TEHL Analysis of Rough Surface Spur Gears with Non-Newtonian Lubricants under Sudden Overloads*

Mongkolwongrojn MONGKOL** and Panichakorn JESDA**

** Department of Mechanical Engineering, Faculty of Engineering,
King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand
E-mail: kmmongko@kmitl.ac.th

Abstract

The time-dependent modified Reynolds equation, elasticity equation, and energy equation with initial conditions were formulated and solved numerically using a multi-grid multilevel with full approximation technique for an involute spur gear. In this analysis, the normal load and sudden overload are applied on either two pairs or one pair of gear teeth. The transition from two pairs to one pair and vice versa are modeled as a step variation of load. The effects of overload, surface roughness, non-Newtonian lubricant properties of the meshing gear in the region along the line of action are examined. The results show that lubricant properties and surface roughness have significant effects on film thickness, film temperature and friction coefficient for spur gears with rough surfaces. The minimum film thickness is decreased rapidly with the decrease of lubricant power law index. For gears operated at a sudden overload condition, the film temperature and friction coefficient are severely increased.

Key words: Spur Gears, TEHL, Non-Newtonian Lubricant, Surface Roughness, Sudden Overload Condition

1. Introduction

The TEHL analysis of involute spur gear under non-stationary condition is still a very complex problem. It is very hard to predict the film thickness and film temperature of lubricant in the contact region. In this case, it is not only the load that varies, but entrainment velocity and contact geometry also varies, as the gear teeth come into action. Moreover, the surface roughness of gear teeth, behavior of lubricant properties undergoing high shear stress and the load may all vary over a wide range along the line of the action. So it is very hard to get all these effects into the transient thermal elastohydrodynamic convergent solution of an involute spur gear.

The gear problem has historically been solved with different simplifications. The numerical solution of elastohydrodynamic lubrication (EHL) problems was solved by Dowson and Higginson[1]. Many numerical analyses have been obtained in the area ranging from thermoelastohydrodynamic (TEHD) lubrication problems to transient EHL problems. Wang and Cheng [2,3] made a Grubin-type analysis of involute spur gear transmissions and were able to calculate the minimum film thickness at several points along the line of action. Lee and Hamrock [4] used the Newton-Raphson method to calculate time-dependent EHL problems under low load conditions. Khonsari, Wang, and Qi[5] formulated Reynolds and energy equations for non-Newtonian liquid-solid lubricants in line contact. Hua and Khonsari [6] gave an isothermal full transient solution of involute spur
gear but did not consider the dynamic load in the model. More recently, a full isothermal transient non-Newtonian EHL solution to the Reynolds equation was given by Larsson [7]. Lubrecht, Ten Napel, and Bosma [8] showed that the multigrid algorithm is more efficient than the Newton-Raphson method in solving EHL with roughness effect. The multigrid technique has been developed to solve transient thermo-elasto-hydrodynamic lubrication (TEHL) by Osborn and Sadeghi [9]. Ai and Cheng[10] presented the formulation of the transient rough EHL problem using a multigrid technique. The results showed that surface roughness induced transient effects significantly in the pressure distribution in line contact. Youqiang Wang[11] present the formulation of the transient TEHL in spur gear problem. The results showed that the transient effect condition has significant effects to the film temperature and film thickness of lubricant in contact region. M. Mongkolwongrojn [12] showed that the non-Newtonian lubricant has significant effects to the film temperature and film thickness of lubricant in contact region under sudden load.

In the present analysis, the non-Newtonian (power law model) and sudden overload conditions are incorporated. Rigid gear teeth rough surfaces are assumed in this full transient thermal EHL analysis. Finite difference multigrid multilevel method with full approximate scheme techniques were implemented to calculate the transient TEHD lubrication with non-Newtonian lubricant properties under gear teeth carrying a sudden overload. Minimum film thickness, maximum film temperature and friction coefficient were determined at different contact points along the line of action.

Nomenclature

\[ b \]: Semi-width of Hertzian contact under load \( W_0 \), \( m \)
\[ C_{RT} \]: Transient dimensionless curvature sum, \( C_{RT} = R_X/R_0 \)
\[ C_{UT} \]: Transient dimensionless entrainment velocity, \( C_{UT} = \bar{u}/u_0 \)
\[ C_{WT} \]: Transient dimensionless load, \( C_{WT} = w'/W_0' \)
\[ D(X) \]: Dimensionless of combined surface roughness of gear and pinion
\[ E_{1/2} \]: Elastic modulus of pinion/gear, \( Pa \)
\[ E' \]: Effective elastic modulus, \( Pa \), \( 1/E' = 1/[2((1 - v_1^2)/E_1 + (1 - v_2^2)/E_2)] \)
\[ f \]: Friction coefficient of pinion
\[ h \]: Lubricant film thickness, \( m \)
\[ h_0 \]: Rigid central film thickness, \( m \)
\[ H \]: Dimensionless film thickness, \( H = h(R_0/b^2) \)
\[ H_0 \]: Dimensionless rigid central film thickness, \( H_0 = h_0(R_0/b^2) \)
\[ K \]: Constant in modified Reynold equation
\[ k \]: Thermal conductivity of lubricant, \( W/m \cdot K \)
\[ k_0 \]: Thermal conductivity of lubricant at ambient pressure, \( W/m \cdot K \)
\[ \bar{k}_p \]: Dimensionless Thermal conductivity of lubricant \( \bar{k}_p = k/k_0 \)
\[ K_{T1} \]: Constant in Energy equation
\[ K_{T2} \]: Constant in Energy equation
\[ K_{T3} \]: Constant in Energy equation
\[ m_0 \]: Apparent viscosity at the shear rate of unit, \( Pa \cdot s \)
\[ n \]: Power law index
\[ p \]: Pressure, \( Pa \)
\[ P \]: Dimensionless pressure, \( P = p/P_0 \)
\[ P_H \]: Maximum Hertzian pressure, \( Pa \), \( P_H = E'(W_0'/2n)^{1/2} \)
\[ r_a \]: Base cycle radius of pinion, \( m \)
\[ r_b \]: Base cycle radius of gear, \( m \)
\[ R_0 \]: Equivalence pitch cycle radius, \( m \), \( 1/R_0 = 1/r_a \sin(\bar{\varphi}) + 1/r_b \sin(\bar{\varphi}) \)
\[ R_1 \]: Radius of curvature of pinion teeth, \( m \)
2. Load, curvature and velocity

The force that is transmitted between two teeth in a meshing gear has a complex behaviour and it is difficult to estimate it. In this analysis, gear teeth load is assumed to act along the line of action. The gear teeth are assumed to be rigid and the dynamics of the gear are neglected. The pitch error is assumed to be very small and will not influence the load. With these assumptions, the load will be constant as long as two pair of gear teeth carry the total load. When only one pair carries the load, the gear teeth load will immediately be doubled. Fig. 1 shows the geometric parameters of an involute spur gear transmission. The contact between gear teeth at a distance $\tilde{S}$ from the pitch line in a pair of involute gear wheel having radii $R_a$ and $R_b$ and a pressure angle $\psi$ can be represented by two circular cylinders rotating with the same angular velocity $\omega_a$ and $\omega_b$ as the wheel themselves. These radii vary along the line of the action as:

\begin{align*}
R_2 & : \text{Radius of curvature of gear teeth, } m \\
R_X & : \text{Curvature sum, } m, \quad 1/R_X = 1/R_1 + 1/R_2 \\
\tilde{S} & : \text{Along Line of action, } m \\
S_0 & : \text{Slip ratio, } S_0 = (u_2 - u_1)/\bar{u} \\
t & : \text{Time, } s \\
\tilde{t} & : \text{Dimensionless time, } \tilde{t} = t(u_0/b) \\
T & : \text{Temperature, } K \\
T_1 & : \text{Surface temperature of pinion, } K \\
T_2 & : \text{Surface temperature of gear, } K \\
T_0 & : \text{Inlet temperature, } K \\
u & : \text{Film velocity, } m/s \\
u_1 & : \text{Pinion teeth surface velocity, } m/s \\
u_2 & : \text{Gear teeth surface velocity, } m/s \\
\bar{u} & : \text{Entrainment velocity, } m/s, \bar{u} = (u_2 + u_1)/2 \\
u_0 & : \text{Reference velocity, } m/s \\
u^* & : \text{Dimensionless film velocity, } u^* = u/u_0 \\
w' & : \text{Transient load, } N/m \\
w_0' & : \text{Reference load, } N/m \\
W'_0 & : \text{Dimensionless reference load, } W'_0 = w'_0/E'R_0^2 \\
x & : \text{Coordinate, } m \\
X & : \text{Dimensionless coordinate, } X = x/b \\
z & : \text{Coordinate, } m \\
Z & : \text{Dimensionless coordinate, } Z = z/h \\
Z_1 & : \text{Viscosity-Pressure index} \\
\theta & : \text{Dimensionless film temperature, } \theta = T/T_0 \\
\theta_1 & : \text{Dimensionless pinion teeth surface temperature, } \theta_1 = T_1/T_0 \\
\theta_2 & : \text{Dimensionless gear teeth surface temperature, } \theta_2 = T_2/T_0 \\
\mu & : \text{Equivalent viscosity, } Pa \cdot s \\
\bar{\mu} & : \text{Dimensionless equivalent viscosity, } \bar{\mu} = \mu/\mu_0 \\
\mu_0 & : \text{Inlet viscosity, } Pa \cdot s \\
\tau & : \text{Shear stress of lubricant, } Pa \\
\rho & : \text{Density of lubricant, } kg/m^3 \\
\rho_0 & : \text{Inlet density of lubricant, } kg/m^3 \\
\bar{\rho} & : \text{Dimensionless density of lubricant, } \bar{\rho} = \rho/\rho_0 \\
\bar{\psi} & : \text{Pressure angle, degree}
\end{align*}
The teeth both roll and slide against each other except at the pitch point, where is a pure rolling condition. The tooth surface velocity are:

\[ u_1(S) = \omega R_1(S) \]

\[ u_2(S) = \omega R_2(S) \]

3. Governing equation

The time-dependent thermo-elastohydrodynamic lubrication of rolling/sliding line contact can be solved simultaneously by using the Reynolds, elasticity, and energy equations to obtain film pressure, film temperature and film thickness distributions and traction coefficient.

3.1 Modified Reynolds equation

The relationship between shear stress and shear rate of non-Newtonian lubrication in this work can be approximated using a power-law viscosity model.

\[ \tau_{xz} = \mu^* \frac{\partial u}{\partial z} \quad \text{and} \quad \tau_{yz} = \mu^* \frac{\partial v}{\partial z} \]

Where the equivalent viscosity

\[ \mu^* = m_o \left[ \left( \frac{\partial u}{\partial z} \right)^{2} + \left( \frac{\partial v}{\partial z} \right)^{2} \right]^{(n-1)/2} \]

The dimensionless modified Reynolds equation for transient thermal line contact problem with consideration of the temperature dependency of viscosity and density across the film can be written as [12]:

\[ \frac{\partial}{\partial X} \left( \varepsilon \frac{\partial p}{\partial X} \right) = K \left( C_{uT} \frac{\partial}{\partial X} (\bar{\rho}H) + C_{uT} \frac{S_0}{2} \frac{\partial}{\partial X} \left( \bar{\rho}H \left( 1 - 2 \frac{\bar{\rho}e_0}{\bar{\rho}e_1} \right) + \frac{\partial}{\partial \bar{\rho}} (\bar{\rho}H) \right) \right) \]
Where

\[ K = \frac{u_0 \mu_0 R_0^2}{b^3 p_H} \]  
\[ \varepsilon = \hat{\rho} H^3 \left( \frac{1}{\mu_{ei}^3} - \frac{\hat{\rho}_0}{\mu_{ei}^3} \right) \]  
\[ \frac{1}{\mu_{ei}} = \int_0^{Z_i} \frac{Z}{\hat{\mu}^3} dZ \]

Where the boundary conditions are

\[ X = X_{\text{inlet}}, P = 0 \quad ; \quad X = X_{\text{exit}}, P = \frac{\partial P}{\partial X} = 0 \]  

### 3.2 The apparent viscosity equation

The apparent viscosity in power-law viscosity model needs to be included as a correction factor for viscosity-pressure-temperature[13]. The dimensionless apparent viscosity can be written as

\[ \hat{\mu}^* = \frac{m_0}{\mu_0} \left[ \frac{u_0 R_0}{b^2} \right]^{n-1} \left[ \frac{1}{H} \frac{\partial \mu}{\partial Z} \right]^{n-1} \exp\left( (\ln(\mu_0) + 9.67) \times (-1 + (1 + 5.1 \times 10^{-9} p_H)^{2.1}) - \gamma T_0(\theta - 1) \right) \] 

### 3.3 The density equation

The dimensionless density of lubricant according to Dowson and Higginson[1] obeys the following relation

\[ \hat{\rho} = \left( 1 + \frac{0.6 \times 10^{-9} p_H P}{1 + 1.7 \times 10^{-9} p_H P} \right) \left( 1 - \beta T_0(\theta - 1) \right) \]  

### 3.4 Thermal conductivity of oil film

The effect of pressure on thermal conductivity has been implemented in the thermal EHD calculation by Wang.[13]

\[ k_p = 1 + \frac{\alpha_{k,1} P_H p}{1 + \alpha_{k,2} P_H p} \]

### 3.5 The film thickness equation

The film thickness, including the deformation of surface under line contact, is given as

\[ H = H_0 + \frac{X^2}{2 C_{R'H}} + D(X) \frac{1}{\pi} \int_{X_{\text{inlet}}}^{X_{\text{exit}}} P(X') \ln|X - X'| dX' \]

Where \( D(X) \) is the dimensionless combined surface roughness of gear and pinion with random roughness distribution.

### 3.6 The load equation

The total load carrying capacity of the lubricant is due to hydrodynamic action. The dimensionless form of load balance equation is

\[ \int_{X_{\text{inlet}}}^{X_{\text{exit}}} P dX = C_{WT} \left( \frac{\pi}{2} \right) \]
3.7 Energy equation

The surface in the contact region can be simplified as semi-infinite bodied. Following Carslaw and Jaeger, the heat conduction in the surface can be analyzed by ignoring the heat conduction in the $X$ and $Y$ directions. The time-dependent dimensionless energy equation for the oil film is then given as [12]:

$$\frac{\partial^2 \theta}{\partial Z^2} = K_{T1} \left(\frac{\partial H^2}{k_p} \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial X} \right) - K_{T2} \left(\frac{\mu u}{k_p} \left(\frac{\partial u^*}{\partial t}\right)^2 \right) - K_{T3} \left(\frac{\theta H^2}{k_p} \frac{\partial P}{\partial t} + u^* \frac{\partial P}{\partial X} \right) \tag{17}$$

Where

$$K_{T1} = \frac{u_0 \rho_0 c_p b^3}{k_0 R_0^2} \tag{18}$$

$$K_{T2} = \frac{\mu \partial u_0^2}{k_0 T_0} \tag{19}$$

$$K_{T3} = \frac{\beta u_0 b^3 p_h}{k_0 R_0^2} \tag{20}$$

The boundary conditions of the energy equation are [15]

$$\theta_{1/2} = 1 \pm \frac{k_0 R_0}{\sqrt{\pi \rho p_1/2 c_{p,1/2} b^3 u_0 c_{UT}}} \left(1 - \frac{z_0}{2}\right) x_{inlet}^{x_{exit}} \int \frac{k_p}{H} \left(\frac{\partial \theta}{\partial Z}\right)_{z=0/1} dX' \tag{21}$$

$$\theta(x_{inlet}) = 1 \tag{22}$$

3.8 The friction coefficient

The friction coefficient on pinion tooth surface is defined as

$$f = \frac{\mu_0 \partial u_0 R_0}{\omega_0 b_{WT}} \int_{x_{inlet}}^{x_{exit}} \left(\frac{\bar{u}^*}{H}\right) \left(\frac{\partial u^*}{\partial Z}\right)_{z=0} dX \tag{23}$$

4. Computational procedure and numerical

The numerical solution of the governing equations described above is carried out by the pressure-temperature iteration between transient modified Reynolds, elasticity and energy equations of the thermo-elastohydrodynamic lubrication with non-Newtonian lubricants, using multi-grid with full approximation scheme technique. Modified Reynolds equation is solved on a uniform grid by the finite difference method. The multi-grid technique has been utilized to improve the convergence rate.

The coupled time-dependent modified Reynolds equation, energy equation, film thickness equation, and load balance of the two surfaces in line contact with non-Newtonian lubricants are simultaneously solved using multi-grid multi-level and Newton-Raphson techniques to obtain fast convergence. The convergence criteria of pressure, temperature, and hydrodynamic load are adopted as follows:

$$\sum_{i=0}^{N} \left| P_i^{k+1} - P_i^k \right| / \sum_{i=0}^{N} p_i^{k+1} \leq 0.0001 \tag{24}$$

$$\sum_{i=0}^{N} \left| \theta_i^{k+1} - \theta_i^k \right| / \sum_{i=0}^{N} \theta_i^{k+1} \leq 0.0001 \tag{25}$$
\[ C_{WT} \left( \frac{R}{2} \right) - \frac{X_{exit}}{X_{inlet}} \int_{X_{inlet}}^{X_{exit}} PdX \left/ C_{WT} \left( \frac{R}{2} \right) \right| \leq 0.0001 \]  \hspace{1cm} (26)

During each time interval, the modified Reynolds equation, elasticity equation and energy equation are calculated using boundary conditions and initial conditions to obtain pressure and temperature distributions. In this problem, the boundary condition \( X_{inlet} = -6.0 \) and \( X_{exit} = +2.0 \).

5. Results and discussion

The gear data and the properties of lubricant used in the analysis are given in Table 1. These gear data correspond to commercially available gearbox.

<table>
<thead>
<tr>
<th>Gear Material</th>
<th>UNB C61300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth (pinion : gear)</td>
<td>35:140</td>
</tr>
<tr>
<td>Contact ratio</td>
<td>1.786</td>
</tr>
<tr>
<td>Module, mm</td>
<td>2</td>
</tr>
<tr>
<td>Pinion speed, rpm</td>
<td>1,000</td>
</tr>
<tr>
<td>Nominal pressure angle, degree</td>
<td>20</td>
</tr>
<tr>
<td>Teeth width, mm</td>
<td>20</td>
</tr>
<tr>
<td>Transmitted power, kW</td>
<td>10.0</td>
</tr>
<tr>
<td>Elastic modulus of pinion and gear, GPa</td>
<td>117.0</td>
</tr>
<tr>
<td>Density of the teeth of pinion and gear, kg/m³</td>
<td>7950.0</td>
</tr>
<tr>
<td>Poisson ratio of the teeth of pinion and gear</td>
<td>0.28</td>
</tr>
<tr>
<td>Specific heat of pinion and gear, J/kg·K</td>
<td>736.8</td>
</tr>
<tr>
<td>Combined surface roughness amplitude ( (R_{rms}) ), µm</td>
<td>0.05, 0.10</td>
</tr>
<tr>
<td>Inlet temperature of lubricant, K</td>
<td>313.15</td>
</tr>
<tr>
<td>Inlet density of the lubricant, kg/m³</td>
<td>892.80</td>
</tr>
<tr>
<td>Inlet viscosity of the lubricant, Pa·s</td>
<td>0.195</td>
</tr>
<tr>
<td>Viscosity-Pressure index( (Z_1) )</td>
<td>0.5685</td>
</tr>
<tr>
<td>Viscosity-Temperature coefficient, K⁻¹</td>
<td>0.05763</td>
</tr>
<tr>
<td>Coefficient of thermal expansivity, K⁻¹</td>
<td>0.00074</td>
</tr>
<tr>
<td>Thermal conductivity of lubricant, W/m·K</td>
<td>0.126</td>
</tr>
<tr>
<td>Specific heat of lubricant, J/kg·K</td>
<td>1870</td>
</tr>
<tr>
<td>Power law index( (n) )</td>
<td>0.975 for pseudoplastic fluid, 1.000 for Newtonian fluid</td>
</tr>
</tbody>
</table>

Fig. 2 shows the variation of the dimensionless idealistic load, central pressure, dimensionless equivalent curvature and entrainment velocity along the line of action. The dimensionless load\( (C_{WT}) \) is 0.5 at the approach point A and suddenly increased to 1.0 at point B \((\delta = -1.10 \text{ mm})\) when the load carried by one pair of teeth at that moment. The next pair of teeth comes into action at point D \((\delta = 0.90 \text{ mm})\) and the load is suddenly decreased to 0.5 again. The amount of sliding reaches its highest level at the approach point. There is no sliding at the pitch point. The slip ratio reaches its highest level at the approach point. The entrainment velocity and the equivalent curvature have their lowest levels at that point too. That means this point might be the most critical from a lubricant film thickness point of view.

Fig. 3 shows the variation of minimum film thickness along the line of action and it
can be seen that the film thickness reaches its minimum at the approach point \( \bar{S} = -4.97 \text{ mm} \). The film thickness of the rough surface gear teeth is smaller than that for the smooth surface gear teeth. The minimum film thickness is \( 0.355 \mu m \) for smooth surface but the minimum film thickness is \( 0.286 \mu m \) and \( 0.218 \mu m \) for rough surface gear teeth with \( R_{rms} = 0.05 \mu m \) and \( R_{rms} = 0.10 \mu m \) respectively. This is because the high pressure fluctuation from asperities and the slip ratio result in higher temperature and lower lubricant viscosity. The transient effect is the most pronounced after points B and D. When the load is suddenly increased at point B \( \bar{S} = -0.90 \text{ mm} \), the minimum film thickness first increases from \( 0.725 \mu m \) to \( 0.742 \mu m \) for smooth surface, and from \( 0.727 \mu m \) to \( 0.737 \mu m \) for rough surface with \( R_{rms} = 0.05 \mu m \) and from \( 0.713 \mu m \) to \( 0.718 \mu m \), for rough surface with \( R_{rms} = 0.10 \mu m \), then decreases to \( 0.700 \mu m \) for smooth surface, and \( 0.673 \mu m \) for rough surface \( R_{rms} = 0.05 \mu m \) and \( 0.629 \mu m \) for rough surface \( R_{rms} = 0.10 \mu m \), and then increases again. It never reaches an equilibrium state until the load is halved again. When the next pair of teeth comes into action at point D \( \bar{S} = 0.40 \text{ mm} \), the load is suddenly decreased. The minimum film thickness first increases from \( 0.780 \mu m \) to \( 0.853 \mu m \) for smooth surface, \( 0.769 \mu m \) to \( 0.843 \mu m \) for rough surface \( R_{rms} = 0.05 \mu m \) and \( 0.733 \mu m \) to \( 0.819 \mu m \) for rough surface \( R_{rms} = 0.10 \mu m \), then decreases until it recovers at approximately \( 1.20 \text{ mm} \), the minimum film thickness value is \( 0.699 \mu m \) for smooth surface, the minimum film thickness are \( 0.700 \mu m \) and \( 0.724 \mu m \) for rough surface \( R_{rms} = 0.05 \mu m \) and \( R_{rms} = 0.10 \mu m \) respectively. This behavior can be understood by the mechanism of the squeeze effect.

The variation of friction coefficient on pinion tooth and oil film temperature rise for smooth surface gear teeth and for rough surface gear teeth along the line of action are presented in Fig. 4(a) and Fig. 4(b). The friction coefficient and temperature rise of oil film for the rough surface gear have higher value along the line of action than those for smooth surfaces. At the approach point (point A, \( \bar{S} = -4.97 \text{ mm} \)), the friction coefficient and maximum film temperature are \( 0.102 \) and \( 67.47 \text{ deg.C} \) for smooth surface, the friction coefficient and maximum film temperature are \( 0.119 \) and \( 81.48 \text{ deg.C} \) for rough surface \( R_{rms} = 0.05 \mu m \) and the friction coefficient and maximum film temperature are \( 0.184 \) and \( 116.47 \text{ deg.C} \) for rough surface \( R_{rms} = 0.10 \mu m \), respectively. When the line of action increases, the friction coefficient and maximum film temperature increase. The first peak value of friction coefficient occurs at \( -4.48 \text{ mm} \) and the first peak value of film temperature occurs at \( -4.00 \text{ mm} \), the friction coefficient and the maximum film temperature are \( 0.171 \) and \( 109.62 \text{ deg.C} \) for smooth surface and \( 0.183 \) and \( 179.53 \text{ deg.C} \) for rough surface \( R_{rms} = 0.05 \mu m \) and \( 0.260 \) and \( 256.17 \text{ deg.C} \) for rough surface \( R_{rms} = 0.10 \mu m \), then decreases until the load is suddenly increased. At \( \bar{S} = -0.70 \text{ mm} \), the friction coefficient and maximum film temperature of smooth surface and rough surface reach the
peak values 0.154, 48.00 deg.C, 0.178, 113.24 deg.C, 0.411 and 268.36 deg.C, respectively, then they decrease to the minimum values at the pitch point. The friction coefficient is approximately zero and the maximum temperature is near the inlet temperature because there is no sliding. It can be seen that the highest temperature rise occurs near the approach point, which is caused by the large slip ratio. Just after the load increase occurs at point B, the friction coefficient and temperature rise reach high levels because of large load, even though the slip ratio is small. When the line of action shifts from pitch point, film temperature and friction coefficient rise up again. At $\tilde{S} = +0.30 \text{ mm}$, the maximum values are 0.083 and 46.11 deg.C for smooth surface, 0.099 and 49.03 deg.C for rough surface ($R_{\text{rms}} = 0.05 \mu\text{m}$) and 0.358 and 187.94 deg.C for rough surface ($R_{\text{rms}} = 0.10 \mu\text{m}$), respectively. When the next pair of teeth comes into action at point D, the film temperature and friction coefficient suddenly decrease but they increase again when the contact moves forward along the line of action. The maximum temperature rise at the approach point is higher than that of the recess point. That is because the slip ratio of the approach point is larger than those of the recess point. The friction coefficient and maximum film temperature of rough surface gear teeth increased more rapidly than the value of smooth surface gear teeth when load is suddenly increased at point B and suddenly decreased at point D because the film pressure and film viscosity were suddenly increased from the effects of asperities.

Fig. 3 Variation of minimum film thickness along the line of action when lubricant is a pseudoplastic fluid.

Fig. 5 shows the variation of minimum film thickness, maximum film temperature and friction coefficient along the line of action for various lubricant properties (Pseudoplastic fluid ($n=0.975$), Newtonian fluid ($n=1.0$)) for rough surface with $R_{\text{rms}} = 0.05 \mu\text{m}$. It can be seen that the minimum film thickness was dependent on power law index value. The minimum film thickness was rapidly decreased with the decrease of the lubricant power law index value; the minimum film thickness of pseudoplastic fluid lubricant is lower than the minimum film thickness for Newtonian fluid along the line of contact as show in Fig. 5(a). At approach point, the minimum film thickness is $0.286 \mu\text{m}$ for pseudoplastic fluid and it is $0.289 \mu\text{m}$ for Newtonian fluid. When the load is suddenly increased at point B, the film thickness decreases to $0.673 \mu\text{m}$ for pseudoplastic fluid and it is $0.855 \mu\text{m}$ for Newtonian fluid. When the load is suddenly decrease at point D, the film thickness increases to $0.844 \mu\text{m}$ for pseudoplastic fluid and it is $1.054 \mu\text{m}$ for Newtonian fluid.

The maximum film temperature and friction coefficient of pseudoplastic fluid and Newtonian fluid have similar variation along the line of action as shown in Fig. 5(b) and Fig. 5(c). At the point close to the approach point (point A, $\tilde{S} = -4.28 \text{ mm}$), the friction coefficient and maximum film temperature are 0.183 and 179.53 deg.C for pseudoplastic
fluid and 0.178 and 149.35 deg. C for Newtonian fluid. When load is suddenly increased to double at point B, the maximum film temperature and friction coefficient rapidly increase. The friction coefficient and maximum film temperature increase to 0.178 and 113.24 deg. C for pseudoplastic fluid and 0.179 and 174.54 deg. C for Newtonian fluid; then they rapidly decrease before rapidly increasing again. The peak value of friction coefficient and maximum film temperature are 0.099 and 49.02 deg. C for pseudoplastic fluid and 0.098 and 49.81 deg. C for Newtonian fluid due to the rising of film temperature, decreasing of film viscosity and increasing of lubricant shear stress. Therefore, the friction coefficient of the lubricant film is increased. At pitch point, the friction coefficient is approximately zero and the maximum temperature is near the inlet temperature.

(a) Friction coefficient on pinion surface when lubricant is pseudoplastic fluid.

(b) Maximum temperature rise when lubricant is pseudoplastic fluid.

Fig. 4 Variation of friction coefficient on pinion surface and maximum temperature rise along the line of action

(a) Friction coefficient on pinion surface, (b) maximum temperature rise
Fig. 5 Variation of minimum film thickness, maximum temperature rise and friction coefficient on pinion surface along the line of action when lubricant properties are changed
(a) Minimum film thickness  (b) Maximum temperature rise  and (c) Friction coefficient on pinion.
During operation of spur gears, at the approach point where the gear teeth are in the first contact and at the point of the load is suddenly carried by one pair of teeth. If the load of gear teeth is increased more than the normal operating condition due to a sudden overload or impact of gear teeth as shown in Fig. 6.

For illustration purposes in this paper, the characteristics of spur gears were investigated when subjected to a sudden overload of 25% of the normal operating load. During sudden overload or impact load at the approach point, the minimum film thickness is close to the normal operating condition because at the approach point the minimum film thickness depends on the mechanism of the squeeze effect; the minimum film thickness at the approach point is 0.286 $\mu m$ for all loads conditions. When load is suddenly increased at point B, the minimum film thickness of impact load at pitch point is smaller than normal operating condition; they are 0.673 $\mu m$ for normal operating condition or for impact load at inlet point; but for the operating condition with impact load at the pitch point, the minimum film thickness is 0.651 $\mu m$. Near the pitch point, the minimum film thickness for impact load at pitch point is greater than the minimum film thickness for normal operating condition; the peak minimum film thickness is 0.780 $\mu m$ which is close to normal operating condition. When the load is suddenly decreased at point D, the minimum film thickness increase to 0.843 $\mu m$ for normal operating condition and it is 0.817 $\mu m$ for impact load at pitch point, as shown in Fig 7(a).

The maximum film temperature and friction coefficient for normal operating condition are compared with the sudden overload at inlet point operating condition, as shown in Fig 7(b) and Fig 7(c). At the approach point, the friction coefficient and maximum film temperature with sudden overload at the inlet point is greater than the normal operating condition, they are 0.146 and 94.50 deg.C for sudden overload at the inlet point operating condition and they are 0.119 and 81.48 deg.C for normal operating condition. When the contact moves forward along the line of action, the friction coefficient and maximum film temperature rapidly increase, the peak value of maximum film temperature is 281.18 deg.C for impact load at inlet point operating condition and 179.53 deg.C for normal operating condition. Then it decreases until the load is suddenly increased. When load is suddenly increased at point B, the friction coefficient and maximum film temperature rapidly increase; the peak values are 0.320 and 200.85 deg.C for sudden overload at pitch point and are 0.179 and 113.24 deg.C for normal operating condition, then the friction coefficient decrease to approximately zero and the film temperature also decrease to inlet temperature at the pitch point and rapidly increasing again. The friction coefficient and maximum film temperature are 0.208 and 71.73 deg.C for a sudden overload at the pitch point and 0.099 and
49.02 deg.C for normal operating condition because the film pressure and film viscosity were increased when load increased. When load is suddenly decreased at point D, the friction coefficient and maximum film temperature are rapidly decreased before rising up again.

Fig. 7 Variation of minimum film thickness, maximum temperature rise and friction coefficient on pinion surface along the line of action when sudden overload occurs at approach point or at the point where the load being to be carried by one pair of teeth.

(a) Minimum film thickness, (b) Maximum temperature rise and (c) Friction coefficient on pinion.
In sudden overload at the point where the load starts being carried by one pair of teeth, the minimum film thickness is decreased, then it increases more than that for the normal operating condition due to the squeeze effect as shown in Fig 7(a). The oil film thickness and oil film temperature under sudden overload obtained from this present calculation are compared with those from the experiment presented in [16]. The results of the present scheme are in good agreement with the results from the experiment on the measured oil film thickness and the operational temperature under increase in load conditions. The oil film temperature increases with increasing in load but the oil film thickness decreases with the increasing in load. Therefore, oil film temperature and oil film thickness are the critical parameters in gear design. The maximum film temperature and friction coefficient are increased significantly as shown in Fig. 7(b) and Fig. 7(c) because near the pitch point, the motion of gear teeth is nearly pure rolling; then the film pressure and film viscosity are primarily dependent on gear tooth load being carried. That means this operating condition might be the most critical.

6. Conclusion

The time dependent Reynolds equation and energy equation were solved to obtain minimum film thickness, maximum film pressure, friction coefficient and film temperature rise in the contact region between two involute spur gears teeth with non-Newtonian fluid. The effects of rough surfaces gear teeth and two different lubricants are studied. The load is kept constant as long as two pair of teeth carried the total load and it is doubled when only one pair is in action. Normal load condition and sudden overload condition were examined. The following can be concluded as:

1. Considering the rough surface effect, the minimum film thickness is thinner than that for smooth surface gear teeth. The maximum film temperature and friction coefficient are seriously greater than for the smooth surfaces gear teeth. The gear teeth with rough surfaces could be dangerous due to high film temperature. Therefore, the surface roughness of gear teeth is significantly affected for gear design application.

2. Considering the effect of the power law index, the minimum film thickness are clearly depended on power law index value, the pseudoplastic fluid lubricant give lowest values of film thickness, film temperature and friction coefficients.

3. For spur gears under sudden overload near the pitch point, the film temperature and friction coefficient are increased significantly, that means this operating condition might be the most critical.

Acknowledgment

The authors wish to acknowledge Thailand Research Fund for the financial support under Grant No. BRG-5180019.

References

(1) Dowson D., Higginson G. R., Elastohydrodynamic lubrication, the fundamentals of roller and gear lubrication., Oxford: Pergamon; 1966.


