Dynamic Characteristics of The Spool-Type Constant-Flow Valves*

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Abstract

Constant flow valves have been presented in industrial applications or academic studies, which compensate recess pressures of a hydrostatic bearing to resist load fluctuating. The flow rate of a constant-flow valve can be constant in spite of the pressure changing in a recess. However the design parameters of this type valve must be specified. This paper analyzes the dynamic responses of the spool-type constant-flow valve that is designed as a spool-type restrictor but with matched preloading spring and supply pressure. In this study the static analysis presents the specific relationships among design parameters for constant flow rate and the dynamic analyses give the variations around the constant flow rate as the working pressure fluctuates.

Key words: Constant-Flow Valve (CFV), Hydrostatic Bearing, Spool-Spring System, Dynamic Characteristics

1. Introduction

The principle of constant-flow valves are based on that valve resistance can be decreased to compensate the decreasing of difference between supply pressure and recess pressure for keeping flow rate. Rippel ⁽¹⁾ has included a DSI type that is named according to double pistons with single feedback and regulating restriction by inlet piston. Bassani and Piccigallo ⁽²⁾ presented another one named DDE type because that is designed as double pistons with double feedbacks and regulation by the piston at exit. The spool-type restrictors can be constant-flow, which regulate restriction by spool and without using flow-meter valve like DDE-type and DSI type. However, the exact constant of flow rate cannot be attained unless selecting appropriate parameters to meet the specific requirement in valve design. Rowe ⁽³⁾ has introduced the mechanism design and written elementary theory of control valves for designing hydrostatic bearings. However, who mainly related the relationships between static characteristics of various bearing types and contact restrictions belong to capillary, slot, orifice, flow control valves, and constant flow pump; pressure-sensing valves. Bassani ⁽⁴⁾ analyzed that the compensation behaviors of double-action type cylindrical-spool and tapered-spool restrictors for static characteristics of hydrostatic bearings which are installed on slideway, lead screw, thrust bearing, et al. However, his study didn't consider negative and infinite stiffness in the compensation design of hydrostatic bearings. Kang et al. ⁽⁵⁾ presented the influences of design parameters belonging to the variable compensations of single-action restrictors respectively on static stiffness of hydrostatic bearing and utilized the continuity equations formulated by film flow to study both open-type and closed-type hydrostatic thrust bearings. Kang et al. ⁽⁶⁾ presented the specific requirements of constant flow rate for various type designs. Kang et al. ⁽⁷⁾ utilized Runge-Kutta method to investigate the dynamic responses of DSI

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types constant-flow valves as working pressure fluctuating. This study uses the same method to simulate the dynamic responses of spool-type constant-flow valves. The influence of design parameters on the deviations of flow rate and spool displacement due to working pressure ratio being fluctuating can be yielded. Also, this paper presented the specific requirement of constant flow rate for this type valves and verified by both analyses and experiments.

Nomenclature

A_1	: projected area of piston $A_1 = \pi d_p^2 / 4$
$A_2, \bar{A}_2$	: projected area of spool $A_2 = \pi d_s^2 / 4$ and area ratio $\bar{A}_2 = A_2 A_1^{-1}$
$A_{12}, \bar{A}_{12}$	: area difference $A_{12} = A_1 - A_2, \bar{A}_{12} = A_{12} A_1^{-1}$
c	: damping coefficient
C_s	: displacement coefficient of spool due to action of restoring spring
$\bar{C}_s$	: dimensionless displacement coefficient (ddc)
d_n, d_s	: diameters of valve set and spool
d_p	: actual diameter of piston
g_s	: spool clearance, $g_s = (d_n - d_s) / 2$
$k, \bar{k}$	: spring constant, dimensionless spring constant $\bar{k} = k \ell_0 (P_s A_1)^{-1}$
ℓ_0	: initial restriction length of spool
P_r, P_{rs}, P_{rd}	: working pressure, the second subscript denotes for static and dynamic parts
P_s	: supply pressure
P_0	: equivalent pressure acting on piston by spring preload
$\bar{P}_r, \bar{P}_0$	: pressure ratio $\bar{P} = P P_s^{-1}$
$Q, \bar{Q}$	: flow rate, dimensionless flow rate (dfr) $\bar{Q} = R_0 Q / P_s$
Q_s, Q_d	: static and dynamic parts of flow rate
$\bar{Q}_d^*$	: dimensionless amplitude of dynamic flow rate ratio
R	: flow resistance with respect to spool displacement x
R_0	: constant of flow resistance with respect to zero displacement of spool
$x_s, \bar{x}_s$	: static displacement of spool, $\bar{x}_s = x_s \ell_0^{-1}$
$x_d, \bar{x}_d$	: dynamic displacement of spool, $\bar{x}_d = x_d \ell_0^{-1}$
$\hat{x}_d, \hat{x}_d^*$	: displacement ratio $\hat{x}_d = \bar{x}_d / \bar{x}_s$, dimensionless amplitude of $\hat{x}_d$
$x_0, \bar{x}_0$	: initial deformation of spring due to preload $\bar{x}_0 = x_0 \ell_0^{-1}$
α	: dimensionless parameter product $\alpha = R_0 A_1 C_s \omega_n$
μ	: absolute viscosity of oil
ω, ω_n	: frequency of fluctuating pressure, natural frequency of spool-spring system
ζ	: damping ratio
Subscript	
d, s	: denote for static and dynamic part, respectively of pressures, flow rates, and spool displacements

2. Principle of Static and Dynamic Flow Rates

This type valve has the same components and layout of single-action spool-type

restrictor ⁽³⁾ as shown by Fig. 1. This figure illustrates that the right end of piston is subjected to preload applied by spring and the pressure that is restricted from P_s applying on the left side of piston. Thus the preload pressure P_0 is determined by the resultant force of both actions per piston area. The pressurized oil can flow into inlet and restricted by the cylindrical spool located on left side of piston to leave valve from outlet with working pressure P_r to a recess of hydrostatic bearing. As soon as $P_r > P_0$, the spool leaves its initial position which is drawn by dashed lines. The piston locates on the right end and manufactured more smooth than spool surface for sliding and less clearance for sealing. A vent is manufactured on right chamber in order to relief air pressure such as only spring force acts on the piston. The spring can be preloaded by tightening the adjusting screw.

Fig. 1 Design of constant-flow valve

According to basic equation of laminar flow in a circular annulus, the flow rate of this valve can be determined by

$$Q = \frac{\pi d_s g_s^3}{12\mu(\ell_0 - x)} (P_s - P_r) - A_1 \dot{x} = \frac{\pi d_s g_s^3 P_s}{12\mu \ell_0} \left(1 - \frac{x}{\ell_0}\right)^{-1} (1 - \bar{P}_r) - A_1 \dot{x}$$

$$= \frac{P_s (1 - \bar{P}_r)}{R_0 (1 - \bar{x})} - A_1 \dot{x} \quad (1)$$

where $R_0 = \frac{12\mu \ell_0}{\pi d_s g_s^3}$ is resistance constant of flow passing through the clearance between spool and valve, and x is spool displacement that is proportional to the net pressure and $\bar{x} = x/\ell_0$ which can be described by

$$\bar{x} = C_s \ell_0^{-1} (P_r - P_0) = C_s \ell_0^{-1} P_s (\bar{P}_r - \bar{P}_0) = \bar{C}_s (\bar{P}_r - \bar{P}_0) \quad (2)$$

where $\bar{C}_s = \frac{C_s P_s}{\ell_0}$ is dimensionless displacement coefficient (*ddc*). Substituting Eq. (2)

into Eq. (1) and divided by $P_s R^{-1}$ gives

$$\bar{Q} = \frac{R_0 Q}{P_s} = \frac{1 - \bar{P}_r}{1 - \bar{C}_s (\bar{P}_r - \bar{P}_0)} \quad (3)$$

The function of pre-compression of spring not only give preload P_0 to spool but also resist the action of $P_s A_{12}$. The spool can just constant valve set without reaction when pressurized lubricant is supplied with P_s and restricted by spool to apply P_0 on the end of spool as shown in Fig. 2. This figure shows that initial equilibrium at working pressure being equal to P_0 . Thus

$$P_s A_{12} + P_0 A_2 = kx_0 \quad (4a)$$

From Eq. (4a), we can get

$$P_0 = A_2^{-1} (kx_0 - P_s A_{12}) \quad (4b)$$

Fig. 2 Equilibrium of static forces applied on spool-piston body.

When $P_r > P_0$, spool can leave valve set to reduce the restriction length with amount x , the flow resistance can be expressed by

$$R = \frac{12\mu}{\pi d_s^3} (\ell_0 - x) = R_0 (1 - \bar{x}) \quad (5a)$$

and displacement ratio of spool is

$$\bar{x} = \frac{x}{\ell_0} = \frac{(P_r - P_0)A_2}{k\ell_0} = \frac{P_s A_2 (\bar{P}_r - \bar{P}_0)}{k\ell_0} = \frac{\bar{P}_r - \bar{P}_0}{\bar{k}} = \bar{C}_s (\bar{P}_r - \bar{P}_0) \quad (5b)$$

where $\bar{k} = \frac{k\ell_0}{P_s A_2} > \bar{C}_s^{-1}$ is dimensionless spring constant. The flow rate passing through the restriction length $\ell_0 - x$ along spool is

$$Q = \frac{P_s - P_r}{R} = \frac{P_s (1 - \bar{P}_r)}{R_0 (1 - \bar{x})} = \frac{P_s (1 - \bar{P}_r)}{R_0 [1 - \bar{C}_s (\bar{P}_r - \bar{P}_0)]} - A_1 \dot{x} \quad (6a)$$

From above equation, we can yield dimensionless flow rate as shown by

$$\bar{Q} = \frac{R_0 Q}{P_s} = \frac{1 - \bar{P}_r}{1 - \bar{C}_s (\bar{P}_r - \bar{P}_0)} - \frac{R_0 A_1}{P_s} \dot{x} \quad (6b)$$

Because working pressure have both static and dynamic parts: $\bar{P}_r = \bar{P}_{rs} + \bar{P}_{rd}$, that induce static and dynamic parts of flow rate, substituting $\bar{Q} = \bar{Q}_s + \bar{Q}_d$ into above equation yields

$$\bar{Q}_s + \bar{Q}_d = \frac{1 - (\bar{P}_{rs} + \bar{P}_{rd})}{1 - \bar{C}_s (\bar{P}_{rs} + \bar{P}_{rd} - \bar{P}_0)} - \frac{R_0 A_1}{P_s} \dot{x}_d \quad (6c)$$

where $\dot{x}_d$ is the dynamic part of velocity $\dot{x}$.

3. Design for Static Flow

Equation (3) can be expressed for only static part as

$$\bar{Q}_s = \frac{1 - \bar{P}_{rs}}{1 - \bar{C}_s (\bar{P}_{rs} - \bar{P}_0)} \quad (7a)$$

where $\bar{P}_{rs}$ denotes for static part. For constant flow $\bar{Q}_s = \text{constant}$, $(1 - \bar{P}_{rs})$ should be eliminated from both numerator and denominator. The denominator of Eq. (7a) can be rearranged into:

$$\begin{aligned} 1 - \bar{C}_s (\bar{P}_{rs} - \bar{P}_0) &= 1 - \bar{P}_{rs} + (1 - \bar{C}_s) \bar{P}_{rs} + \bar{C}_s \bar{P}_0 \\ &= (1 - \bar{P}_{rs}) + \bar{C}_s \bar{P}_0 \left(1 - \frac{\bar{C}_s - 1}{\bar{C}_s \bar{P}_0} \bar{P}_{rs} \right) \end{aligned} \quad (7b)$$

where the terms in the second parentheses on the right side of above equation becomes $1 - \bar{P}_{rs}$ if

$$\bar{C}_s - 1 = \bar{C}_s \bar{P}_0 \quad (8)$$

Substituting Eq. (4b) into Eq. (8) gives

$$\bar{C}_s - 1 = \frac{P_s A_2}{k \ell_0} - 1 = \frac{P_0 A_2}{k \ell_0} = \frac{k x_0 - P_s A_{12}}{k \ell_0} = \frac{x_0}{\ell_0} - \frac{P_s A_{12}}{k \ell_0} \quad (8a)$$

Above equation can be rearranged into

$$P_s A_1 = k(\ell_0 + x_0) \quad (8b)$$

Thus, the constant flow rate can be yielded as design parameters satisfying Eq. (8b). Furthermore, the condition described by Eq. (8) makes the denominator of Eq. (7a) becomes $(1 + \bar{C}_s \bar{P}_0)(1 - \bar{P}_{rs})$ and results into

$$\bar{Q}_s = \frac{1 - \bar{P}_{rs}}{(1 + \bar{C}_s \bar{P}_0)(1 - \bar{P}_{rs})} = \frac{1}{1 + \bar{C}_s \bar{P}_0} = \bar{C}_s^{-1} = \text{constant} \quad (9a)$$

Then, static flow rate can be yielded as

$$Q_s = \frac{P_s \bar{Q}_s}{R} = \frac{P_s}{R \bar{C}_s} = \frac{\pi d_s^3 g_s^3 P_s k \ell_0}{12 \mu \ell_0 P_s A_2} = \frac{\pi d_s^3 g_s^3 k}{12 \mu A_2} \quad (9b)$$

Furthermore, static displacement of spool can be yielded from Eq. (5b) as shown by

$$x_s = \ell_0 \bar{x}_s = \ell_0 \bar{C}_s (\bar{P}_{rs} - \bar{P}_0) = C_s (P_{rs} - P_0) \quad (10)$$

The determinations of Eqs. (8) and (9a) yield that the collocation of $\bar{P}_0$ and $\bar{C}_s$ for constant flow and corresponding constant of flow rate as shown in Fig. 3. Furthermore, the dimensionless static displacements of spool for various designs as static pressure ratio changing are shown in Fig. 4.

Fig. 3 Dimensionless flow rate $\bar{Q}_s$ with corresponding collocations of preload $\bar{P}_0$ and dimensionless displacement coefficient

Fig. 4 The dimensionless spool displacement versus $\bar{P}_{rs}$ for various designs

We can use a hydrostatic planar bearing to test the valve performance. This bearing has at least three recesses to support a horizontal worktable that is loaded to change recess pressures which are compensated by constant-flow valves individually. The inlet pressures of each recess are controlled to keep constant and the outlet pressures are changed by worktable loads. Both pressures and the temperatures of flow of each recess and the flow

rates passing through each recess are measured. A practical valve as shown by Fig. 1 is used for experiments. An experimental equipment used in this study as shown in Fig. 5 is composed by a hydrostatic system supplying pressurized oil to recesses of the hydrostatic bearing, and a worktable mounting on the hydrostatic bearing compensated by eight recesses. Four upward recesses compensated by restrictors will be identified are used for calibrating outlet pressure of restrictor and support worktable vertically, and four recesses compensated by constant restrictors and located on the sides of worktable are used only to equilibrate the worktable position in the horizontal directions of both sides. The inlet pressure P_s can be calibrated by tuning pressure control valve of supply circuit, and a worktable is used to support load for tuning the outlet pressure of restrictors by changing the recess pressures. The pressures of four recesses can be determined simultaneously by changing load of worktable.

This hydrostatic system utilizes overall and individual accumulators to stabilize pressures in this system. The individual accumulators provide supplied pressures of restrictor inlets that is measured as P_{si} for the i -th recess then flow into the i -th recess with outlet pressure P_{ri} and flow rate Q_i , and finally flow out from the clearance and return to reservoir. The measurement range and sensitivity of pressure meters used to measure the pressures of upstream and downstream of restrictors are 0~50 bar and 0.15682 Volt (kg/cm), respectively, the output is electric current which is necessarily transformed from 4~20 mA by resistance 250 Ω to voltage 1V~5V for computer input. The measurement range and sensitivity of flow meters used to measure the flow rate of each restrictors are 1L/h~100L/h and 0.006253 Volt (cm/min), respectively, the output is voltage from 0 to 10V with error $\pm 0.5\% \sim \pm 1\%$. The IC type temperature sensors are used with thermometers and amplifier AD590J, which have the measurement range and sensitivity are $-55 \sim 150^\circ\text{C}$ and $1 \mu\text{A}/^\circ\text{K}$ respectively, the output is voltage from 0 to 10 Volt corresponding to the temperature range from 0 to 100°C with repeatable accuracy $\pm 0.1^\circ\text{C}$.

Fig. 5 Experimental equipment.

The valve used for experiments has fixed parameters as given by $d_s=3.155$ cm, $d_p=3.198$ cm, $d_n=3.2025$ cm, $g_s=0.2375$ mm, $A_1=8.0324$ cm², $A_2=7.814$ cm², set

$\ell_0 = 3.891$ cm, lubricant SAE 20 with absolute viscosity for typical average temperature between outlet and inlet at 20.5°C which is kept at the inlet of valves.

Experiment 1

Other design parameters $k=18.4$ Kg/cm, $x_0=3$ mm, and $\mu=0.16$ Pa·s are given, substituting these parameters into Eqs. (8b) and (9b) to determine the supply pressure of constant flow valve respectively by

$$P_s = k(\ell_0 + x_0) A_1^{-1} = \frac{18.4(3.891 + 0.3)}{8.0324} = 9.6 \text{ Kg/cm}^2$$

and

$$Q = \frac{\pi d_s g_s^3 k}{12 \mu A_2} = \frac{3.14 \times 3.155 \times 0.02375^3 \times 18.4 \times 9.8}{12 \times 0.16 \times 10^{-4} \times 7.814} \frac{\text{cm}^3}{\text{s}} = 957.1 \text{ cm}^3/\text{min}$$

According to results measured from the test as mentioned above, the constant flow as shown in Fig. 6 can be obtained as supply pressure is 9.6 Kg/cm². In this figure, the flow rate keeps almost constant as P_r varying or loading is changed. The variations of supply pressure and both temperatures are shown also. Constant flow cannot be attained other than supply pressures being 9.6 Kg/cm². Fig. 7 shows that the variations of both temperatures and flow rate with respect to load changing from 0 to 200 Kg with 20 Kg increment by using supply pressures 7·8·9·10 Kg/cm². Both figures reveal that the flow rate is influenced by temperatures variety mainly and supply pressure can be kept with small error less than 1%.

Fig. 6 The variations of (a) flow rate and (b) both temperatures with respect to load changing from 0 to 200 Kg with 20 Kg increment and (c) pressure ratio of recess and (d) supply pressure of nominal 9.6 Kg/cm² ($\times$: 1st measurement, $\triangle$: 2nd measurement, $\bullet$: 3rd measurement, $\square$: 4th measurement)

Experiment 2

Other design parameters $k=26$ Kg/cm, $x_0=1$ mm, and $\mu=0.18$ Pa·s are given, substituting these parameters into Eqs. (8b) and (9b) to determine the supply pressure of constant flow valve respectively by

$$P_s = k(\ell_0 + x_0) A_1^{-1} = \frac{26(3.891 + 0.1)}{8.0324} = 12.92 \text{ Kg/cm}^2$$

and

$$Q = \frac{\pi d_s g_s^3 k}{12 \mu A_2} = \frac{3.14 \times 3.155 \times 0.02375^3 \times 26 \times 9.8}{12 \times 0.18 \times 10^{-4} \times 7.814} \frac{\text{cm}^3}{\text{s}} = 1202.1 \text{ cm}^3/\text{min}$$

Fig. 7 The variations of both temperatures and flow rate with respect to load changing from 0 to 200 Kg with 20 Kg increment for experiment 1 by using supply pressures 7 · 8 · 9 · 10 Kg/cm² (× : 1st measurement, □ : 2nd measurement, ● : 3rd measurement, ○ : 4th measurement)

According to results measured from the test as mentioned above, the variations of both temperatures and flow rate with respect to load changing from 0 to 200 Kg with 20 Kg increment by using supply pressures 12, 12.5, 13, and 14 Kg/cm² are shown in Figure 8. The constant flow as shown in Fig. 8(c) can be obtained as supply pressure is 13 Kg/cm². Constant flow cannot be attained or larger variations of flow rate as load change other than supply pressures being 13 Kg/cm². This experiment reveals that supply pressure can be kept with small error less than 1% and shows also that the flow rate is influenced by temperatures variety mainly.

Fig. 8 The variations of both temperatures and flow rate with respect to load changing from 0 to 200 Kg with 20 Kg increment for experiment 2 by using various supply pressures ($\times$: 1st measurement, $\triangle$: 2nd measurement, $\bullet$: 3rd measurement, $\square$: 4th measurement)

4. Dynamic characteristics

From dynamic equilibrium as shown in Fig. 8, the motion equation of spool is expressed by

$$m\ddot{x}_d + c\dot{x}_d + kx_d = P_{rd}A_2 \quad (11a)$$

Using $\hat{x}_d = x_d/x_s$, $\omega_n = \sqrt{\frac{k}{m}}$, $\tau = \omega_n t$, $\bar{\omega} = \omega/\omega_n$, $\zeta = \frac{c}{2m\omega_n}$, $\bar{k} = \frac{k\ell_0}{P_s A_2}$,

$\frac{dx_d}{dt} = x_s \omega_n \frac{d\hat{x}_d}{d\tau}$ 及 $\frac{d^2 x_d}{dt^2} = x_s \omega_n^2 \frac{d^2 \hat{x}_d}{d\tau^2}$, above equation can be expressed in

Fig. 9 Equilibrium of dynamic forces

dimensionless form as

$$\ddot{\hat{x}}_d + 2\zeta\dot{\hat{x}}_d + \hat{x}_d = (\bar{k}\bar{x}_s)^{-1}\bar{P}_{rd} = \bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd} \quad (11b)$$

When $\bar{P}_{rd} = \bar{P}_{rd}^* \sin \bar{\omega}\tau$

$$\hat{x}_d = \hat{x}_{dc}^* \cos \bar{\omega}\tau + \hat{x}_{ds}^* \sin \bar{\omega}\tau$$

$$\dot{\hat{x}}_d = \bar{\omega}(-\hat{x}_{dc}^* \sin \bar{\omega}\tau + \hat{x}_{ds}^* \cos \bar{\omega}\tau)$$

$$\ddot{\hat{x}}_d = -\bar{\omega}^2(\hat{x}_{dc}^* \cos \bar{\omega}\tau + \hat{x}_{ds}^* \sin \bar{\omega}\tau)$$

Substituting above $\hat{x}_d$ and its derivatives into Eq. (11b) gives

$$\begin{aligned} & \left[(1 - \bar{\omega}^2)\hat{x}_{dc}^* + 2\zeta\bar{\omega}\hat{x}_{ds}^* \right] \cos \bar{\omega}\tau + \left[-2\zeta\bar{\omega}\hat{x}_{dc}^* + (1 - \bar{\omega}^2)\hat{x}_{ds}^* \right] \sin \bar{\omega}\tau \\ &= \bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \sin \bar{\omega}\tau \end{aligned} \quad (12)$$

$$\begin{bmatrix} 1 - \bar{\omega}^2 & 2\zeta\bar{\omega} \\ -2\zeta\bar{\omega} & 1 - \bar{\omega}^2 \end{bmatrix} \begin{bmatrix} \hat{x}_{dc}^* \\ \hat{x}_{ds}^* \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \end{bmatrix}$$

$$\Delta = (1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2$$

$$\hat{x}_{dc}^* = -2\zeta\bar{\omega}\bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \left[(1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2 \right]^{-1}$$

$$\hat{x}_{ds}^* = (1 - \bar{\omega}^2)\bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \left[(1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2 \right]^{-1}$$

$$\sqrt{(\hat{x}_{dc}^*)^2 + (\hat{x}_{ds}^*)^2} = \bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \left[(1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2 \right]^{-\frac{1}{2}}, \quad \phi = \frac{2\zeta\bar{\omega}}{1 - \bar{\omega}^2}$$

The harmonic solution can be yielded as

$$\hat{x}_d = \bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \left[(1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2 \right]^{-\frac{1}{2}} \sin(\bar{\omega}\tau + \phi) \quad (13)$$

Thus, the amplitude and phase of dynamic response $\hat{x}_d$ are

$$\hat{x}_d^* = \bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \left[(1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2 \right]^{-\frac{1}{2}} \quad (14a)$$

$$\phi = \frac{2\zeta\bar{\omega}}{1 - \bar{\omega}^2} \quad (14b)$$

Differentiating Eq. (13) with respect to time gives velocity of $\hat{x}_d$ as shown by

$$\dot{\hat{x}}_d = \bar{C}_s\bar{x}_s^{-1}\bar{P}_{rd}^* \bar{\omega} \left[(1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2 \right]^{-\frac{1}{2}} \cos(\bar{\omega}\tau + \phi) \quad (15)$$

Since $\frac{1 - (\bar{P}_{rs} + \bar{P}_{rd})}{1 - \bar{C}_s(\bar{P}_{rs} + \bar{P}_{rd} - \bar{P}_0)} = \bar{C}_s^{-1}$, substituting Eqs. (8) and (15) into Eq. (6c) gives

the dynamic response of flow rate due to working pressure fluctuating as shown:

$$\bar{Q}_d = -\alpha\bar{P}_{rd}^* \frac{\bar{\omega} \cos(\bar{\omega}\tau + \phi)}{\sqrt{(1 - \bar{\omega}^2)^2 + 4\zeta^2\bar{\omega}^2}} \quad (16a)$$

where $\alpha = R_0 A_1 C_s \omega_n$. Consequently, the relationship of amplitudes of dynamic responses between flow rate and spool displacement can be obtained by substituting Eq. (14a) into Eq. (16a) and shown by

$$\bar{Q}_d^* = \alpha \hat{x}_d^* \bar{C}_s^{-1} \bar{\omega} \quad (16b)$$

which reveals that the amplitude of dynamic flow rate is proportional to the product of spool amplitude and fluctuating frequency. Dimensionless parameters for various designs of the spool-type constant-flow valve are shown in Table 1.

Table 1 Dimensionless parameters of six design cases

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
$\bar{P}_0$	0	0.1	0.2	0.3	0.4	0.5
$\bar{C}_s$	1	1.11	1.25	1.43	1.67	2
$\bar{Q}_s$	1	0.9	0.8	0.7	0.6	0.5

Fig. 10 Amplitudes of $\hat{x}_d$ and $\bar{Q}_d$ for case 1 ($\bar{Q}_s = 1$) as $\bar{P}_{rd}^* = 0.01$

Fig. 11 Amplitudes of $\hat{x}_d$ and $\bar{Q}_d$ for case 1 ($\bar{Q}_s = 1$) as $\bar{P}_{rd}^* = 0.1$

The analytical results of case 1 subjected to various static pressure ratios are illustrated in Figs. 10 and 11 for $\bar{P}_{rd}^* = 0.01$ and 0.1 , respectively. Dimensionless parameter product $\alpha = 0.1$ and damping ratios $\zeta = 0.01, 0.1, 0.8$, and 1 are selected to determine.

In both figures, the amplitudes of spool displacement and flow rate are shown on left and right column, respectively. Which show that the amplitude of spool displacement is suppressed by static pressure ratio but proportional to the amplitude of dynamic pressure ratio. The amplitude of flow rate cannot be influenced by static pressure ratio but can be influenced by damping ratio, excitation frequency and proportional to the amplitude of dynamic pressure ratio.

The analytical results of cases 4 and 5 subjected to static pressure ratios $\bar{P}_{rd}^* = 0.01$ are illustrated in Figs. 12 and 13, respectively. Dimensionless parameter product $\alpha = 0.1$ and damping ratios $\zeta = 0.01, 0.1, 0.8$, and 1 for both cases are selected also. Both figures reveal not only the same behavior as shown by Figs. 11 and 12 but also show that the amplitude of spool displacement can increase as static flow rate is decreasing but the amplitude of flow rate cannot be influenced by static flow rate.

Fig. 12 Amplitudes of $\hat{x}_d$ and $\bar{Q}_d$ for case 4 ($\bar{Q}_s = 0.7$) as $\bar{P}_{rd}^* = 0.01$

From Fig. 10 to 13 reveals that (1) the resonance occurs as excitation frequency is close to the natural frequency of the spool-spring system, which induces large amplitudes of $\hat{x}_d$ and $\bar{Q}_d$, when damping ratio is smaller than 1, (2) in companions of Figs. 10, 12, and 13 for a specific $\bar{P}_{rd}^*$, $\hat{x}_d^*$ gets smaller as $\bar{Q}_s$ is increasing, but $\bar{Q}_d^*$ can be invariant as $\bar{Q}_s$ changes, (3) resonant peaks of both $\hat{x}_d^*$ and $\bar{Q}_d^*$ are reduced as damping is increasing, when damping ratio $\zeta \geq 1$ resonant peaks are suppressed to disappear, (4) $\hat{x}_d^*$ gets larger as $\bar{\omega}$ is decreasing, but maximum of $\bar{Q}_d^*$ presents near natural frequency since $\bar{Q}_d^*$ is proportional to the product of $\bar{\omega}$ and $\hat{x}_d^*$.

5. Conclusions

This paper verifies that constant flow cannot be attained by arbitrary collocations of design parameters. The design requirements for constant flow must satisfy that dimensionless displacement coefficient (*ddc*) of spool minus one can be equal to the

Fig. 13 Amplitudes of $\hat{x}_d$ and $\bar{Q}_d$ for case 5 ($\bar{Q}_s = 0.6$) as $\bar{P}_{rd}^* = 0.01$

product of ddc and equivalent pressure ratio of spring preload. Constant dimensionless flow rate is equal to inverse ddc and constant flow rate is proportional to cubic clearance and spring constant, but inversely to viscosity of oil and spool diameter. The results of dynamic analysis reveal that the amplitude of spool displacement is suppressed by static pressure ratio but proportional to the amplitude of dynamic pressure ratio. The amplitude of flow rate cannot be influenced by static pressure ratio but can be influenced by damping ratio, excitation frequency and proportional to the amplitude of dynamic pressure ratio.

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