Effect of Underlay Stiffness on Cutting Profile of Polycarbonate Sheet during Wedge Indentation Process*

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Abstract
This paper reports on the cutting characteristics of a polycarbonate (PC) sheet stacked on a flexible underlay by a wedge indentation. In order to investigate the effect of underlay stiffness on sheared profile of PC sheet, the stiffness ratio \( k_r \) in the thickness direction was varied. Indentation of a 42° center bevel blade into a 0.5mm thickness PC sheet that was stacked on the flexible underlay was carried out experimentally and numerically. Deformation profile of wedged PC sheet was observed by a CCD camera in order to reveal the effect of underlay on the cutting performance of the PC sheet. On the experimental works, it was found that the underlay mechanical properties affected the cutting load response and deformation features of the PC sheet. To discuss the effect of underlay stiffness on the deformation profile of the PC sheet, a finite element method (FEM) analysis with elasto-plastic model was conducted. Through the experiment and FEM simulation, it was revealed that the deformation profile of wedged PC sheet was remarkably related to the bent-up angle of the PC sheet, which was mainly caused by sinking and lateral elongation of the underlay. Moreover, in order to obtain a smart profile of the PC sheet, the stiffness ratio \( k_r \) must be chosen in a suitable range.

Key words: Shearing, Elasto-Plastic FEM, Wedge Indentation, Friction, Flexible Underlay

1. Introduction

A cutting method with a center bevel blade into a sheet material on a counter plate is widely used in many packaging or printing industries for converting of paperboard, labels, laminated resin sheets, ductile metal film and other similar materials\(^{(3),(4)}\). The cutting method is recently noticed in order to cut any shapes of precision parts for a lead-frame, an insulation thin film on LSI chip. However, several problems that affect to the product quality and cutting possibility of thin sheet are caused by the variation of blade tip such as crushing or abrasion of the wedge profile during the cutting process.

Numerical and experimental researches to understand the deformation characteristics of a center bevel blade during the cutting process of a white-coated paperboard have been reported in recent years \(^{(3),(4),(5)}\). It was found that the cutting blade tip became trapezoidal (as a crushed form) when the cutting blade tip contacted to the counter plate, and the cutting tip shape affects the breaking characteristics and cutting load response of the paperboard. Murayama et al.\(^{(6),(7)}\) have reported the cutting mechanism of a 42° wedge indentation into an aluminum sheet with the thickness of 0.4mm by using a trapezoidal cutting blade.
imitated as a crushed tip. In that study, a couple of separation modes were estimated using the ratio of blade tip thickness $w$ and the worksheet thickness $t$. In case of $w/t<0.23$, the second mode necking occurred and the string-like burrs were generated beneath the blade tip. Chaijit et al. have investigated the cutting processability of a trapezoidal blade on the aluminum worksheet of 0.4mm for $w/t>0.23$. The separation limit of worksheet was confirmed as exceeded to $w/t=0.28$ and when $w/t>0.3$, the wedge surface was detached from the deformed worksheet and the blade tip mainly pushed the worksheet in an upsetting mode without separation. Several research works were based on a hard counter plate without any flexible underlay. In that reports, a cutting mechanism of trapezoidal blade was mainly discussed with respect to $w/t<0.3$ ($t=0.4mm$), whereas the thinner sheets, that is less than 15μm thickness, are often required to cut off in the pattern die-cutting of resin sheets or thin metal sheets. As the blade tip thickness $w$ is empirically larger than 5μm, it is generally difficult to maintain the blade tip in keen without any damage. Unsuccessful cutting of a thin sheet seems to be caused by the blade tip crushing, the abrasion and the un-uniformity of tip pressure distribution with a hard counter plate. This situation indicates that the cutting mechanism of a thin sheet on a flexible underlay becomes interesting, because to use a flexible underlay enables to keep keen the cutting blade tip. Such a combination of a keen wedge blade and a flexible underlay is supposed to be empirically used in several fields.

Pelletie et al. has studied determining the mechanical properties of thin films on substrate with a rigid indenter loaded for a nano-indentation test. This report is useful for understanding the influence of the Young’s modulus ratio and the yielding stress ratio of a worksheet with a substrate. However, since this model was based on a half infinite thickness of the substrate, it is insufficient for explaining the cutting deformation of a thick worksheet mounted on a finite thickness underlay. Chaijit et al. has also studied the effect of underlay rigidity on an Aluminum foil cutting, which was relatively thinner than the underlay. There are not almost any studies about the cutting mechanism of a thick resin worksheet on a flexible underlay during wedge shearing process. Nagasawa et al. has reported the contact stability and the cutting characteristics of a polycarbonate worksheet stacked on a polycarbonate underlay. However, that study did not discuss about the effect of the underlay stiffness.

In this paper, therefore, the indentation of a center bevel blade which had the tip thickness $w=1\sim2\mu m$ into a 0.5mm thickness polycarbonate (PC) sheet that stacked on an underlay was analyzed experimentally and numerically in order to reveal the deformation behaviors of the PC sheet stacked on arbitrary flexible underlay.

### 2. Analysis condition

#### 2.1 Experimental method and specimens

For a cutting experiment, a polycarbonate (PC, NF2000) sheet of $t_S=0.5mm$ thickness was used as a worksheet material. Specimen was a rectangle sheet with $l=40mm$ length and $b=20mm$ width. As the PC sheet was initially coated with a water-soluble glue layer which was used for attaching a masking film, all the specimens of PC sheet were sufficiently washed and dried before cutting. Five kinds of underlay material were chosen: (1) a copper plate “CU” with thickness $t_U=0.3mm$, (2) a polycarbonate sheet “PC” with $t_U=0.5mm$, (3) a hard rubber sheet “RS” of $t_U=0.81mm$, (4) a laminated rubber mat “RCM” with $t_U=2.0mm$ and (5) a hard foam polypropylene sheet “HFP” with $t_U=2.1mm$. Figure 1 shows a schematic of experimental apparatus and specimen configuration. Measurements were carried out 20 times for each case. On the experimental apparatus (a compression testing equipment), the upper crosshead had a cutting blade mounted in a load cell with the maximum load 10 kN. Attitude of the blade was vertical to the worksheet. Each specimen of the PC worksheet was attached to the underlay, and then it was placed onto a stainless steel counter plate (SUS630 with the hardness of 510VHN (100gf, 10s), the surface roughness of...
$R_s=0.2\,\mu m$) fixed on the lower crosshead. The center bevel blade, made from cemented carbide (W-C-Co, FM10K), had the length of $L=30\,mm$, the height of $H=10\,mm$, the thickness of $S=0.9\,mm$, and the initial tip thickness of $w=1\,\mu m$ in average. The bevel face of the blade had a surface roughness of $R_a=0.014\,\mu m$ ($0.019 \sim 0.008\,\mu m$). The apex angle $\alpha$ of the blade was $42^\circ$. The upper crosshead moved downward with a velocity of $V=0.05\,mm\cdot s^{-1}$.

![Fig.1 Schematic of experiment apparatus](image1)

Fig.1 Schematic of experiment apparatus

The compressive stress-strain relationship in the thickness direction TD (out-of-plane) of the RS, RCM and HFP underlay was measured based on JIS-K7220 for $\varepsilon \approx 0.5$ with $V=0.0017\,mm\cdot s^{-1}$. Where, the $\varepsilon$ is the true strain in TD. The mechanical properties of the PC worksheet and the underlay sheets are shown in Table 1 and 2, respectively.

**Table 1 In-plane mechanical properties of polycarbonate (PC) and copper (CU) plate**

<table>
<thead>
<tr>
<th>Sheet symbol</th>
<th>Thickness $t_s, t_U$/ mm</th>
<th>Young’s modulus $E$/ GPa</th>
<th>Yield strength $\sigma_Y$/ MPa</th>
<th>Tensile strength $\sigma_B$/ MPa</th>
<th>Breaking strain $\varepsilon_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC (MD)</td>
<td>0.5</td>
<td>2.65</td>
<td>61.8</td>
<td>153.7</td>
<td>0.57</td>
</tr>
<tr>
<td>Cu</td>
<td>0.3</td>
<td>9.27</td>
<td>210</td>
<td>225</td>
<td>0.12</td>
</tr>
</tbody>
</table>

**Table 2 Mechanical properties of other underlay materials**

<table>
<thead>
<tr>
<th>Sheet symbol</th>
<th>Thickness $t_s$/ mm</th>
<th>$E_{IPD(tens)}$/ MPa $(3.3x10^{-4} / s^{-1})$</th>
<th>$E_{TD(comp)}$/ MPa $(2x10^{-4} / s^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCM</td>
<td>2.04</td>
<td>423.0</td>
<td>1246</td>
</tr>
<tr>
<td>RS</td>
<td>0.81</td>
<td>27.0</td>
<td>915</td>
</tr>
<tr>
<td>HFP</td>
<td>2.08</td>
<td>62.0</td>
<td>21.0</td>
</tr>
</tbody>
</table>

$E_{IPD(tens)}$: In-plane direction, tensile Young’s modulus  
$E_{TD(comp)}$: Thickness direction, Young’s modulus

**Table 3 Friction coefficients of underlays (at 4.9 kPa)**

<table>
<thead>
<tr>
<th>PC : $\mu_U$</th>
<th>RS : $\mu_U$</th>
<th>RCM : $\mu_U$</th>
<th>HFP : $\mu_U$</th>
<th>CU : $\mu_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.28</td>
<td>0.27</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>0.03–0.08</td>
<td>0.25–0.32</td>
<td>0.27–0.29</td>
<td>0.08–0.12</td>
<td>0.03–0.10</td>
</tr>
</tbody>
</table>

Steel counter plate: $\mu_P$

<table>
<thead>
<tr>
<th>PC : $\mu_P$</th>
<th>RS : $\mu_P$</th>
<th>RCM : $\mu_P$</th>
<th>HFP : $\mu_P$</th>
<th>CU : $\mu_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07</td>
<td>1.02</td>
<td>0.25</td>
<td>0.13</td>
<td>0.05</td>
</tr>
<tr>
<td>0.06–0.08</td>
<td>0.96–1.07</td>
<td>0.23–0.26</td>
<td>0.10–0.17</td>
<td>0.02–0.07</td>
</tr>
</tbody>
</table>

The applied pushing force $F$ was measured by the load cell. The indentation displacement of the blade into the PC worksheet $c$ was measured as the upper crosshead displacement, and $f (=F/b)$ $kN/m$ was the line force applied in vertical to the blade. To ensure non-lubricated contact between the PC worksheet and the blade, the tool surface was washed with alcohol before indenting. The three kinds of friction coefficient $\mu_C$, $\mu_U$ and $\mu_P$ were experimentally measured by the horizontal method based on JIS-K7125. The $\mu_C$ between the blade and the PC worksheet was 0.06 in average ($0.05–0.08$)\(^{12}\). The $\mu_U$
between the PC worksheet and the underlay and the \( \mu_p \) between the underlay and the counter plate were shown in Table 3. Here, they were measured with the applied contact pressure \( p = 4.9 \) kPa among the blade, the PC worksheet and the counter plate, respectively. When the applied contact pressure was increased with the PC worksheet, those friction coefficients were empirically increased \(^{11}\).

In this experimental investigation, the inclined angle \( \beta \), the elevation (bottom-up) angle \( \gamma \), the necked elevation height \( \eta_n \), the necked elevation length (a half width of necking) \( b_n \) and the bent-up angle \( \theta_{BU} \) at the peak line force position as shown in Fig.2 were measured from CCD video photographs. Over here, \( \theta_{BU} \) was evaluated as a gradient of distance \( 2t_s \) on the upper surface of the PC worksheet.

### 2.2 FEM simulation

A general purpose finite element code, MSC.MARC 2010.1.0, was employed for simulating the indentation of a 42 degree center bevel blade into the PC worksheet stacked on a flexible underlay. A non-linear analysis using the updated Lagrange procedure and a large strain analysis were considered. The two-dimensional symmetric model as shown in Fig.3 was constructed in a half-length of the worksheet and the underlay by using four-node plain strain quadrilateral element type. The worksheet and the underlay were assumed to be deformable while the blade and the counter plate were assumed to be rigid. The thickness of worksheet \( t_S = 0.5 \)mm was fixed and that of underlay \( t_U = 0.05, 0.1, 0.5 \) and \( 1.0 \)mm. Here, \( t_U = 0.5 \)mm was mainly used as the referenced condition. The blade indentation depth \( d \) was defined with reference to the upper side surface of the worksheet mounted on the underlay. When we discuss with the blade indentation compared to the experimental, a certain elastic gap must be considered \(^{12}\), as shown in Eq.(1). Here, \( k_v \) is the equivalent spring constant of the experimental apparatus, and \( c \) is the indentation displacement of blade.

\[
d/t = c/t - f/k_v \tag{1}
\]

The half-length of worksheet and underlay was assumed to be 20mm, which was same as the experiment. In order to successfully calculate the cutting simulation, the worksheet and the underlay beneath the blade tip were constructed with fine meshes in the lateral span of 1 and 3 mm, respectively. The minimum, initial side length of elements with the worksheet and underlay was 5 and 10 \( \mu \)m, respectively. A large side-length of elements was used for the area far from the center of the blade. Those fine and large-subdivided areas (meshes) were attached with each other by using the GLUE CONTACT function.

### Table 4 Finite element analysis conditions

<table>
<thead>
<tr>
<th></th>
<th>Worksheet</th>
<th>Underlay sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object type</td>
<td>Elasto-plastic with linear</td>
<td>Elasto-plastic with linear</td>
</tr>
<tr>
<td></td>
<td>work hardening</td>
<td>work hardening</td>
</tr>
<tr>
<td>Young's modulus ( /\text{GPa} )</td>
<td>( E_S = 2.65 )</td>
<td>( E_U = 0.1325, 0.265, 2.65 ) and 26.5</td>
</tr>
<tr>
<td>Thickness ( /\text{mm} )</td>
<td>( t_S = 0.5 )</td>
<td>( t_U = 0.05, 0.1, 0.5 ) and 5.0</td>
</tr>
<tr>
<td>Bevel apex angle ( /\text{degree} )</td>
<td>( \alpha = 42 )</td>
<td></td>
</tr>
<tr>
<td>Tip thickness ( /\mu \text{m} ): Friction coefficient:</td>
<td>( w = 2.0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \mu_C = 0.23 ) and ( \mu_p = 0.05 ) (constant)</td>
<td>( \mu_U = 0.05, 0.2 ) and 0.4</td>
</tr>
</tbody>
</table>

\( \mu_C \) : friction coeff. of blade, \( \mu_U \) : friction coeff. of underlay, \( \mu_P \) : Friction coeff. of counter plate

During the blade indentation, a few elements are largely deformed beneath the blade tip, and consequently; the calculation tends to be fault due to crushing of each element. The center-side elements of the worksheet were automatically re-generated in the deformed
region using an automatic meshing function with the ADVANCING FRONT QUAD in order to overcome the crushing of any elements. The re-meshing was performed when i) the inner angle of element was greater than 175° or less than 5° and/or ii) the strain change of element was greater than 0.4. In this simulation, any crack and fracture models were not considered. Because any cracks were not detected in the experimental observation using a CCD camera.

![Simulation model with respect to initial profile of worksheet and underlay.](image)

The coulomb tan^1 friction model \(^{(12)}\) with the relative-slipping velocity threshold of 0.01 was assumed for each contact interface. Coefficients of the friction were assumed as Table 4. The material properties of the worksheet and underlay were assumed to be isotropic elasto-plastic with work-hardening. The plastic flow curve of the worksheet was referred from a uni-axial tensile test of PC specimens \(^{(12)}\). The plastic behavior of underlay was also assumed to be same as the worksheet.

In order to discuss the effect of underlay stiffness in TD on the sheared profile of wedged worksheet, several conditions were assumed as shown in Table 4. For estimating the friction state of the blade surface, the friction coefficient \(\mu_c\) was determined after fitting the gradient of simulated load response in the experiment for a shallow indentation of the blade to the worksheet \((d/t_s<0.3)\). The measured value of \(\mu_c = 0.06\) did not match with the load response at all. According to the privilege investigation, the value of \(\mu_p\) did not affect the load response at all. Since \(\mu_p\) may be arbitrary in this simulation, it was assumed to be 0.05 (slippery state). Values of \(\mu_u\) were fairly sensitive to determine the load response. Effect of \(\mu_u\) variation on cutting load response will be discussed in section 3.2.

In order to investigate the deformation and sheared profile of the wedged worksheet, the bent-up angle \(\theta_{BU}\), which was similar to Fig.2 (b) but measured at the lower side of the worksheet, was measured from the peak point of simulated load response. The inclined angle\(\beta\), the elevation angle\(\gamma\), the necked elevation height\(\eta_n\) and the necked elevation length\(b_n\) were also measured. Those parameters were measured at the near separation stage and the unloading condition.

3. Results and Discussion

3.1 Cutting load response and deformation of worksheet on experiment

In order to discuss the effect of underlay stiffness in the following sections, we introduce the stiffness of TD \(k_l = E_U/t_U\), \(k_S = E_S/t_S\) and the stiffness ratio of \(k_r = k_U/k_S\) expressed as Eq.(2). Here, \(E_{TD,\text{comp}}\) (Table 2) was used as \(E_U\) for RS, RCM and HFP underlays, while \(E\) shown in Table 1 was used for \(E_U\) of CU and PC underlays.

\[
k_r = (E_U/E_S)/(t_U/t_S)
\]

Eq.(2)

Figure 4 shows the experimental relationship between cutting line force \(f\) kN/m and blade indentation displacement \(c\) mm on five kinds of underlay material. These \(f-c\) curves were representing the cutting load response of worksheet that mounted on a CU, PC, RS,
RCM and HFP underlay. The symbols PC-CU, PC-PC, PC-RS, PC-RCM, PC-HFP denote the PC worksheet stacked on the corresponding underlay, respectively. The stiffness ratio $k_r$ of PC-CU, PC-PC, PC-RS, PC-RCM and PC-HFP were calculated as 61.8, 1.0, 0.225, 0.123 and 0.002, respectively.

Figure 4 showed that the break down (cutting off) position of $c/t_s$ was postponed when $k_r$ was decreased for $k_r < 0.1$, while it was relatively fast for $k_r > 0.1$.

Regarding the friction coefficients, the estimation of $\mu_C$ was determined from experimental result as shown in Fig.4. In the case of PC worksheet stacked on PC underlay, the value of $\mu_C = 0.23$ appeared to be suitable for comparing the FEM simulation on the cutting load response with the experimental result.

Figure 5 shows the peak position $c_{\text{peak}}/t_s$, the breakdown position $c_{\text{break}}/t_s$ and the peak cutting line force $f_{\text{peak}}$ with respect to the stiffness ratio $k_r$. From this figure, there was a minimum condition of $f_{\text{peak}}$, $c_{\text{peak}}/t_s$ and $c_{\text{break}}/t_s$ at $k_r \approx 1$. Through this result, it was confirmed that the cutting load response behavior remarkably depended on the underlay stiffness $E_U/t_U$. For the experimental relationships between $\theta_{BU}$, $\beta$, $\eta_n$ and $b_n$, they are discussed in the next section.

Figure 6 shows several representative CCD side views of a sheared section of the PC worksheet stacked on the five underlay materials. From Fig.4 and 6, the following features can be detected. Compared with PC-CU, the breaking down position of PC-PC was roughly 10% faster. The breaking down position of PC-RS was roughly 12% later and that of PC-RCM was 24% later, respectively. In case of HFP, the underlay was simply sunk for $c/t_s < 2$ without any large indentation depth of the blade into the PC worksheet. Namely, the PC worksheet was in a state of floating phenomena.
For the PC-CU, the PC-PC and the PC-RS combination, the load response in the first stage \((f < f_{\text{max}})\) was almost same tendency, while the tendency of the PC-RCM combination was different from those. As the RCM was a laminated composite which had a thick-coated layer of a hard resin and its substrate was relatively mild, the wedge indentation resistance seemed to be different from the uniform compressive resistance which was shown in Table2. The RCM's gradient of \((\partial f / \partial (c/t_s))\) was not so different from the PC-RS's gradient for \(c/t_s < 0.5\). Comparing the load response of PC-PC with other cases, it seems that several mechanical parameters such as the Young's modulus, the out-of-plane bending rigidity, the stiffness in TD and the friction affect the cutting processability of PC worksheet.

3.2 Effect of Young's modulus of underlay

In order to discuss the effect of underlay stiffness on the FEM simulation, the Young's modulus ratio \(E_U/E_S\) was chosen as 0.05, 0.1, 1, and 10. Here, \(E_U/E_S\) is equal to \(k_r\) when \(t_U = t_S = 0.5\) mm. The friction coefficients \(\mu_U\), \(\mu_P\) and \(\mu_C\) were assumed to be 0.23, 0.2 and 0.05, respectively.

Figure 7 shows the deformation profile of wedged worksheet and underlay with respect to \(k_r\) for \(d/t_s > 0.6\). When the blade was indented to the worksheet, in case of \(k_r \leq 0.1\), a little camber of the worksheet and a slight dent of underlay were detected for \(d/t_s = 0.7\), as shown in Fig.7 (a) and (b). In case of \(k_r \geq 1\) as shown in Fig.7 (c) and (d), the worksheet was necked for \(d/t_s > 0.6\) without any apparent camber of worksheet, and also a sinking deformation of underlay was not observed.

Figure 8 shows the relationship between cutting line force and the normalized indentation depth of blade. When \(k_r \geq 1\), the load response was not almost affected by \(k_r\). When \(k_r \leq 0.1\), the gradient of cutting line force \((\partial f / \partial (d/t_s))\) in the early stage \((d/t_s < 0.3)\) increased with \(k_r\) due to the stiffness variation of underlay in the thickness direction (TD). Through this result, it was confirmed that the variation of gradient \((\partial f / \partial (d/t_s))\) at the early stage was affected by \(k_r\) \((t_U = t_S)\) when \(k_r\) was less than a certain value.
From the peak load point in Fig. 4, the experimental blade displacement $c_{\text{peak}}/t_S$ can be converted to the corresponded indentation depth $d_{\text{peak}}/t_S$ by using Eq. (1). The peak position $d_{\text{peak}}/t_S$ was roughly estimated as 10% shifted to the left side from $c_{\text{peak}}/t_S$ (becomes a small number). As the result, it was found that the experimental peak position almost matched to the simulation. The simulated gradient in case of $k_r \leq 1$ in the early stage ($\partial f/d/\partial (d/t_S)=63.6 \, \text{kN} \cdot \text{m}^{-1}$) fairly matched to the experimental results of PC-PC ($\partial f/d/\partial (d/t_S)=69.5 \, \text{kN} \cdot \text{m}^{-1}$) and PC-CU ($\partial f/d/\partial (d/t_S)=68.6 \, \text{kN} \cdot \text{m}^{-1}$). However, the experimental breaking down position and the peak load response of PC-CU ($c_{\text{peak}}/t_S=1.01$, $f_{\text{peak}}=26.6 \, \text{kN} \cdot \text{m}^{-1}$) were not similar to the simulation result ($c_{\text{peak}}/t_S=0.78$, $f_{\text{peak}}=25.4 \, \text{kN} \cdot \text{m}^{-1}$) when $k_r > 1$. This mismatching seems to be caused by the difference of friction at the worksheet/underlay interface. In order to discuss the effect of friction at the worksheet/underlay interface, the cutting load response was shown in Fig. 9 by varying $\mu_U$. Over here, other friction coefficients $\mu_C$, $\mu_P$ were assumed to be 0.23, 0.05 as shown in Table 4. When $\mu_U$ was increased, the peak load $f_{\text{peak}}$ was increased, while the peak position $d_{\text{peak}}/t_S$ and the breaking down position $d_{\text{break}}/t_S$ were postponed. Seeing the simulated load response in case of $\mu_U=0.4$, the simulation result was fairly similar to the experimental result of PC-Cu. Hence, it seemed that the difference of cutting load response between the PC-PC and PC-Cu cases was caused by the difference of friction state at the worksheet/underlay interface. A slippery state of the contact surface superiors to cut off the PC worksheet successfully.

Next, the effect of the stiffness ratio $k_r$ on $\theta_{BU}$, $\beta$, $\gamma$, $\eta_n$ and $b_n$ was discussed. Figure 10 shows the effect of $k_r$ on $\theta_{BU}$. As shown in Fig. 10, it was $1-2^\circ$ for $k_r > 1$, while it was approximated with Eq. (3) for $0.005 < k_r < 1$ in case of the experiment. Figure 11 shows the effect of $k_r$ on the profile parameters $\beta$, $\gamma$, $\eta_n$ and $b_n$, while Fig. 12 shows the profile parameters $\beta$, $\gamma$, $\eta_n$ and $b_n$ with respect to $\theta_{BU}$. It was found that the correlations between those profile parameters and $\theta_{BU}$ were linearly related. The correlations were approximated
by Eq.(4) ~ (7) for $0<\theta_{BU}<35^\circ$.

From the experimental results as shown in Fig.11 and Fig.12, it was confirmed that the sheared profile of wedged worksheet was remarkably affected by the stiffness ratio $k_r$.

$$\theta_{BU} = 1.854(k_r)^{-0.482} \quad \text{(for } 0.005<k_r\leq1) \quad \text{Eq.(3)}$$

From the simulated result as shown in Fig.10, the relationship between $\theta_{BU}$ and $k_r$ was expressed by Eq.(8) for $0.05<k_r\leq1$. Seeing Fig.7, Eq.(8) and Fig.10, when $k_r$ becomes a small number, it was found that sinking of the underlay remarkably contributes to increase $\theta_{BU}$. From Fig.11, it showed that the simulated profile parameters $\beta$, $\gamma$, $b_n/t_S$, $\eta_n/t_S$ were close to the experimental result, although the simulated range of $k_r$ ($t_U=t_S$) was restricted to $0.05<k_r<10$. These results confirmed that the $k_r$ affected the sheared profile of the wedged worksheet. The profile parameters were linearly approximated with $\theta_{BU}$ in Eq.(9) ~ (12), as shown in Fig.12.

$$\theta_{BU} = 0.959(k_r)^{-0.427} \quad \text{(for } 0.05<k_r\leq1) \quad \text{Eq.(8)}$$

$$\beta = 0.80\theta_{BU} + 13.8 \quad \text{Eq.(4)}$$

$$\gamma = 0.41\theta_{BU} + 8.4 \quad \text{Eq.(5)}$$

$$b_n/t_S = 0.0011\theta_{BU} + 0.61 \quad \text{Eq.(6)}$$

$$\eta_n/t_S = 0.0073\theta_{BU} + 0.09 \quad \text{Eq.(7)}$$

From the simulated result as shown in Fig.10, the relationship between $\theta_{BU}$ and $k_r$ was expressed by Eq.(8) for $0.05<k_r\leq1$. Seeing Fig.7, Eq.(8) and Fig.10, when $k_r$ becomes a small number, it was found that sinking of the underlay remarkably contributes to increase $\theta_{BU}$. From Fig.11, it showed that the simulated profile parameters $\beta$, $\gamma$, $b_n/t_S$, $\eta_n/t_S$ were close to the experimental result, although the simulated range of $k_r$ ($t_U=t_S$) was restricted to $0.05<k_r<10$. These results confirmed that the $k_r$ affected the sheared profile of the wedged worksheet. The profile parameters were linearly approximated with $\theta_{BU}$ in Eq.(9) ~ (12), as shown in Fig.12.

$$\theta_{BU} = 0.959(k_r)^{-0.427} \quad \text{(for } 0.05<k_r\leq1) \quad \text{Eq.(8)}$$

$$\beta = 0.761\theta_{BU} + 17.887 \quad \text{Eq.(9)}$$

$$\gamma = 0.858\theta_{BU} + 5.59 \quad \text{Eq.(10)}$$

$$b_n/t_S = 0.062\theta_{BU} + 0.550 \quad \text{Eq.(11)}$$

$$\eta_n/t_S = 0.0203\theta_{BU} + 0.140 \quad \text{Eq.(12)}$$

All of those simulated profile parameters were close to the experiment for $\theta_{BU} < 3^\circ$. For the range of $\theta_{BU} > 3^\circ$ ($k_r<0.05$), there were not any experimental data in this work, except for the special case of PC-HFP combination, a further investigation was required. Regarding the simulation in case of flexible underlay such as $k_r=0.002$ or 0.01, the calculation was fairly difficult to converge in a balanced state. A successful simulation is required for estimating such the flexible underlay combination.

### 3.3 Effect of thickness of underlay

Since the underlay stiffness in TD is also varied with the thickness of underlay $t_U$, thus the effect of $t_U$ on the sheared profile of wedged worksheet was discussed here. The thickness of underlay $t_U$ was chosen as 0.05, 0.1, 0.5 and 5.0mm, while the thickness of worksheet $t_S$ was fixed to be 0.5mm. The $E_U$ and $E_S$ were assumed to be 2.65GPa. Here, the stiffness ratio $k_r=0.1, 1, 5$ and 10 were obtained. The friction coefficients $\mu_C$, $\mu_U$ and $\mu_P$ were also assumed to be 0.23, 0.2 and 0.05, respectively.
Figure 13 shows the deformation profile of wedged worksheet and underlay with respect to \( k_r \) for \( d/t_s > 0.6 \). When the blade was indented to the worksheet for \( d/t_s > 0.6 \), a camber of the worksheet and a dent of the underlay were not apparently observed for all cases as shown in Fig.13 (a) ~ (d). However, seeing the necking of worksheet at \( d/t_s = 0.7 \) in case of \( k_r = 5 \), the necked elevation length was remarkably smaller than that other cases. This non-monotonous tendency of necking/detaching seemed to be caused by a dent and a lateral-sliding behavior of the underlay. In order to observe the dent and lateral-sliding of the underlay, the displacement of a representative node which located on the underlay was observed and recorded. This tracking of an arbitrary node at the underlay surface was possible and worthy for seeing a sliding state of the worksheet on the contact surface, owing that the motion tendency (displacement) of primary nodes on the underlay was almost same, and any re-meshing of the underlay was not carried out.

Figure 14 shows the displacement of a representative node in the lateral (in-plane), thickness directions, when the blade indentation \( d/t_s \) was increased from 0 up to 0.75. The measured point was indicated by a red point, which was initially 0.25 mm distant from the center, 0.02mm depth inside of the surface. In Fig.14, the lateral (x-directional) and thickness-directional (y-directional) displacement indicated positive and negative, respectively. Namely, the underlay surface moved in the lateral-outward direction and sunk in the thickness direction.

For a small value of \( k_r \) (= 0.1 and 1), a dent of the underlay was relatively large while a lateral elongation of the underlay was limited. In case of a large value \( k_r = 10 \), the dent of the underlay was tiny while the lateral elongation appeared to be large. In case of the middle value \( k_r = 5 \), the dent and the lateral elongation of the underlay were not so large, compared to the other cases of \( k_r = 0.1, 1 \) and 10. Seeing the result, it was confirmed that a large dent and a large lateral elongation of the underlay contributed to the large necking of the worksheet. Moreover, it was revealed that the thickness effect with \( k_r \) non-linearly differed
from the stiffness-modulus effect on $k_r$.

Figure 15 shows the relationship between the line force $f$ and the blade indentation depth $d/t$, with respect to $k_r$. When the ratio $k_r$ was chosen as 0.1, 1, 5 and 10, the gradient of the cutting load ($\partial f/\partial (d/t)$) was not almost varied for $d/t < 0.3$. However, seeing the peak point of the cutting load and the breakdown position, in case of $k_r=5$, they were fairly different from that of other cases. Namely, an occurrence delay of the peak position and an increasing of peak cutting load were caused by the necking suppression of the worksheet for a certain range of $k_r$ when $t_U$ was varied. It was originally caused by the suppression of the underlay deformation in terms of lateral (in-plane) elongation and out-of-plane sinking.

Regarding the relationship between the simulated bent-up angle $\theta_{BU}$ and $k_r$ in case of $E_U=E_S$ as shown in Fig.16, it was found that $\theta_{BU}$ was decreased for $k_r > 5$; while the variance tendency (gradient) of $\theta_{BU}$ by $k_r$ was remarkably decreased for $k_r < 1$. Calculated bent-up angle $\theta_{BU}$ was about 1° for $0.1<k_r<1$, while $\theta_{BU}$ was monotony decreased down to 0 for $1<k_r<10$. Namely, $\theta_{BU}$ almost saturated for $k_r < 1$, while $\theta_{BU}$ was remarkably affected by a certain thin underlay for $k_r > 5$.

The sheared profile parameters $\beta$, $\gamma$, $\eta_{n/t_S}$ and $b_{n/t_S}$ were investigated with respect to $k_r$ ($E_U=E_S$) and $\theta_{BU}$. They were linearly approximated with $\theta_{BU}$ by Eq. (13) ~ (16).

$$\beta = 0.799\theta_{BU} + 17.981$$  \hspace{1cm} \text{Eq.(13)}

$$\gamma = 0.823\theta_{BU} + 5.747$$  \hspace{1cm} \text{Eq.(14)}

$$b_{n/t_S} = 0.150\theta_{BU} + 0.459$$  \hspace{1cm} \text{Eq.(15)}

$$\eta_{n/t_S} = 0.023\theta_{BU} + 0.136$$  \hspace{1cm} \text{Eq.(16)}

It was found that the approximated equations of $\beta$, $\gamma$, and $\eta_{n/t_S}$ had the gradient and the constant terms similar to the case of Eq.(9),(10) and (12). Although the gradient coefficient of $b_{n/t_S}$, Eq.(15), was slightly different from that of Eq.(11), the estimated value of Eq.(15) was similar to that of Eq.(11) for $\theta_{BU} < 3^\circ$.

So far, it was found that the sheared-profile parameters were determined by the bent-up angle $\theta_{BU}$, which was caused by the in-plane elongation and the out-of-plane sinking of the underlay. Through the experiment and the FEM simulation, a range of stiffness ratio $k_r=0.1$~1.0 seems to be superior for cutting off the worksheet by using the 42° center bevel blade. By choosing $k_r=0.1$~1.0, a certain sinking and lateral elongation of the underlay cause a small bent-up of the worksheet, but do not vary the sheared profile of the worksheet.

4. Conclusions

In this study, a 42° center bevel blade indentation to a 0.5mm thickness polycarbonate (PC) worksheet stacked on finite thickness underlays was experimentally and numerically carried out in order to reveal the effect of underlay stiffness in TD on the cutting profile of the worksheet. By varying the Young’s modulus $E_U$ and the thickness $t_U$ of the underlay, several features on the deformation mode were obtained as follows: 1) The sheared-profile parameters, such as the inclined angle $\beta$, the elevation angle $\gamma$, the necked elevation length $b_n$ and the necked elevation height $h_n$, were characterized with the stiffness ratio $k_r = (E_U/E_S)/(t_U/t_S)$ and the bent-up angle $\theta_{BU}$. 

![Fig.16 The bent-up angle $\theta_{BU}$ of worksheet with respect to $k_r$ in case of $E_U=E_S$.](image-url)
2) The FEM simulation on the deformation profile of wedged PC worksheet and the cutting load response showed a good agreement with experimental results.

3) Through the simulation, it was revealed that the sheared-profile parameters were primarily determined by $\theta_{BU}$, while the effects of $k_r$ on the sheared-profile parameters were a little different for the thickness $t_U$ and the Young’s modulus $E_U$.

4) By varying the thickness $t_U$, it was revealed that the out-of-plane sinking and the lateral (in-plane) elongation of the underlay remarkably affected the lower necking of the PC worksheet during the blade indentation.

5) For the specified range of $t_U$ and $E_U$ in this work, a preferable sheared profile of wedged PC worksheet was obtained for $k_r=0.1\sim1.0$.

6) When $t_U$ was chosen as thinner than 20% of the worksheet thickness $t_S$ in this work, the cutting performance was estimated as better condition.

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References


