \((\sum a_{im}^k x_i)\) exceeds its ideal quantity \((r_m^k t)\) by period \( t \). \( r_m^k \) is given by

\[
r_m^k = \frac{\sum_{i=1}^{T} a_{im}^k d_i}{T}, \quad (m = 1, 2, \ldots, M).
\]
In terms of this idea, the line stoppage time due to the part shortage \( P^t_k \) is given.

(2) Imbalance of workload at the workstation causes either of another line stoppage, \( A^t_k \) or induces the idle time, \( W^t_k \).

(3) Workload leveling is taken place in forward direction only for the consecutive injection periods. If there is still excess workload, the line stoppage occurs owing to it.

Furthermore, by viewing equivalently the product from the paint line as the parts from sub-line in the mixed-model assembly line, the line stoppage time \( P^t_0 \) due to the delay from the paint line is given just similarly in the case of part shortage.

\[
P^t_i = \max \{ \max \left( \frac{y_i^t - r_{pi}^t}{r_{pi}} \right) CT, 0 \}, \quad (t = 1, 2, \ldots, T).
\]

where \( r_{pi} \) is the average estimated supply rate of product model \( i \) from the paint line.

Under uncertain environment, we can update this value adequately during the planning horizon based on the procedure proposed previously. The detailed description of the model should be referred to the reference (21).

3. Methods for Multi-objective Analysis

3.1 General Statements
The generic problem under consideration can be described as follow.

\[
\text{min} \quad \{f_1(x), f_2(x), \ldots, f_N(x)\} \quad \text{subject to} \quad x \in X,
\]

where \( x \) denotes an \( n \)-dimensional decision variable vector, \( X \) an admissible region and \( f_i \), \( (i=1, \ldots, N) \) objective functions. We call it multi-objective optimization (MOP) when we intend to derive a unique solution termed as preferentially optimal solution of DM. Meanwhile, multi-objective analysis (MOA) tries to reveal a certain trade-off relation among the conflicting objectives by presenting Pareto optimal solution set or Pareto front. MOA is probably one of the most straightforward approaches for engaging in trade-off. In this sense, MOA can provide fruitful information available for MOP performed at the next stage.

3.1.1 Weighting method
The weighting method is a conventional method of MOA besides \( \varepsilon \)-constraint and weighted min-max methods. Since the weighting method outperformed the other methods for the problem under consideration (8), we will focus on the weighting method below. In general, we can use an appropriate one from above-mentioned method depending on the problem under consideration. Under a few mild conditions, we can derive the Pareto optimal solution set by solving the following single-objective optimization problem repeatedly for a set of weighting coefficient \( w_i \).

\[
\text{min} \quad \sum_{i=1}^{N} w_i f_i(x) \quad \text{subject to} \quad x \in X, \quad \left( \sum_{i=1}^{N} w_i = 1 \right)
\]

For the sequencing problem outlined in Section 2, the resulting problem is solved by SA (Simulated Annealing) (22). Hence, we call this solution method as SA weighting method hereinafter.

3.1.2 Multi-objective evolutionary algorithms (MOEA)
After the proposal of multi-objective genetic algorithms (MOGA), various MOEAs have been developed recently. However, many of them do not necessarily exhibit a good performance in real-world applications (23) and for many-objective optimization problems (24)-(26). In developing a new MOA method, we note SPEA2 in this study besides PSA employed in our previous studies.
SPEA2 is an improved version of its predecessor method SPEA (Strength Pareto Evolutionary Algorithm), which incorporates a fine-grained fitness assignment strategy, a density estimation technique, and an enhanced archive truncation method. Since E-SPEA2 mentioned below is deployed from SPEA2 and both algorithms are quite similar, the algorithm of SPEA2 will be referred thereat.

3.2 Elite Induced MOEA

3.2.1 Basic concept

The basic principle behind the general idea termed E-MOEA (Elite Induced MOEA) is rather simple and straightforward from the original MOEA: a combination of weighting method and MOEA. It makes use of advantages inherited by both MOEA and weighting method while virtually eliminating shortcomings of each method (Refer to Table 1). The original MOEA algorithm serves as a main solving algorithm in E-MOEA. One of the major differences is that instead of using all randomly generated initial solutions, it introduces some number of the elite solutions into a set of random initial solutions. Here, the elite solutions are obtained from the weighting method. Accordingly, E-MOEA algorithm is composed of two parts:

2. Application of MOEA.

Eventually, the major concerns that will affect on performance refer to the number of elite solutions or its rate to the whole size and their locations. General scheme of E-MOEA is demonstrated in Fig.2.

As depicted there, various locations of elites are predefined by a set of weighting coefficient rules referred to normal, northwest/southeast corners, and Pareto corner distribution, guiding dominated solutions to the elites. In other words, the elite solution set will induce the Pareto front at the direction toward its preexisting region. By adjusting the number of elites and the weighting coefficient rule appropriately, DM is able to manipulate a location so that the solutions would lie on a specific region. This makes E-MOEA unique since it provides more degree of freedom and flexibility to satisfy diversified value system of DM.

In a summary of this section, straightforward selection pressure from the elites makes the algorithm simple and computation load smaller. This is a great advantages over the approaches\(^\text{(24)-(26)}\) developed recently for many-objective optimization problem. Due to the
existence of the elites, selection pressure that might contribute to the convergence speed is always kept at high level. Moreover, the induction ability that will easily guide the solution set to DM’s preferable region is a unique function not concerned at all for them. Since there exists no king method, option of selecting the most relevant MOEA from the conventional ones is also favorable aspect of the proposed approach.

### Table 1 Comparison of revealed properties between weighting method and MOEA

<table>
<thead>
<tr>
<th>Method</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighting</td>
<td>A highly accurate Pareto front can be obtained.</td>
<td>Dispersion adjustment of the Pareto front is necessary and rather difficult.</td>
</tr>
</tbody>
</table>
| MOEA         | No adjustment in dispersion of the Pareto front is required | • Poor accuracy of the Pareto front  
• Slow convergence |

#### 3.2.2 Meaning of induction

All MOA algorithms give only a set of solutions though we are willing to have at most several candidate solutions in real world applications. This is because MOA is out of concern about any preference information imbedded by DM, and highlights the diversity of solutions over the entire Pareto front as a technique for MOA. However, even in MOA, we should address the interest of the DM’s preference more elaborately. Let us consider this assertion by taking the following $\varepsilon$-constraint method as an example:

\[
(p.4) \quad \min f_p(x) \quad \text{subject to} \begin{cases} x \in X \\ f_i(x) \leq f_i^* + \varepsilon_i, \quad (i = 1, 2, \ldots, N, i \neq p). \end{cases}
\]

If a value function conceived implicitly by the DM is described by $V(f(x))$, MOA must be concentrated within the particular extent where the DM prefers. According to this intention, the above problem should be re-described as

\[
(p.5) \quad \min f_p(x) \quad \text{subject to} \begin{cases} x \in X \\ f_i(x) \leq f_i^* + \varepsilon_i, \quad (i = 1, 2, \ldots, N, i \neq p) \\ \partial V / \partial f_i \leq 0, \quad (i = 1, 2, \ldots, N, i \neq p). \end{cases}
\]

while $f_i^*$ and $\varepsilon_i$ denote the minimum value of $f_i(x)$ and its relaxed amount, respectively.

In terms of this idea, a discussion of diversification is meaningful over the entire front in the case of (a) in Fig.3, because the preference will increase everywhere on the front if we reduce either objective function. Under a different value system shown in (b), however, it is enough to emphasize the diversity only in the limited extent of the front crossing with the painted triangle in the figure. This is because outside of this region, we can obtain a more preferable solution by leaving from the front. The elite induction technique can readily manage this prior discrimination that becomes helpful in the next decision making phase.

![Fig.3 Two cases of meaningful Pareto front: (a) over the entire front, (b) in the limited front](image-url)
Moreover, such induction can contribute to an adaptive decision making when single elite is treated as the preferential point since we can easily obtain various solutions around it. This provides an effective approach for the post-optimal analysis for multi-objective optimization problems.

3.3 E-SPEA2 algorithm
Here, we show only the detailed algorithm of E-SPEA2. That of E-PSA should be referred to the previous study (1).

1. Set \( t = 0 \) and empty archive (external set) \( A_0 \). Note that \( 1 < t \leq T \) (Maximum number of generation). Then, select the desired weighting coefficient rule.

2. Select the desired elitism ratio \( \mu = NE/NS \) deciding how many elites will be inserted into the initial population. Here, \( NE \) denoted as desired number of elites and \( NS \) is total number of solutions, respectively.

3. Generate most of the initial population \( P_0 \) randomly while some numbers are generated by applying the weighting method (according to the input \( \mu \)). Then, import elites into \( P_0 \).

4. Calculate fitness values of individuals in \( P_t \) and \( A_t \).

5. Copy non-dominated individuals of \( P_t \) and \( A_t \) into \( A_{t+1} \). If size of \( A_{t+1} \) is more than \( N \) (archive size) then reduce members in \( A_{t+1} \) by truncation, else if size of \( A_{t+1} \) is less than \( N \) then fill \( A_{t+1} \) with dominated individuals in \( P_t \) and \( A_t \). If \( t > T \) then output the non-dominated set of \( A_{t+1} \). Stop.

6. Fill mating pool by binary tournament selection with replacement on \( A_{t+1} \).

7. Apply recombination and mutation operators to the mating pool and set \( P_{t+1} \) to the resulting population. Set \( t = t + 1 \) and go back to Step (4).

3.4 Performance Metrics (PMs)
Generally, performance metrics to evaluate the algorithm for MOA are divided into three classes: distance to the Pareto optimal front, the number of non-dominated elements in the obtained set, and the spread of solutions.

In this study, we noted three different performance metrics mentioned below.

3.4.1 Generational distance; GD
The generational distance (27) is a way of estimating the average distance of the elements in the set of non-dominated vectors found so far from those in the Pareto front. \( GD \) is defined as

\[
GD = \frac{\sqrt{\frac{\sum_{i=1}^{m} d_i^2}{m}}}{m}
\]  

(7)

where \( m \) is the number of non-dominated solutions, \( d_i \) is the Euclidean distance between each of these solutions and the nearest member of the Pareto optimal set. A value of \( GD = 0 \) indicates that all the elements generated are in the Pareto front. Therefore the smaller GD is, the more accurate Pareto front is obtained.

3.4.2 Spacing; SP
Spacing metric (28) calculates the spread (distribution) of solutions on the Pareto front. \( SP \) judges how well the solutions are distributed in such front and is defined as

\[
SP = \frac{1}{m - 1} \sum_{i=1}^{m} (\bar{d} - d_i)^2
\]

(8)
where \( d'_i = \min_{j \in \{1, \ldots, m\}} \left( \sum_{k=1}^{K} f'_i(x) - f'_j(x) \right), (i = 1, \ldots, m) \)

and \( \overline{d} \) is the mean of all \( d'_i \). A value of zero for this metric indicates all members of the Pareto front currently available are equidistantly spaced.

### 3.4.3 Degree of Induction; DI

Induction efficiency (induction strength) of the elite is the ability to draw dominated solutions (initial population) in the direction close to the elite itself. For this measure, we invented newly the following metrics.

A. In the case of single elite induction

This metric is calculated from the following steps (See Fig.4).

1. Determine the centroid of the initial population \( g_I \).
2. Designate the point where the elite is located. Such point is denoted as \( g_E \).
3. Compute the distance \( L \) between \( g_I \) and \( g_E \) and let \( \delta = L / 100 \).
4. Calculate the distances \( l_i \) from \( g_E \) to each individual \( x_i \) \((i = 1, \ldots, m)\).
5. Define degree of induction \( DI \) (Single) by

\[
DI(\text{Single}) = \frac{1}{m} \sum_{i=1}^{m} \frac{l_i}{\delta / m} 
\]

where \( m \) denote the number of solution involved in the archive of non-dominated solutions.

B. In the case of multi-elite induction

Here, each step is almost similar to that of single elite case. Since the induction is due to the combined effect from the plural elites, it is relevant to consider the elites in the neighborhood will affect much more than those in the remote. Hence, geometric average of the distance \( l_{ij} \) between each \( x_i \) and elite \( g^k_E \) is used in this evaluation. As a result, induction efficiency is modified into

\[
DI(\text{Multiple}) = \frac{1}{m} \sum_{i=1}^{m} \prod_{k=1}^{E} \left( \frac{l_{ik}}{\delta_k} \right)^{|E|/m} 
\]

where \( \delta_k = L_k / 100 \). \((k = 1, \ldots, |E|)\). Note that \( L_k \) and \( E \) denotes the distance between \( g_I \) and each elite and an index set of elites, respectively (See Fig.5).
4. Results and Discussion

In Tables 2 and 3, we show various parameters used in the numerical experiments whose results are evaluated by averaging over 20 sample data sets.

**Table 2** Input parameters for the case study

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cycle time $C_T$ [min]</td>
<td>5</td>
</tr>
<tr>
<td>Number of workstation $K$</td>
<td>100</td>
</tr>
<tr>
<td>Number of product model $I$</td>
<td>10</td>
</tr>
<tr>
<td>Total productions $\sum_i d_i$</td>
<td>100</td>
</tr>
<tr>
<td>Injection period $T$</td>
<td>30</td>
</tr>
<tr>
<td>Defective rate from paint line</td>
<td>0.2</td>
</tr>
<tr>
<td>Correction time at paint line [min]</td>
<td>[15,25]</td>
</tr>
<tr>
<td>Elite number</td>
<td>5</td>
</tr>
<tr>
<td>Size of population (Total solution numbers)</td>
<td>30</td>
</tr>
</tbody>
</table>

**Table 3** Parameters used in E-SPEA2 and SA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population pool size</td>
<td>200</td>
</tr>
<tr>
<td>Archive size (Total solution numbers)</td>
<td>30</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>0.7</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>0.01</td>
</tr>
<tr>
<td>Tournament size</td>
<td>2</td>
</tr>
<tr>
<td>Number of generations</td>
<td>400</td>
</tr>
<tr>
<td>Sample size</td>
<td>30</td>
</tr>
<tr>
<td>Initial temperature*</td>
<td>100</td>
</tr>
<tr>
<td>Annealing factor*</td>
<td>0.8</td>
</tr>
<tr>
<td>Temperature update frequency*</td>
<td>30</td>
</tr>
<tr>
<td>Neighbor search iteration*</td>
<td>250</td>
</tr>
</tbody>
</table>

*Parameters used in SA

![Fig.6 Results of E-SPEA2 versus SPEA2](image)
4.1 E-SPEA2 under Multiple Elite Inductions

From the comparison depicted in Fig.6, we can clearly observe that E-SPEA2 outperforms SPEA2 in both ways: frontal distribution and closeness to optimality (Note that \( \mu = 5/30 \) in this case). Adding the superiority of PSA over E-PSA shown previously, we can confirm the ability of induction in terms of the elites for E-SPEA2.

Figure 7 depicts various Pareto frontier locations drawn by different elite placements. Such placements are done by imposing the weighting coefficient rules (normal dist., southeast dist., and northwest dist.). Different elite placements can effectively draw the dominated solutions toward their preexisted locations. At this point, multiple elite induction ability of E-SPEA2 has been realized.

4.2 E-SPEA2 under Single Elite Induction

We conduct further experiment to investigate how Pareto front behave if it is being induced by single elite. Such investigation leads to the post-optimal analysis where single elite is treated as a preferential point chosen by DM. For example, let us suppose three preferential points (A, B and C) which are obtained respectively by an appropriate MOP method for the DMs who have different value systems.

As shown in Fig.8, DM who decides point A as his/her preferentially optimal solution can obtain a set of solutions around it, and we can say the same thing for the others. Hence, it is helpful for DM to engage in the advanced decision making required in the practical application. Moreover, we notice that the resulting fronts from single elite placement are denser than ones obtained by the multiple elite inductions. In other word, single elite induction has higher induction strength compared with the case of multiple elite’s induction.

4.3 Comparison of Performance Metrics and Convergence Issue

We summarized the results of performance metrics in Table 4. Regarding to GD, result of normal distribution of E-SPEA2 reveals quantitatively its superiority shown in Fig.6 to the SPEA2. Generally speaking, it is not necessarily meaningful to compare SP between the methods since the induction is rather opposite idea to the spacing. However, noticing that point A corresponds to northwest, point B to Pareto corner, point C to southeast distributions, respectively, the single elite induction has stronger effect compared with the multiple cases as supposed beforehand. Since these values represent meaningful and plausible reasoning, we can claim these indices are appropriate for evaluating the quality of the resulting Pareto front.
From Fig.9, we can observe that the set of solutions is moving towards the Pareto front in southwest direction relative to its initial location. Significant improvements can be observed from the 1st generation until the 150th generation. After the 150th generation, the front tends to spread so as to cover more frontal area until reaching a stopping criterion at the 400th generation. Generally speaking, in evolutionary optimization, there is no guarantee that the solution method will eventually provide us with true Pareto front. Hence, it is not rational to say that E-SPEA2 has reached its optimality. However, the above profile of convergence is fairly acceptable.

5. Conclusion

As same as the case of E-PSA, E-SPEA2 is proven to be superior compared with its predecessor (SPEA2). This is also confirmed quantitatively through evaluating performance metrics. Such improvements are achieved by introducing the elites that will guide the dominated solutions towards their preexisted locations. Moreover, E-SPEA2 possesses a certain adaptively in decision making when single elite is treated as the preferential point since we can easily obtain various solutions around it. This provides an effective approach for the post-optimal analysis of multi-objective optimization problems.
Although we have already applied some combinations between the weighting method or the $\varepsilon$-constraint method and PSA, we can consider a variety of combinations in advance. Through extensive investigations both in benchmark problems and applications, it is meaningful to find out the most suitable hybridization in future studies. Moreover, it is interesting to examine the effectiveness for the many-objective problems while extending the system boundary more widely.

### Table 4 Comparison of Performance Metrics

<table>
<thead>
<tr>
<th>MOP Methods</th>
<th>Metrics</th>
<th>GD</th>
<th>SP</th>
<th>DI</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-SPEA2</td>
<td>Single Elite ($\mu = 1/30$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hyper decrease in WIP</td>
<td>N/A</td>
<td>24.39</td>
<td>0.366</td>
</tr>
<tr>
<td></td>
<td>inventory (Point A)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Both objectives are at</td>
<td>N/A</td>
<td>5.20</td>
<td>0.809</td>
</tr>
<tr>
<td></td>
<td>optimal (Point B)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reduction of line stoppage</td>
<td>N/A</td>
<td>23.28</td>
<td>0.446</td>
</tr>
<tr>
<td></td>
<td>(Point C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiple Elites</td>
<td>Normal distribution</td>
<td>29.35</td>
<td>65.04</td>
<td>0.042</td>
</tr>
<tr>
<td>($\mu = 5/30$)</td>
<td>Northwest distribution</td>
<td>41.03</td>
<td>208.57</td>
<td>0.112</td>
</tr>
<tr>
<td></td>
<td>Pareto corner</td>
<td>36.46</td>
<td>43.13</td>
<td>0.227</td>
</tr>
<tr>
<td></td>
<td>Southeast distribution</td>
<td>60.28</td>
<td>22.39</td>
<td>0.146</td>
</tr>
<tr>
<td>SPEA2</td>
<td>205</td>
<td>53.62</td>
<td>N/A</td>
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<td>SA weighting method</td>
<td>Ref.</td>
<td>143.57</td>
<td>N/A</td>
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</table>

*To make use of the performance metric (GD), result from SA weighting method is treated as a true Pareto optimal set.

### List of Abbreviation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>DM</td>
<td>Decision Maker</td>
</tr>
<tr>
<td>E-MOEA</td>
<td>Elite Induced MOEA</td>
</tr>
<tr>
<td>E-SPEA2</td>
<td>Elite-induced SPEA 2</td>
</tr>
<tr>
<td>MOEA</td>
<td>Multi-Objective Evolutionary Algorithm</td>
</tr>
<tr>
<td>PSA</td>
<td>Pareto Simulated Annealing</td>
</tr>
<tr>
<td>SP</td>
<td>SPacing;</td>
</tr>
<tr>
<td>WIP</td>
<td>Work In Process</td>
</tr>
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<td>DI</td>
<td>Degree of Induction</td>
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<td>E-PSA</td>
<td>Elite-induced PSA</td>
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<td>GD</td>
<td>Generational Distance</td>
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<td>MOA</td>
<td>Multi-objective Analysis</td>
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<td>MOGA</td>
<td>Multi-Objective Genetic Algorithm</td>
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<td>SA</td>
<td>Simulated Annealing</td>
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<td>SPEA2</td>
<td>Strength Pareto Evolutionary Algorithm 2</td>
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</table>

### References


