Daily Planning for Three Echelon Logistics Considering Inventory Conditions*

Yoshiaki SHIMIZU** and Dicky FATRIAS**
** Department of Mechanical Engineering, Toyohashi University of Technology, 1-1, Hibarigaoka, Tenpaku-cho, Toyohashi, 441-8580, Japan
E-mail: shimizu@me.tut.ac.jp

Abstract
Under recent global competition and short product life cycle, we need to undertake agile decision making in every manufacturing process. With this point of view, this study provides a practical hybrid meta-heuristic method for a three echelon logistic network optimization. It supports decision making at a tactical and operational level associated with inventory management for daily planning. That includes a new idea to solve practical multi-depot VRP efficiently. Its solution procedure is developed by means of two modified heuristic methods known as the saving method and tabu search together with the graph algorithm of minimum cost flow problem. The resulting procedure is expected to play a role of optimization engine in the total system incorporated with information system for inventory management. Moreover, to enhance usability of the method, visualization of result is also realized by virtue of Google map API. Numerical experiments are carried out to validate effectiveness of the proposed approach.

Key words: Three Echelon Logistics Network, Dynamic Optimization, Hybrid Metaheuristic Method, VRP, Inventory Information, Google Map

1. Introduction
Due to agility, greenness and service innovation, daily logistics optimization is becoming important especially for small businesses like convenience stores or supermarkets in Japan. Recently, a review of articles published in the last decade within the context of supply chain management has been published1). Thereat, they reveal a scarcity of models that capture dynamic aspects relevant to real-world applications, and emphasize an increasing need for extensive studies on this topic.

Noticing such circumstance, in this paper, we have extended our strategic approach so as to include a certain decision at a tactical and operational level. Actually, we associate with vehicle routing problem (VRP) considering a substantial inventory management issue. Moreover, taking into account dynamic demand and inventory of warehouse, we try to give a practical approach amenable to innovative resolution to such daily logistics optimization. The final scope of this study aims at developing an integrated decision support system with information system that can dynamically manage appropriate data bases on the inventory of resources and the demand of products (See Fig.1).

The rest of the paper is organized as follows. In Section 2, we formulate the problem after giving a brief review of the associated studies. Section 3 outlines the proposed solution procedure. Numerical experiments are provided in Section 4. Finally, we give conclusions.
2. Problem Statement

2.1 Brief Review of the Related Studies

Regarding transportation among depots and customers, each vehicle must take a circular route from its depot as a starting point and a destination at the same time. This generic problem has been studied popularly as VRP\(^2\). The VRP is a well-known NP-hard combinatorial optimization problem which minimizes the total distance traveled by a fleet of vehicles under various constraints. Recent studies on VRP application can be roughly classified into the following four kinds.

One of them is an extension from generic customer demand satisfaction and vehicle payload limit. For examples, practical conditions such as customer availability or time window\(^3\),\(^4\), pick up\(^5\), split and mixed deliveries\(^6\) are concerned not only separately but in a combined manner\(^7\). The second is known as the multi-depot problem that tries to deliver from multiple depots\(^8\)-\(^10\). The thirds are interested in the multi-objective formulation for the single depot and multi-depot problems\(^11\)-\(^16\). Though these three classes might belong to an operational level, the last one\(^17\)-\(^20\) corresponds to a tactical concern. That is, decision on the allocations of depot is involved besides VRP.

Though many of those studies are solved by using a certain meta-heuristic method\(^21\),\(^22\), a certain local search is applied in the literature\(^23\). To effectively reduce the computational difficulty, a hybrid algorithm of Bender’s decomposition with genetic algorithm is also proposed\(^24\). Due to the difficulty of solution, however, only small problems with no less than a hundred customers are solved to validate its effectiveness except for the literature\(^25\). Moreover, though actual transportation cost depends not only the distance but also load (Ton-Kilo basis), those studies consider only distance (Kilo basis) to derive the route. Hence, the tactical concerns mentioned above are unfavorable to make a generic and consistent dealing over two decision levels, i.e., allocation problem and VRP.

2.2 Problem Formulation

Taking a global logistics network composed of major distribution center (DC), sub-DCs or depots (RS), and customers (RE), we try to decide the available depots, paths from DCs to depots and circular routes from every depot to its client customers (See Delivery section in Fig.1). The goal of this problem is to minimize total cost for daily logistics over planning horizon \(T\). This problem is formulated as the following mixed-integer programming problem under mild assumptions, e.g., round-trip transport between DC and depot; uni-modal transport; averaged time invariant unit costs and system parameters except for demand and inventory, independency or separable decision per each planning period, etc.
(p.1) Minimize for every $t \in T$
\[
\sum_{i=1}^{n} \sum_{j=1}^{m} \left( C_{ij} d_{ij} + H_{ij} f_{ij}(t) \right) + \sum_{v \in V} \sum_{p \in P} \sum_{p' \in P} c_{ij} d_{pp'} \left( g_{pp'}(t) + q_{pp'} z_{pp'}(t) \right) \\
+ \sum_{j=1}^{m} \left( H_{0, j} r_{j}(t) + H_{j, i} y_{j}(t) + H_{j, s}(t) + \sum_{i=1}^{n} f_{ij}(t) \right) + \sum_{i=1}^{n} F_{i} y_{i}(t)
\]
subject to
\[
\sum_{p \in P} z_{pp}(t) \leq 1, \quad \forall k \in K; \forall v \in V, \quad \exists t \in T \\
\sum_{p \in P} z_{pp}(t) - \sum_{p' \in P} z_{pp'}(t) = 0, \quad \forall p \in P; \forall v \in V, \quad \exists t \in T \\
\sum_{j=1}^{m} s_{j}(t) + \sum_{i=1}^{n} f_{ij}(t) = \sum_{v \in V} \sum_{k \in K} g_{jkv}(t), \quad \forall j \in J, \quad \exists t \in T \\
r_{j}(t) + s_{j}(t) + \sum_{i=1}^{n} f_{ij}(t) \leq Q_{j} x_{j}(t), \quad \forall j \in J, \quad \exists t \in T \\
g_{pp'}(t) \leq W_{v} z_{pp'}(t), \quad \forall p \in P; \forall p' \in P; \forall v \in V, \quad \exists t \in T \\
\sum_{p \in P} z_{pp}(t) \leq M y_{v}(t), \quad \forall v \in V, \quad \exists t \in T \\
\sum_{k \in K} g_{kp}(t) = 0, \quad \forall j \in J; \forall v \in V, \quad \exists t \in T \\
\sum_{v \in V} \sum_{p \in P} g_{jkv}(t) - \sum_{p' \in P} g_{jkv}(t) = D_{k}(t), \quad \forall k \in K, \quad \exists t \in T \\
\sum_{p \in P} \left( g_{pp'}(t) - D_{k}(t) z_{pp'}(t) \right) = \sum_{p \in P} g_{kkv}(t), \quad \forall k \in K, \forall v \in V, \exists t \in T \\
\sum_{j=1}^{m} z_{jkv}(t) = y_{v}(t), \quad \forall v \in V, \quad \exists t \in T \\
\sum_{j=1}^{m} z_{jkv}(t) = y_{v}(t), \quad \forall v \in V, \quad \exists t \in T \\
\sum_{p \in P} z_{pp}(t) \leq | \Omega |-1, \forall \Omega \subseteq P \setminus \{1\}, | \Omega | \geq 2, \forall v \in V \\
P_{i}^{\min} \leq \sum_{j=1}^{m} f_{ij}(t) \leq P_{i}^{\max}, \quad \forall i \in I, \quad \exists t \in T \\
r_{j}(t) + s_{j}(t) \leq S_{j} x_{j}(t), \quad \forall j \in J, \quad \exists t \in T
\]

Variables
\[ f_{ij}(t): \text{ load from DC } i \text{ to depot } j \text{ at period } t \]
\[ g_{pp}(t): \text{ load of vehicle } v \text{ on the path from } p \in P \text{ to } p' \in P \text{ at period } t \]
\[ r_{j}(t): \text{ take over inventory at depot } j \text{ at period } t \]
Journal of Advanced Mechanical Design, Systems, and Manufacturing

Vol. 7, No. 3, 2013

488

In (p.1), the objective function is composed of round-trip transportation costs between every DC and the opening depot (hereinafter just depot), circular transportation costs for traveling to every customer, shipping costs at DC, holding, handling and shipping costs at depot, and fixed charges of vehicles. Several constraints are applied: vehicles cannot visit a customer twice (Eq.(1)); vehicle visiting a certain depot must leave it (Eq.(2)); no travel between distribution centers (DCs) (Eq.(3)); material balance (Eq.(4)); upper bound capacity at depot (Eq.(5)); upper bound load capacity for vehicle (Eq.(6)); each vehicle must travel on a certain path (Eq.(7)); vehicles return to the depot empty (Eq.(8)); customer demand is satisfied (Eq.(9)); sum of inlet good must be greater than that of outlet by its demand (Eq.(10)); each vehicle leaves only one depot and returns there (Eqs.(11) and (12)); sub-tour elimination constraint (Eq.(13)) and the amounts of good available from DC are bounded (Eq.(14)); and the amounts of inventory are upper bounded (Eq.(15)). We also assume the following inventory control policy.

\[
\begin{align*}
\lambda_j(t) = & \begin{cases} 
(1-\zeta_j)(t-1) & \text{if } (1-\zeta_j)(t-1) \geq R_j \\
S_j - (1-\zeta_j)(t-1) & \text{if } (1-\zeta_j)(t-1) < R_j,
\end{cases} \\
& \forall j \in J, \forall t \in T
\end{align*}
\]

where \(\zeta (\leq 1)\) and \(R_j\) are a fouling rate of unsold goods and an ordering point at depot \(j\), respectively.

We know that it is almost impossible to solve the above problem with practical size using any currently available commercial software. Against this, under complicated situations resulting from a variety of real-life conditions, we have successfully solved various logistics optimization problems by using a method called hybrid tabu search (HybTS)\textsuperscript{27,28}. This is a two-level solution method in which the upper level sub-problem
optimizes the selection of available depots while the lower level sub-problem optimizes the paths from DCs to customers via depots so as to minimize the total cost. HybTS is not only a practical and powerful method but also flexible and suitable for a variety of extensions. Hence, we will deploy the similar idea to solve the above problem while being free from serious computational difficulty.

3. Daily Decision Associated with Inventory Conditions

3.1 Multi-level approach incorporating vehicle routing problem

For the daily logistics optimization, it is meaningful to take into account the inventory control at every depot. To make the foregoing hierarchical approach available for the present case, we have majorly invented two new ideas and integrated them into the framework of our hybrid method. In our best knowledge, such global approach has not been reported elsewhere.

In its first level, we choose the available depots using the modified tabu search. Then, in the second level, we tentatively obtain round trip paths from DCs to customers via depots using a graph algorithm for the minimum cost flow (MCF) problem. Assigning the customers thus allocated as the clients for each depot, we derive every vehicle route of depot using the modified saving method and modified tabu search. The result thus obtained is fed back to the first level to evaluate another candidate of available depots. This procedure will be repeated until a given convergence condition has been satisfied. The procedure of this algorithm is illustrated in Fig.2.

In developing the above algorithm, we need to obtain the MCF graph that considers the inventory at each depot. For example, the case where \(|I| = |J| = |K| = 2| is illustrated in Fig.3. In Table 1, we summarized the information required to put on the edges and nodes in the graph. In terms of the MCF graph thus derived, we can solve the optimal allocation problem extremely fast through a graph algorithm like RELAX429) together with its sensitivity analysis. The sensitivity analysis is amenable to repeatedly solve the problem with slightly different parameters one after another. After all, we can efficiently allocate the client customers to each depot on the Ton-Kilo basis.

Then, to solve the VRP in terms of the Ton-Kilo basis, we applied an approach composed of the modified saving method and the modified tabu search in a hybrid manner25),26). Thereat, we noted the fixed operational cost for the working vehicle should be involved in the economic evaluation. After all, the algorithm of the modified saving method is outlined as follows.

Step 1: Create round trip routes from the depot to all customers. Compute the saving value by \(s_{ij} = (d_{0j} - d_{0i} - d_{ij})D_J^{+}(d_{0j} + d_{0i} - d_{ij})q\), where \(D_J\), \(q\) and \(d_{ij}\) denotes the demand at location \(j\), weight of vehicle itself and distance between locations \(i\) and \(j\), respectively.

Step 2: Order these pairs in descending order of the saving values.

---

**Fig.2** Flow chart of the solution procedure

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Create round trip routes from the depot to all customers. Compute the saving value by (s_{ij} = (d_{0j} - d_{0i} - d_{ij})D_J^{+}(d_{0j} + d_{0i} - d_{ij})q), where (D_J), (q) and (d_{ij}) denotes the demand at location (j), weight of vehicle itself and distance between locations (i) and (j), respectively.</td>
</tr>
<tr>
<td>2</td>
<td>Order these pairs in descending order of the saving values.</td>
</tr>
</tbody>
</table>
Step 3: Merge the path following the order as long as the feasibility is satisfied and the saving value is greater than \(-F_v/c_v\), where \(F_v\) denotes the fixed operational cost of vehicle.

However, since the modified saving method derives only an approximated solution, we move on the modified tabu search to update such solution. The modified tabu search is a variant that probabilistically accepts the degraded candidate like simulated annealing in its local search. Here, we emphasize such an advantage that transportation costs are able to be accounted on the same Ton-kilo basis over the procedures at the upper level (first and second) and lower one (third) in Fig. 2.

### 3.2 Analysis of inventory level on demand variation

It is commonly known that too much inventory slips the economical efficiency while the stock-out or opportunity loss will happen in the opposite case. For the daily logistics, therefore, it is of special importance to correctly estimate the demand and properly manage the inventory. Generally speaking, however, estimating demand correctly is almost impossible in many cases while it is possible to roughly estimate the extent of deviation from the experience.
Under such circumstance, it is relevant and practical to try to reveal the relation between the extent of demand deviation and the inventory level through parametric approach. Through such analyses, we can setup a reliable inventory level to maintain the economically efficient logistics while preventing from the state of stock-out. Though such consideration is able to reveal many prospects for the robust and reliable logistic systems, it has not been almost concerned in the network optimization of logistics so far due to computational difficulties.

4. Numerical Experiments

4.1 Setup of test problem

To examine some performance of the proposed method, we provided several benchmark problems with different problem sizes, i.e., $|I|$, $|J|$, $|K|$. Every system parameter is set randomly within the respective prescribed interval as summarized in Table 2. Location of every member is also generated randomly, and distances between them are calculated as the Euclidian distance.

Table 2 Notes on parameter setup

<table>
<thead>
<tr>
<th>Member</th>
<th>Item</th>
<th>Range</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>$Hp$: Shipping cost</td>
<td>$100\times[0.2, 0.8]$</td>
<td>$&lt;3&gt;$</td>
</tr>
<tr>
<td>$P_{\text{max}}$: Available (Max)</td>
<td>$1000\times[0, 1] + P_{\text{min}}$</td>
<td>$&lt;5&gt;$, Total $P_{\text{max}} &gt;$ Total capacity of RS</td>
<td></td>
</tr>
<tr>
<td>$P_{\text{min}}$: Available (Min)</td>
<td>$1000\times[0.2, 0.8]$</td>
<td>$&lt;5&gt;$, Total $P_{\text{min}} &gt;$ Total demand</td>
<td></td>
</tr>
<tr>
<td>RS</td>
<td>$Hs$: Shipping cost</td>
<td>$100\times[0.2, 0.8]$</td>
<td>$&lt;3&gt;$</td>
</tr>
<tr>
<td>$Ha$: Handling cost</td>
<td>$50\times[0.2, 0.8]$</td>
<td>$&lt;3&gt;$</td>
<td></td>
</tr>
<tr>
<td>$Ho$: Holding cost</td>
<td>$100\times[0.2, 0.8]$</td>
<td>$&lt;5&gt;$</td>
<td></td>
</tr>
<tr>
<td>$Q$: Capacity</td>
<td>$p\times[0.2, 0.8]$</td>
<td>$&lt;5&gt;$, $p=100\times</td>
<td>K</td>
</tr>
<tr>
<td>$S$: Allowable inventory</td>
<td>$x\times[0.5, 0.7]$</td>
<td>$&lt;3&gt;$, Varing at each time</td>
<td></td>
</tr>
<tr>
<td>RE</td>
<td>$D$: Demand</td>
<td>$100\times[0.2, 0.8]$</td>
<td>$&lt;3&gt;$, Total demand $&lt;$Total capacity of RS$^*$</td>
</tr>
</tbody>
</table>

$C_y = 3$, $c_v = 1$, $W_v = 500$, $F_v = 50 000$, $q_v = 10$; $<n>$ multiple of $n$

4.2 Results for leading condition

Regarding daily demand, we randomly changed the amount within $100(1\pm\alpha)$% from the foregoing day. On the other hand, the unsold goods at each depot are stored as the inventory and it is possible to use them in the following days. However, it is supposed to be spoiled randomly within $10\%$ ($\zeta=0.1$) and the goods are supplied to the upper limit when the inventory level becomes below the prescribed safety level ($R_j=\beta S_j$), i.e., the fixed order quantity policy.

* Under this condition, stock-out status will not occur.
First, we solved the smaller size problems like $|I|=3$, $|J|=10$ and $|K|=100$ over 30 days and $|I|=5$, $|J|=20$ and $|K|=200$ over 10 days. Parameters $\alpha$, $\beta$ and $\zeta$ are set at 0.3, 0.5 and 0.1, respectively. Figures 4 and 5 illustrate the changes of demand and inventory during the planning horizon. Under these conditions, we derive the optimal cost that broadly changes with the demand fluctuation as shown in Fig.6. In Fig.7, we can see that the change of working numbers of depot is moderated and kept nearly constant (around 60%). However, working rate of each depot differs greatly as shown in Fig.8. This observation is available for considering the restructuring of logistics at the next stage. That is, the depots suffering low working rate may be integrated into the other higher ones.

Moreover, we solved the larger problems to examine the necessary computation time. Fixing the planning horizon at 1, $|I|=10$, and $|J|=30$, we solved the problems like $|K|\in\{250, 500, 1000, 1500, 2000, 2500\}$. As expected a priori, the required CPU time increases exponentially with the size as shown in Fig.9. Even for these larger size problems, however, we can obtain the result within a reasonable time or around several hours.
From the convergence profile for the largest problem in Fig.10, we can confirm the sufficient convergence. By means all of these results, we can claim the significance of the approach and computational effectiveness of the proposed method.

4.3 Results over wide range of deviations

To analyze effect of the inventory condition against demand deviations, we carried out a parametric study regarding ordering points using a small model like $|I|=2$, $|J|=5$, $|K|=100$ over 30 days. Actually, we solved every pair of problem with five different ordering points ($\beta=\{0.1, 0.2, 0.3, 0.4, 0.5\}$) and four different ranges of demand variation ($\alpha=\{0.2, 0.3, 0.4, 0.5\}$). This comes to that 600 optimization problems were totally solved. Under the same conditions as before, we applied these values in turn to derive each solution. Now, we show the results in Figs.11 and 12.

Figure 11 shows a feature of total cost regarding the range of demand deviation and the level of ordering point. Due to the non-deterministic parameter setting, complicatedly winding profile is observed. But its trend is plausible since the region where the minimum cost locates will move on the higher ordering point according to the increase in the deviation ranges as a whole. This fact suggests that it is important to control the ordering point or inventory level according to the demand deviation in the cost management. When we cut off only the inventory cost from the total cost, its changes are rather simple as shown in Fig.12. Since higher stock level needs more holding cost, the cost increases proportionally with the ordering point regardless of the deviation ranges of demand.

Finally, from these parametric studies, we claim the adequateness of the applied model behind the mathematical formulation. Plausibility of the results can support the significance of the approach if the actual parameters were used in real world optimization.
4.4 Prospect as a working tool

To realize the planning system illustrated in Fig. 1 as a final goal, it is essential to provide a user friendly interface to manage the system. At the planning section in production side, this goal is closely related to data handling and visualization of the circumstance at hand. Regarding this matter, we can effectively utilize some software developed by Google Maps API. By now, we have deployed the following step-wise procedure by using Java scripts and appropriate free software.

Step 1: Collect the address of members in an Excel spread sheet or text sheet.
Step 2: Add the information of longitude and latitude of every one into the sheet. We can utilize an appropriate free software for this purpose.
Step 3: Calculate the distance between every pair of members using a procedure of Google Map API known as Gio-coding.
Step 4: Solve the optimization problem by the proposed method.
Step 5: Display the routes obtained from Step 4 in Google map.

In the result of the illustrative problem with \(|I|=1\), \(|J|=3\) and \(|K|=17\), every depot has a single route. In Fig.13, for simplicity, only the routing paths from depot 1 are shown in using the marks (A – B – – – – K – A; dotted arrow is super-imposed to easily capture the straight path of the actual circular route). We can see this kind of visual information is very helpful for some tasks at operational level. However, there still remain many possibilities to add more amenable service information using GIS applications and Google Map API.

5. Conclusion

We have proposed a hierarchical approach to optimize a daily logistics problem including inventory control at depots and vehicle routing for customer delivery. For this purpose, we have extended our two-level method by virtue of the modified saving method and the modified hybrid tabu search together with the graph algorithm to solve the MCF problem in a hybrid manner. Through this approach, we can evaluate the transportation cost not only practically but consistently in terms of the Ton-Kilo basis.

In the numerical experiments, we have shown the proposed method can solve the complicated and manifold problem that has never been solved previously within a reasonable computation time. To enhance the solution speed for larger problems in advance, we can apply the parallel computing technique deployed previously\(^{30}\). We also mentioned about the Web application referred to Google Maps API to enhance a practical usability. Eventually, this claims the effectiveness of the proposed idea and the prospect of the present challenge.
Future studies should be devoted to relax the conditions assumed here. The step-wise procedures for visualization are favored to be integrated into the single Java script. Eventually, we aim at establishing a total decision support system as illustrated in Fig.1 for daily optimization associated with low carbon logistics.

Acknowledgements

This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid, 25350442, Japan. We also appreciate Mr. Tomoaki Yoshioka for his good help when developing a Google Map application.

References


(29) Massachusetts Institute of Technology, Lab. For Information and Decision Systems. Available