Development of Optimized Adaptive Feed-Forward Cancellation with Damping Function for Head Positioning System in Hard Disk Drives*

Shota YABUI**, Atsushi OKUYAMA***, Takenori ATSUMI† and Masaki ODAI††

** Japan Research Laboratory, HGST Japan, Ltd.
1, Kirihara, Fujisawa, Kanagawa 252–8588, Japan
E-mail: shota.yabui@hgst.com

*** School of Engineering, Tokai University.
4–1–1, Kita-Kaname, Hiratsuka, Kanagawa 259–1292, Japan
E-mail: okuyama@tokai-u.jp

† Japan Research Laboratory, HGST Japan, Ltd.
1, Kirihara, Fujisawa, Kanagawa 252–8588, Japan
E-mail: takenori.atsumi@hgst.com

†† Hitachi Research Laboratory, Hitachi Ltd.
832–2, Horiguchi, Hitachinaka, Ibaraki, 312–0034, Japan
E-mail: masaki.odai.eo@hitachi.com

Abstract
This paper presents an adaptive feed-forward cancellation (AFC) technique with a damping function based on optimized AFC. The proposed AFC includes a mechanical resonance characteristic as an internal model that is realized by using the damping function and that achieves the best performance in the frequency domain. This makes it possible to apply the proposed AFC to suppress not only repeatable run-out (RRO) but also non-repeatable run-out (NRRO) of the head positioning system in hard disk drives. Simulation results showed that the proposed AFC can suppress NRRO such as disk flutter vibration and that the stability of the feedback loop was improved by the damping function.

Key words : Hard Disk Drive, Repeatable Run-Out, Non-Repeatable Run-Out, Adaptive Feed-Forward Cancellation, Damping Function, Disk Flutter

Introduction

Increasing the recording density of hard disk drives is very important. However, as the recording density of hard disk drives increases, the spacing between data tracks decreases. This means that the head positioning accuracy of hard disk drives must be improved to meet the demand for larger data capacity. In head positioning systems of hard disk drives, position error signals are classified as either repeatable run-out (RRO) or non-repeatable run-out (NRRO). RRO is a periodic disturbance caused by disk rotation. NRRO is a non-periodic disturbance caused by factors such as torque noise, mechanical vibrations, and aerodynamic drag forces from the airflow induced by the spinning disks. Various control methods have been developed to suppress RRO and NRRO in order to improve positioning accuracy. A resonant filter was developed as one means of suppressing them\(^{[1][2]}\). The filter can suppress RRO and NRRO by using loop shaping techniques based on a vector locus.

Adaptive feedforward cancellation (AFC) is a disturbance suppression method that was developed to suppress sinusoidal disturbances\(^{[3]}\). AFC has been shown to suppress RRO in hard disk drives\(^{[4][5][6][7]}\). A linear time-invariant (LTI) model of AFC was derived from an adaptive algorithm\(^{[4][7]}\). The LTI model is equal to a sinusoidal disturbance model, and RRO
is observed as a sinusoidal disturbance in the position error signal. Therefore, AFC is mainly applied to suppress RRO.

In our previous studies, we introduced an optimized AFC\(^{(8)(9)}\) to suppress RRO. The performance of AFC is optimized in the frequency domain. However, the optimized AFC was not used to suppress NRRO, because NRRO is not a sinusoidal disturbance. For example, a model of disk flutter vibration caused by flow induced vibration is defined as a mechanical resonance characteristic. A model of a disturbance generation system based on the internal model principle must be included in the feedback system\(^{(10)}\). Therefore, AFC is not effective for suppressing disk flutter vibration.

We propose here an optimized AFC with a damping function. The proposed AFC includes a mechanical resonance characteristic as an internal model through the use of the damping function. This makes it possible to apply AFC to suppress not only RRO but also NRRO of the head positioning system in hard disk drives. Simulation results showed that the proposed AFC was able to suppress NRRO in the form of disk flutter vibration. Moreover, the stability of the feedback loop was improved by the damping function.

### 1. Head positioning systems of hard disk drives with AFC

Figure 1 shows the basic structure of a hard disk drive. It consists of a voice coil motor (VCM), several magnetic heads, several disks, and a spindle motor. The control input in a head positioning system is the voltage supplied to the power amplifier that drives the VCM and the magnetic heads. The control variable is the head positioning signal. Figure 2 is a block diagram of the control system with the AFC, where \( P \) is the controlled object, \( C \) is the stabilizing controller, and \( u(t) \) is the AFC output that compensates for disturbance.
An adaptive algorithm enables AFC to suppress periodic disturbances such as RRO. The form of the algorithm is expressed as follows.

\[ u(t) = p(t) \cos(\omega t) + q(t) \sin(\omega t) \]  

Equation (1) indicates the AFC output; the coefficients \( p(t) \) and \( q(t) \) are updated by the adaptive law below.

\[
\begin{align*}
\dot{p}(t) &= \lambda \cdot PES(t) \cos(\omega t + \theta) \quad (2) \\
\dot{q}(t) &= \lambda \cdot PES(t) \sin(\omega t + \theta) \quad (3)
\end{align*}
\]

Here, \( t \) is time, \( \omega \) is the desired compensation frequency, \( PES(t) \) is the position error signal, \( \lambda \) is the learning rate of the algorithm, and \( \theta \) is a phase parameter of AFC output. The values for \( \lambda \) and \( \theta \) are defined by a designer in advance. The method used to design \( \theta \) was introduced by using a vector locus(8)(9). The \( \lambda \) value is decided so that it does not cause instability in a closed loop system. The disturbance is exactly cancelled when the estimates of coefficients \( p(t) \) and \( q(t) \) have nominal values. Here, \( d(t) \) is the disturbance of the head positioning control system.

The AFC algorithm can be converted into an LTI model(3)(4)(7). Equation (4) is the LTI model from \( PES(t) \) to \( u(t) \). A block diagram of the control system is shown in Fig. 3.

\[
F(s) = \lambda \frac{s \cos(\theta) + \omega \sin(\theta)}{s^2 + \omega^2} \quad (4)
\]

\[
\text{Fig. 3} \quad \text{Head positioning control systems with LTI model of AFC}
\]

From Eq. (4), \( \lambda \) determines the gain of AFC, and \( \theta \) determines the zero of the transfer function. \( F(s) \) is equal to a sinusoidal disturbance model. That is, AFC includes the model as an internal model. Therefore, AFC is mainly used to suppress RRO, and \( d(t) \) is RRO in Fig. 3. However, AFC is considered ineffective for suppressing NRRO because a disturbance model of NRRO is not a sinusoidal transfer function. Disk flutter vibration is one type of NRRO caused by flow induced vibration. In this case, \( d(t) \) is a disturbance signal caused by disk flutter vibration. A disturbance model of disk flutter vibration is defined as Eq. (5)(11),

\[
D(s) = \frac{2\eta^2 \Omega^2}{s^2 + 2\eta \Omega s + \Omega^2}, \quad (5)
\]

where \( \eta \) is the damping ratio, \( \Omega \) is the central frequency, and \( \rho \) is the gain of the model. AFC does not have such a model as an internal model. Therefore, we developed an enhanced AFC to suppress NRRO.

2. Development of AFC with a damping function

This section introduces the AFC technique with the damping function. The AFC includes a mechanical resonance characteristic as an internal model. As a result, the proposed AFC can be applied to suppress not only RRO but also NRRO such as disk flutter vibration. We derived the AFC with a damping function from a state space model.
2.1. Relationship between state space model of AFC and state space model of sinusoidal transfer function

This section introduces the method using a state space model. Equation (6) indicates the state space of AFC.

\[
\begin{bmatrix}
\dot{p}(t) \\
\dot{q}(t)
\end{bmatrix} = \hat{A}(t) \begin{bmatrix}
p(t) \\
q(t)
\end{bmatrix} + \hat{B}(t)e(t)
\]

\[
= \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
p(t) \\
q(t)
\end{bmatrix} + \begin{bmatrix}
\lambda \cos(\omega t + \theta) \\
\lambda \sin(\omega t + \theta)
\end{bmatrix} e(t)
\]

\[
u(t) = \hat{C}(t) \begin{bmatrix}
p(t) \\
q(t)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\cos(\omega t) & \sin(\omega t)
\end{bmatrix} \begin{bmatrix}
p(t) \\
q(t)
\end{bmatrix}
\]

(6)

The state space model is a linear time-variant (LTV) model. However, Eq. (4) gives an LTI model of AFC. We investigated the relationship between the LTV model and the LTI model by using the state space model. The state space model of Eq. (4) is described by Eq. (7).

\[
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} = A \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + Be(t)
\]

\[
= \begin{bmatrix}
0 & \omega \\
-\omega & 0
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + \begin{bmatrix}
\lambda \cos(\theta) \\
\lambda \sin(\theta)
\end{bmatrix} e(t)
\]

\[
u(k) = C \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0 & 1
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}
\]

(7)

In this form, if state variables are transformed as the equation below:

\[
\begin{bmatrix}
p(t) \\
q(t)
\end{bmatrix} = R(t) \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}, \quad R(t) = \begin{bmatrix}
\cos(\omega t) & -\sin(\omega t) \\
\sin(\omega t) & \cos(\omega t)
\end{bmatrix},
\]

then, matrices \( A, B, C \) are derived from matrices \( R(t) \) and \( \hat{A}(t), \hat{B}(t), \hat{C}(t) \):

\[
A = R^{-1}(t)(\hat{A}(t)R - \hat{R}(t)) = \begin{bmatrix}
0 & \omega \\
-\omega & 0
\end{bmatrix},
\]

\[
B = R^{-1}(t)\hat{B}(t) = \begin{bmatrix}
\lambda \cos(\theta) \\
\lambda \sin(\theta)
\end{bmatrix},
\]

\[
C = \hat{C}(t)R(t) = \begin{bmatrix}
0 & 1
\end{bmatrix}.
\]

The calculation indicates that Eq. (6) is the same as Eq. (7). The relationship between AFC and a sinusoidal transfer function is derived from the transformation of state variables.

2.2. Development of proposed AFC from state space model

The previous section introduced the relationship between the LTV and LTI models of AFC. To derive AFC with a damping function, the relationship is applied to an LTI model that is defined as a mechanical resonance characteristic, as shown in Eq. (8). Here, \( \xi \) is a damping ratio, and \( \omega \) is a central frequency. The LTI model has the same numerator as AFC.

\[
U(s) = \frac{s \cos(\theta) + \omega \sin(\theta)}{s^2 + 2\xi \omega s + \omega^2}
\]

(8)

Equation (9) indicates the state space model of Eq. (8).

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} = \begin{bmatrix}
0 & \omega \\
-\omega & -2\xi \omega
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix} + \begin{bmatrix}
\lambda \cos(\theta) \\
\lambda \sin(\theta)
\end{bmatrix} e(t)
\]

\[
u(k) = \begin{bmatrix}
0 & 1
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}
\]

(9)
In the state space, the state variables are also converted to
\[
\begin{bmatrix}
  p(t) \\
  q(t)
\end{bmatrix} = R(t) \begin{bmatrix}
  x_1(t) \\
  x_2(t)
\end{bmatrix}.
\]

Then, the matrices \( \hat{A}(t), \hat{B}(t), \hat{C}(t) \) of AFC with a damping function are derived from the following equations.
\[
\begin{align*}
\hat{A}(t) &= R^{-1}(t)(R(t)A + \dot{R}(t)), \\
\hat{B}(t) &= R(t)B, \\
\hat{C}(t) &= CR^{-1}(t)
\end{align*}
\]

The state space model of AFC with a damping function is expressed by Eq. (10),
\[
\begin{bmatrix}
  \dot{p}(t) \\
  \dot{q}(t)
\end{bmatrix} = -2\zeta \omega \begin{bmatrix}
  \cos^2(\omega t) & \sin(\omega t) \cos(\omega t) \\
  \sin(\omega t) \cos(\omega t) & \sin^2(\omega t)
\end{bmatrix} \begin{bmatrix}
  p(t) \\
  q(t)
\end{bmatrix} + \begin{bmatrix}
  \lambda \cos(\omega t + \theta) \\
  \lambda \sin(\omega t + \theta)
\end{bmatrix} e(t)
\]
\[
\begin{bmatrix}
  u(t) = \begin{bmatrix}
  \cos(\omega t) & \sin(\omega t)
\end{bmatrix}
\end{bmatrix} \begin{bmatrix}
  p(t) \\
  q(t)
\end{bmatrix}.
\]

Equation (10) expresses a state space model of AFC with a damping function. In Eq. (10), the state equation indicates the update law of coefficients \( p(t) \) and \( q(t) \). We can see that the A matrix has a damping function. The AFC has a mechanical resonance characteristic defined as Eq. (8) by a damping function.

3. Design and simulation results

We conducted simulations on head positioning in order to verify the characteristics of AFC with a damping function. A technical committee consisting of representatives of hard disk drive manufacturers and major universities doing hard disk drive servo research in Japan has developed an open-source HDD benchmark problem(11) to encourage study of the head positioning control system.

3.1. Simulation conditions

We used the track-following control system in the benchmark problem Ver. 3.1 for the head-positioning simulation. A block diagram of the feedback control system is shown in Fig. 2. The frequency responses of plant \( P \) and of feedback controller \( C \) used in the benchmark problem are respectively shown in Figs. 4 and 5.
We injected disturbance to the feedback control system in order to evaluate the disturbance suppression performance of the proposed AFC. We focused on disk flutter vibration. In the HDD Benchmark Problem, a disturbance caused by disk flutter vibration is defined as the output when white Gaussian noise with a standard deviation = 1 is injected into the disk flutter vibration models. The model of disk flutter vibration is expressed by Eq. (11). Table 1 lists the parameters of these vibration models. Figure 7 plots the disturbance signal caused by the disk flutter vibration in the unit of position, where the signal is injected as disturbance \( d(t) \) in Fig. 2.

\[
F(s) = \sum_{i=1}^{7} \frac{2a_i\eta_i\Omega_i^2}{s^2 + 2\eta_i\Omega_i s + \Omega_i^2}
\]  

(11)

<table>
<thead>
<tr>
<th>( i )</th>
<th>Gain of models: ( a_i )</th>
<th>Damping ratio: ( \eta_i )</th>
<th>Central frequency: ( \Omega_i ) [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.09</td>
<td>0.01</td>
<td>( 2 \times \pi \times 750 )</td>
</tr>
<tr>
<td>2</td>
<td>0.17</td>
<td>0.01</td>
<td>( 2 \times \pi \times 780 )</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>0.01</td>
<td>( 2 \times \pi \times 900 )</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>0.01</td>
<td>( 2 \times \pi \times 1020 )</td>
</tr>
<tr>
<td>5</td>
<td>0.06</td>
<td>0.005</td>
<td>( 2 \times \pi \times 1080 )</td>
</tr>
<tr>
<td>6</td>
<td>0.09</td>
<td>0.01</td>
<td>( 2 \times \pi \times 1230 )</td>
</tr>
<tr>
<td>7</td>
<td>0.06</td>
<td>0.005</td>
<td>( 2 \times \pi \times 1800 )</td>
</tr>
</tbody>
</table>
In this simulation, AFC was applied to suppress the disturbance signal \( d(t) \). In the benchmark problem, AFC is not designed to suppress NRRO. Therefore, we designed AFC to suppress the disturbance signal. In this section, the optimized AFC developed in our previous study\(^{(8)(9)}\) is referred to as the conventional method, and the proposed method is the optimized AFC with the damping function. Both the conventional and proposed methods were designed to be applied to a head positioning system in order to compare them. The state space model of the proposed AFC is given in Eq. (12). Table 2 lists the parameters of the proposed AFC.

\[
\begin{bmatrix}
\dot{p}_i(t) \\
\dot{q}_i(t)
\end{bmatrix}
= -2\zeta_i\omega_i \begin{bmatrix}
\cos^2(\omega_i t) & \sin(\omega_i t) \cos(\omega_i t) \\
\sin(\omega_i t) \cos(\omega_i t) & \sin^2(\omega_i t)
\end{bmatrix}
\begin{bmatrix}
p_i(t) \\
q_i(t)
\end{bmatrix}
+ \begin{bmatrix}
\lambda_i \cos(\omega_i t + \theta_i) \\
\lambda_i \sin(\omega_i t + \theta_i)
\end{bmatrix} e(t)
\]

\[u(t) = \sum_{i=1}^{7} u_i(t) = \begin{bmatrix}
\cos(\omega_i t) & \sin(\omega_i t)
\end{bmatrix}
\begin{bmatrix}
p_i(t) \\
q_i(t)
\end{bmatrix}
\]  

(12)
In order to match the disk flutter model, the parameter values $\zeta_i$ and $\omega_i$ in Table 2 are the same as the values $\eta_i$ and $\Omega_i$ in Table 1. Here, $\theta_i$ was designed in order to achieve the best performance in the frequency domain. The design method is shown in our previous study (8)(9). The papers introduce the optimization method for AFC by using loop shaping techniques based on vector locus. To recede from the critical point $[-1, 0]$ on the Nyquist chart, the proposed method set the parameters of $\theta_i$. $\lambda_i$ was selected so that it matched the width of the frequency response of the disk flutter model. The conventional AFC does not have a damping function; that is, $\zeta_i = 0$. The other parameters were the same as the proposed AFC. The frequency response of each AFC is shown in Fig. 8. The gain of the conventional AFC is infinity at each central frequency $\omega_i$ because the conventional AFC does not have a damping function. In contrast, the gain of the proposed AFC can be determined by $\zeta_i$.

### Table 2  Proposed AFC parameters

<table>
<thead>
<tr>
<th>$i$</th>
<th>AFC gain: $\lambda_i$</th>
<th>Damping ratio: $\zeta_i$</th>
<th>Central frequency: $\omega_i$ [rad/s]</th>
<th>Phase: $\theta_i$ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.15</td>
<td>0.01</td>
<td>$2 \times \pi \times 750$</td>
<td>-103.16</td>
</tr>
<tr>
<td>2</td>
<td>0.17</td>
<td>0.01</td>
<td>$2 \times \pi \times 780$</td>
<td>-105.35</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>0.01</td>
<td>$2 \times \pi \times 900$</td>
<td>-114.33</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>0.01</td>
<td>$2 \times \pi \times 1020$</td>
<td>-113.87</td>
</tr>
<tr>
<td>5</td>
<td>0.10</td>
<td>0.005</td>
<td>$2 \times \pi \times 1080$</td>
<td>-88.88</td>
</tr>
<tr>
<td>6</td>
<td>0.12</td>
<td>0.01</td>
<td>$2 \times \pi \times 1230$</td>
<td>-142.04</td>
</tr>
<tr>
<td>7</td>
<td>0.10</td>
<td>0.005</td>
<td>$2 \times \pi \times 1800$</td>
<td>-188.25</td>
</tr>
</tbody>
</table>

### 3.2. Simulation results

Firstly, the characteristics of AFC with a damping function were verified in a time domain. The time responses of the position error signal when $d(t)$ was injected are plotted in Fig. 9. We can see in the figure that if we compare the position error signal for the cases with and without AFC, the use of AFC is clearly more effective. The effectiveness of the conventional AFC and the proposed AFC are almost the same in the time domain.

However, differences were verified in the frequency domain. Figure 10 shows the frequency response of open loop characteristics. Figure 11 shows the vector locus of the open loop characteristics. The vector locus of the conventional AFC is closer to the critical point than that of the proposed AFC. Figure 12 plots the frequency responses of the sensitivity function. The figure indicates that the $H_\infty$ norm of the sensitivity function of the conventional AFC is larger than that of the proposed AFC. The waterbed effect results in the conventional AFC having a larger frequency response than the proposed AFC because the depth of the frequency
response at the central frequency cannot be designed using a damping function. That is, the conventional AFC results in excessive suppression of the disturbance signal caused by disk flutter vibration.

Fig. 9  Time response of position error signal: $PES(t)$
Fig. 10 Frequency response of open loop with AFC: blue line is the conventional AFC; red broken line is the proposed AFC.

Fig. 11 Vector locus of open loop: blue line is the conventional AFC; red broken line is the proposed AFC.

(a) Vector locus  
(b) Close-up of vector locus around the critical point.
Table 3 lists the numerical data of the simulation results. The value of PES sigma is almost the same for the conventional and the proposed AFC. On the contrary, the $H_\infty$ norm of the sensitivity function is smaller, and the gain margin and phase margin are larger in the proposed AFC than in the conventional AFC. Therefore, the stability of the feedback loop is improved by the damping function.

We also verified parameter variation of $\zeta_i$ and $\omega_i$ for a design guideline. Table 4 lists the numerical data of the simulation results. Figure 13 plots the frequency responses of the sensitivity function. The parameter variation of $\zeta_i$ mainly influences phase margin. $\zeta_i$ should be designed considering with design specification of phase margin. On the other hand, $\omega_i$ mainly influences disturbance suppression of AFC. Table 5 lists the numerical data of the simulation results. In this simulation, PES $3\sigma$ is equal to 18.7nm without AFC. If variation of $\omega$ is more than 10%, AFC can not suppress disturbance caused by disk flutter vibrations. Natural frequencies of disk flutter models should be identified precise from spectrum of position error signal[12].

Table 3 Simulation data

<table>
<thead>
<tr>
<th></th>
<th>Conventional AFC</th>
<th>Proposed AFC</th>
</tr>
</thead>
<tbody>
<tr>
<td>PES $3\sigma$</td>
<td>12.4 nm</td>
<td>12.0 nm</td>
</tr>
<tr>
<td>$H_\infty$ norm of sensitivity function</td>
<td>8.5 dB</td>
<td>6.5 dB</td>
</tr>
<tr>
<td>Gain margin</td>
<td>6.05 dB</td>
<td>7.13 dB</td>
</tr>
<tr>
<td>Phase margin</td>
<td>23.19 deg</td>
<td>30.20 deg</td>
</tr>
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</table>

Table 4 Simulation data for parameter variation of $\zeta$

<table>
<thead>
<tr>
<th>Parameter variation of $\zeta$</th>
<th>−50%</th>
<th>−20%</th>
<th>0%</th>
<th>+20%</th>
<th>+50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>PES $3\sigma$</td>
<td>11.6 nm</td>
<td>11.6 nm</td>
<td>12.0 nm</td>
<td>12.4 nm</td>
<td>12.4 nm</td>
</tr>
<tr>
<td>$H_\infty$ norm of sensitivity function</td>
<td>7.3 dB</td>
<td>6.8 dB</td>
<td>6.5 dB</td>
<td>6.5 dB</td>
<td>6.4 dB</td>
</tr>
<tr>
<td>Gain margin</td>
<td>7.10 dB</td>
<td>7.13 dB</td>
<td>7.13 dB</td>
<td>7.13 dB</td>
<td>7.16 dB</td>
</tr>
<tr>
<td>Phase margin</td>
<td>25.84 deg</td>
<td>28.52 deg</td>
<td>30.20 deg</td>
<td>31.96 deg</td>
<td>34.45 deg</td>
</tr>
</tbody>
</table>

Table 5 Simulation data for parameter variation of $\omega$

<table>
<thead>
<tr>
<th>Parameter variation of $\omega$</th>
<th>−10%</th>
<th>−5%</th>
<th>0%</th>
<th>+5%</th>
<th>+10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>PES $3\sigma$</td>
<td>17.4 nm</td>
<td>15.9 nm</td>
<td>12.0 nm</td>
<td>15.8 nm</td>
<td>17.7 nm</td>
</tr>
<tr>
<td>$H_\infty$ norm of sensitivity function</td>
<td>6.5 dB</td>
<td>6.5 dB</td>
<td>6.5 dB</td>
<td>6.7 dB</td>
<td>7.04 dB</td>
</tr>
<tr>
<td>Gain margin</td>
<td>7.13 dB</td>
<td>7.13 dB</td>
<td>7.13 dB</td>
<td>7.10 dB</td>
<td>7.04 dB</td>
</tr>
<tr>
<td>Phase margin</td>
<td>33.26 deg</td>
<td>31.77 deg</td>
<td>30.20 deg</td>
<td>28.55 deg</td>
<td>27.52 deg</td>
</tr>
</tbody>
</table>
We compared the characteristic of the proposed AFC expressed in Eq. (10) and an LTI model of the mechanical resonance characteristic expressed in Eq. (8) in a simulation. In this simulation, the proposed AFC was defined as Eq. (12), and the LTI model was defined as Eq. (13). The design parameters are given in Table 2. The outputs of the proposed AFC and the resonant filter are shown in Fig. 14. It is clear from the figure that these time responses coincide. Therefore, the performance of the proposed AFC is equivalent to the LTI model defined as a mechanical resonance characteristic.  

\[ U(s) = \sum_{i=1}^{7} A_i \frac{s \cos(\theta_i) + \omega \sin(\theta_i)}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \]  

4. Conclusion  

We proposed an optimized AFC with a damping function. The proposed AFC includes a mechanical resonance characteristic as an internal model and achieves the best performance in the frequency domain. These results mean that the proposed AFC can be applied to suppress not only RRO but also NRRO of head positioning systems in hard disk drives. The simulation results on an HDD benchmark problem showed that the proposed AFC suppressed disk flutter vibration, which is a type of NRRO, and that the stability of the feedback loop was improved by the damping function.
Fig. 14 Time response of filter output $u(t)$: blue line is the resonant filter; red broken line is the proposed AFC

References


(8) S. Yabui, M. Kobayashi, A. Okuyama, and T. Atsumi, ”DEVELOPMENT OF OPTIMIZED ADAPTIVE FEEDFORWARD CANCELLATION FOR HEAD POSITIONING CONTROL SYSTEM IN HARD DISK DRIVES,” , Microsystem Technologies, Online First, (2012)


