Improvement of Convergence for Adaptive Feed-Forward Cancellation Using Variable Gains in a Head Positioning System of Hard Disk Drives*

Shota YABUI**, Itsuro KAJIWARA***, Shigeo NAKAMURA** and Takenori ATSUMI**

** HGST Japan, Ltd.
1, Kirihara, Fujisawa, Kanagawa 252-8588, Japan
E-mail: shota.yabui@hgst.com
*** Division of Human Mechanical Systems and Design, Hokkaido University
Sapporo, Hokkaido 060-8628, Japan

Abstract
We present an enhanced adaptive feed-forward cancellation (AFC) with variable gains to be used in the head positioning control system of a hard disk drive (HDD). The variable gains can help to improve the convergence characteristic of the enhanced AFC after the track seeking control. Therefore, the proposed enhanced AFC can improve the transient response of the head positioning after the track seeking control. The track seeking simulation results from HDDs showed that the proposed control method compensated for the disturbances caused by the repeatable run-out (RRO) and airflow-induced vibration immediately after the track seeking control.

Key words: Hard Disk Drive, Repeatable Run-Out, Non-Repeatable Run-Out, Adaptive Feed-Forward Cancellation, Transient Response

1. Introduction
The rapid growth in demand for a larger data capacity requires an increase in the areal density of hard disk drives (HDDs). The head positioning control system of an HDD must compensate for any disturbances, because increasing the track density requires improving the head positioning accuracy in the head positioning control system of an HDD(1). There are basically two types of disturbances in the head positioning control system of HDDs, repeatable run-out (RRO) and non-repeatable run-out (NRRO). RRO is the periodic disturbance caused by the disk rotation. NRRO is the non-periodic disturbance caused by some other factors. One of the main factors of NRRO is the airflow induced vibrations of disks, which is appropriately called disk flutter(2). The control system must compensate for these disturbances.

Adaptive feed-forward cancellation (AFC) was used to compensate for RRO(3)(4), and(5) in several previous studies. Moreover, an enhanced AFC was used to compensate for disk flutter induced positioning errors(6). The performance was optimized by using loop shaping techniques that were based on the vector locus in the frequency domain. The simulation and experimental results showed the head positioning accuracy was improved for the steady-state responses in a track following control.
However, the enhanced AFC cannot improve the transient responses in a track-seeking control because the transient characteristics of the enhanced AFC may worsen the positioning error after the track-seeking control. Therefore, the calculation of enhanced AFC should stop during the track-seeking control, and restart after the track-seeking control. As a result, the control system cannot compensate for RRO and the disk flutter induced positioning errors immediately after track-seeking control.

Especially, the amplitudes of RROs and disk flutter induced positioning errors are dependent on the disk radial location. RROs occur due to disk distortion caused by the disk clamp fasteners. The distortion affects is serious on the inner side of the disk. The amplitude gradually increases as the magnetic head moves towards the inner side of the disk. On the other hand, disk flutter occurs due to the disk mode shape. The amplitude gradually increases as the magnetic head moves toward the outer side of the disk. The amplitude variations worse the positioning accuracy after the track seeking control. It decreases the data transmission rate of the HDDs because the control system requires time to settle the transient responses.

We present an enhanced AFC with variable gains in a head positioning control system of an HDD in this paper. The enhanced AFCs with variable gains that are optimized according to the head position are used in the control system to compensate for the variation in disturbance amplitudes. The convergence characteristic of the enhanced AFC is improved by using the variable gains in the settling responses after the track seeking control. It increases the data transmission rate of the HDDs by the improvement of the transient responses. The track seeking simulation results of HDDs demonstrated that the proposed method compensates for RROs and disk flutter induced positioning errors immediately after the track seeking control.

2. HDD Head Positioning Control System

2.1. Features of head positioning system

Figure 1 illustrates the head positioning control system in HDDs. The HDD is comprised of a voice coil motor, several magnetic heads, several disks, and a spindle motor. In the head positioning system, the controlled variable is the head-position signal, which is generated from the embedded information in the servo sectors located at regular intervals on the disks. The disturbances in head positioning systems are classified as either RRO or NRRO. RRO is the periodic disturbance caused by the disk rotation. NRRO is the non-periodic disturbance caused by some other factors. One of the main NRRO factors is disk flutter vibrations. Figure 2 shows the power spectrum of a RRO and disk flutter induced positioning errors defined as HDD Benchmark Problems. RRO disturbance may greatly degrade the HDD servo performance, and disk flutter is a major contributor to head positioning errors. Moreover, the amplitude of these disturbances depends on the disk radial location.
2.2. Amplitude variation of RRO

The shapes of servo tracks have certain irregularities due to various disturbances that occur during the servo writing process. These written-in disturbances are the main sources of the RRO. The main root cause of disturbance is disk distortion caused by the disk clamp fasteners\(^{(8)}\). The distortion is bigger on the inner side of the disk. Therefore, the RRO amplitude gradually increases as the magnetic head moves from the disk outer diameter (OD) to the inner diameter (ID) of the disk. Figure 3 shows the relationship between the head position and the normalized RRO amplitude when mimicking the results from a previous study\(^{(8)}\) (normalized amplitude of RRO is 1 at ID). In Fig.3, the most OD is 0mm, most ID is 20mm. The relationship can be calculated from the finite element model (FEM) analysis for disk.

2.3. Amplitude variation of disk flutter induced positioning error

Disk flutter occurs due to the disk mode shape caused by airflow\(^{(9)(12)}\). The notation for disk flutter mode is \((m, n)\), where \(m\) is the number of nodal circles and \(n\) is the number of nodal diameters. For example, the notation \((0, 1)\) represents a disk flutter mode with no nodal circle and one nodal diameter. In the head positioning control system, the amplitude of the disk
flutter induced positioning error is small when the number of nodal circles \( m \) is not 0. We focus on the \((0, n)\) mode of disk flutter in this paper. Figure 4 shows examples of modal disk flutter shapes. The amplitude of the disk flutter induced positioning error becomes relatively large on the outside track of disks because of the modal shape. Figure 5 shows the relationship between the head position and the amplitude of the disk flutter induced positioning error (normalized amplitude of the positioning error is 1 at OD). The relationship can be also calculated from FEM analysis for a disk.

3. Proposed Control System

A conventional enhanced AFC was developed to compensate for RROs and disk flutter induced positioning errors in HDDs\(^{(6)(13)}\) in previous studies. The simulation results showed the head positioning accuracy was improved in the track following control. The head positioning accuracy was improved by the optimization of the AFC parameters in the frequency responses. Therefore, the compensation and convergence performances were improved in the following control. However, the optimization method cannot improve the transient responses after the track seeking control. The design method only consider the steady response and cannot compensate the the amplitude variations of the disturbances in the track-seeking control. To improve the transient responses, we propose a control system that uses an enhanced AFC with variable gains (a proposed enhanced AFC) that are optimized according to the head position. The proposed control system is introduced in this chapter.

3.1. Adaptive algorithm of the conventional enhanced AFC

A conventional enhanced AFC is explained in this section using a block diagram of a
control system. The parameters of the conventional enhanced AFC are estimated by using the error signals in a control system to compensate for any disturbances. Figure 6 shows a block diagram of a control system with the conventional enhanced AFC, where $P$ is the controlled object and $C$ is the stabilizing controller, $r(k)$ is the reference, $e(k)$ is the position error signal, and $d(t)$ is the disturbance. A conventional enhanced AFC can compensate for disturbances by using the adaptive algorithm,

$$u(k) = p(k-1)\cos(\sqrt{1-\zeta^2}\omega Tk) + q(k-1)\sin(\sqrt{1-\zeta^2}\omega Tk).$$

Equation (1) indicates the output of the conventional enhanced AFC; the coefficients $p(k)$ and $q(k)$ are the adaptive parameters that were updated by the adaptive laws as the following equations,

$$p(k) = e^{-\xi T_k} p(k-1) + \lambda e(k)\cos(\sqrt{1-\zeta^2}\omega Tk + \theta),$$

$$q(k) = e^{-\xi T_k} q(k-1) + \lambda e(k)\sin(\sqrt{1-\zeta^2}\omega Tk + \theta).$$

In these equations, $T$ is the sampling time, $k$ is the sample number, $\omega$ is the desired compensation frequency, $\lambda$ is the learning rate of the algorithm, $\theta$ is the phase parameter of the AFC output, and $\zeta$ is the damping ratio. $\lambda$ and $\theta$ are the design parameters. The disturbance $d(t)$ is completely cancelled when the estimates of the coefficients, $p(k)$ and $q(k)$, have nominal values. In the recurrence formula of the conventional enhanced AFC, the adaptive algorithm has a damping function that is used as a forgetting factor, $e^{-\xi T_k}$. If $\zeta$ is equal to 0, the adaptive algorithm is equal to a traditional AFC. The design method of $\theta$ was introduced by using a vector locus\(^{(13)}\)\(^{(14)}\). The conventional enhanced AFC can be used to compensate for RROs and disk flutter induced positioning errors in the track following control. The simulation and experimental results showed that the head positioning accuracy was improved in steady-state responses.

The conventional enhanced AFC in Fig. 6 does not turn on during a track-seeking control because the transient characteristics of the conventional enhanced AFC may worsen the positioning error after the track-seeking control.

The calculation of conventional enhanced AFC should stop during the track-seeking control, and restart after the track-seeking control. The control method is called ‘true track’\(^{(7)}\). As a result, the conventional enhanced AFC cannot compensate for disturbances immediately after track-seeking control.

3.2. The proposed enhanced AFC: the enhanced AFC with variable gain

We proposed an enhanced AFC with variable gains to compensate for amplitude varia-
Fig. 7 Feedback control systems with the proposed enhanced AFC

Fig. 8 Time response of the reference \( r(k) \) and the relationship of the control mode


tions from disturbances. Figure 7 shows the proposed control system. In Fig. 7, \( G_o(x) \) is the variable gain for the output-port of the proposed enhanced AFC, and \( G_i(x) \) is the variable gain for the input-port of the proposed enhanced AFC. \( G_o(x) \) is determined so that it coincides with the amplitude of the disturbances using the following equation:

\[
G_o(x) = f_o(x),
\]

(4)

where \( x \) is the head position, and \( f_o(x) \) is the function with respect to the head position. \( f_o(x) \) is designed to match the variation characteristics of the disturbances. For example, \( f_o(x) \) is set as the polynomial approximation for the relationship between amplitude and head position \( x \). \( G_o(x) \) is 0 during the track seeking control so that it does not have residual vibrations of the enhanced AFCs. At other times, \( G_i(x) \) is given by

\[
G_i(x) = 1/G_o(x) = 1/f_o(x).
\]

(5)

In the track following control, \( G_o(x) \times G_i(x) \) is equal to 1. The variable gains do not affect the stability of the control system. In the track seeking control, \( u(k) \) can compensate for the amplitude variations of the disturbances made by the variable gain \( G_o(x) \). Therefore, the
control system can immediately compensate for the RROs and disk flutter induced positioning errors in the settling responses after the track seeking control.

Figure 8 shows an example of time response of the reference $r(k)$. The $r(k)$ indicates the time responses including the track-seeking control and track-following control (0 - 15ms: track-seeking, other time: track-following control). The track-seeking control moves the head position from the outside track to the inside track. At the start time of the track-seeking control, $G_i(x)$ is switched from $1/f_g(x)$ to 0. The $G_i(x)$ is 0 to prevent the transient response during the track-seeking control. The $G_o(x)$ is $f_g(x)$ to compensate the amplitude variation. The $G_i(x)$ is 0 to prevent the effect of the control system during the track-following control.

4. Verification of Proposed Control System through Simulation

4.1. Controlled Object

We have designed a head positioning control system using an HDD Benchmark Problem\(^{(1)}\) to verify the effectiveness of the proposed control system. The mechanical system $P(s)$ is given by the following equation in the HDD Benchmark Problem, where $i$ is the number of modes under consideration, $\alpha_m$ is the residue of each mode, $\omega_m$ and $\zeta_m$ are the natural angular frequency and damping ratio of each resonance, respectively, and $K_p$ is the plant gain,

$$P_m(s) = K_p \sum_{i=1}^{l} \frac{\alpha_m(i)}{s^2 + 2\zeta_m(i)\omega_m(i)s + \omega_m^2(i)}.$$ (6)

In this model, $l$ is seven, and so the model order is fourteen. $K_p$ is $3.74 \times 10^9$, and the values of the other parameters are listed in Table 1. In the HDD Benchmark Problem, the sampling time $\tau$ is 37.9 $\mu$s, and the time-delay is 10.0 $\mu$s. Figure 9 shows the frequency responses of the controlled object.

---

**Table 1 Parameters of $P(s)$ from HDD Benchmark Problem**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\alpha_m(i)$</th>
<th>$\omega_m(i)$</th>
<th>$\zeta_m(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3000 $\times 2\pi$</td>
<td>0.01</td>
<td>0.005</td>
</tr>
<tr>
<td>3</td>
<td>4100 $\times 2\pi$</td>
<td>-1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>5000 $\times 2\pi$</td>
<td>0.03</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>8200 $\times 2\pi$</td>
<td>1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>6</td>
<td>12300 $\times 2\pi$</td>
<td>-1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>7</td>
<td>16400 $\times 2\pi$</td>
<td>1.00</td>
<td>0.02</td>
</tr>
</tbody>
</table>
4.2. Disturbances Models

We focus on the following disturbances: RRO at 360 Hz and disk flutter induced disturbance signal at 900 Hz. This paper also uses the disturbance model defined in the HDD benchmark problem. This software has been widely used for HDD research\(^{(15),(7),(7),(7),(7),(20,)}\). In the HDD Benchmark Problem, RRO is defined as a sinusoidal signal with an angular frequency of \(360 \times 2\pi \) rad/s. The disk flutter induced positioning error is defined as the output when a white Gaussian noise with a standard deviation \(\sigma = 1\) is injected into the transfer function given by

\[ D(s) = \frac{2a_d \eta_d \omega_d^2}{s^2 + 2\eta_d \omega_d s + \omega_d^2}, \quad (7) \]

where

\[ a_d = 0.20, \quad \eta_d = 0.01, \quad \omega_d = 900 \times 2\pi. \quad (8) \]

In the HDD Benchmark Problem, the amplitude variation of the disturbances is not defined. Therefore, we defined the amplitude variation for each disturbance. For RRO, the variable gain for the output-port \(G_{oR}(x)\) was given as,

\[ G_{oR}(x) = f_{oR}(x) = 196430x^3 - 2393x^2 + 9x + 0.2. \quad (9) \]

\(f_{oR}(x)\) is set as the polynomial approximation for the graph in Fig. 3. For the disk flutter induced positioning error, we defined \(G_{od}(x)\) based on the results of the polynomial approximation for the graph in Fig. 5. \(G_{od}(x)\) was given as,

\[ G_{od}(x) = f_{od}(x) = 2000x^2 - 85x + 1. \quad (10) \]

\(G_{oR}(x)\) and \(G_{od}(x)\) are 0 during the track seeking control so that there are no residual vibrations of the enhanced AFCs. At other times, \(G_{oR}(x)\) and \(G_{od}(x)\) are given by

\[ G_{oR}(x) = 1/G_{od}(x), \quad G_{od}(x) = 1/G_{oR}(x). \quad (11) \]

4.3. Feedback Controller Using Enhanced AFC

The feedback control system is introduced in this section. The stabilizing controller \(C\) is produced through the combination of the PI-lead filter \(C_p\) and the notch filter \(C_n\). \(C_p\) is discretized using a bilinear transformation in order to implement the control system. \(C_n\) is discretized using the bilinear transformation with frequency prewarping. To enlarge the gain

---

Fig. 10 Frequency response of stabilizing filter: \(C\)
Table 2  Enhanced AFC parameters

<table>
<thead>
<tr>
<th>i</th>
<th>AFC gain $\lambda_i$</th>
<th>Damping ratio $\xi_i$ [%]</th>
<th>Angular frequency $\omega_1$ [rad/s]</th>
<th>Phase $\theta_i$ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>0</td>
<td>360°×2π</td>
<td>-150.35</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>0.01</td>
<td>900°×2π</td>
<td>-113.87</td>
</tr>
</tbody>
</table>

margin to more than 5 dB, the phase margin is increased to more than 35°, and to set the open-loop gain 0-dB crossover frequency to 1 kHz, $C_p$ and $C_n$ are given by

$$C_p(s) = \frac{0.0408(s + 2\pi \cdot 10)(s + 2\pi \cdot 250)}{s(s + 2\pi \cdot 4000)}, \quad (12)$$

$$C_n(s) = \frac{s^2 + 2\xi_{dnc} \omega_{dnc} s + \omega_{dnc}^2}{s^2 + 2\xi_{nnc} \omega_{nnc} s + \omega_{nnc}^2}, \quad (13)$$

where

$$\xi_{nnc} = 0.01, \quad \xi_{dnc} = 0.13, \quad \omega_{nnc} = 4500 \times 2\pi. \quad (14)$$

Figure 10 shows the frequency response of $C$.

The control system uses the following two enhanced AFCs to compensate for an RRO and disk flutter vibration:

$$u(k) = \sum_{i=2}^{2} p_i(k - 1) \cos(\sqrt{1 - \xi_i^2} \omega_1 T k) + \sum_{i=2}^{2} q_i(k - 1) \sin(\sqrt{1 - \xi_i^2} \omega_1 T k), \quad (15)$$

$$p_i(k) = e^{-\xi_i \omega_1 T k} p_i(k - 1) + \lambda_i e(k) \cos(\sqrt{1 - \xi_i^2} \omega_1 T k + \theta_i), \quad (16)$$

$$q_i(k) = e^{-\xi_i \omega_1 T k} q_i(k - 1) + \lambda_i e(k) \sin(\sqrt{1 - \xi_i^2} \omega_1 T k + \theta_i). \quad (17)$$

Table 2 lists the parameters of the enhanced AFC. $\theta_i$ was designed in order to achieve the best performance in the frequency domain(13). $\lambda_i$ is gain and $\xi_i$ is function of a damping ratio in the enhanced AFC(6). For RRO, the disturbance model of a damping ratio is 0. $\xi_1$ is 0 and $\lambda_1$ is set so that it matched the width of RRO. For disk flutter induced positioning error, in order to match the disturbance model of the disk flutter, the parameter values $\lambda_2$ and $\xi_2$ are the same as the values $\lambda_1$ and $\eta_1$ in Eq.(7).

The frequency response of the open-loop characteristics is shown in Fig. 11. The vector locus of the open-loop characteristics is shown in Fig. 12, and the frequency response of the sensitivity function is shown in Fig. 13. We can see from these figures that the circles in a vector locus obtained by using the enhanced AFC recede from the critical point $[-1, 0]$ on the Nyquist diagram, and the enhanced AFC can decrease the sensitivity function’s gain at the peak frequencies of the corresponding disk flutter vibrations. The design method was shown in our previous study(13)(14).
Fig. 11 Frequency response of open-loop characteristics

Fig. 12 Nyquist diagram

Fig. 13 Frequency response of sensitivity function
4.4. Performance Evaluation

Simulations are conducted for the track seeking control using the above mentioned control system design results. Figure 14 shows a block diagram of the track seeking control system with the proposed enhanced AFC. Figure 15 shows a block diagram of the track seeking control system with the conventional enhanced AFC. Here, $d(t)$ is the disturbance signal, $F_{acc}$ is the acceleration feed-forward input signal, and $F_{pos}$ is the position feed-forward input signal. In this simulation, the head position $x$ on the outside track was set as 0 m, and $x$ on the inside track was set as 0.02 m. $d(t)$ includes a RRO and disk flutter induced positioning error. We conducted two types of simulations. First, the track seeking control moved the head position from the OD ($x = 0$) to the ID ($x = 0.02$). The target seek time was set from 5 to 12.6 ms. The time length was 200 samples. The acceleration input $F_{acc}$ and the position feed-forward input signal $F_{pos}$ were generated using the sampled-data polynomial, which satisfies the boundary conditions as well as the characteristics of the ZOH\cite{21}. Figure 16 shows the time responses of $F_{acc}$ and $F_{pos}$ when the head position moved from the $x = 0$ to $x = 0.02$. The time response of the $d(t)$ at the head position was given as the result shown in Fig. 17. In this case, $d(t)$ is mainly caused by the RRO after the track seeking control. We compared the proposed method with the conventional one\cite{7}, in which the $p(k)$ and $q(k)$ were not updated during the track-seeking control, and were restarted after the track-seeking control. Figure 18 shows the output signal of the enhanced AFCs, where the dashed-line is that for the conventional enhanced AFC and the solid line is that for the proposed enhanced AFC. Figure 19 shows the positioning error signal around the target track. The proposed enhanced AFCs can compensate for the position error signal immediately after the track seeking control, because the variable gains $G_{dR}$ and $G_{od}$ can control the output match to the disturbance amplitude during the track seeking control. On the other hand, the conventional AFC cannot change the output signal during the track seeking control, because the input of the conventional AFC is 0 (turn off the input of switch in Fig.15) during the track seeking control.
Next, the track seeking control moved the head position from the OD ($x = 0$) to the ID ($x = 0.02$). The target seek time was also set from 5 to 12.6 ms. Figure 20 shows the time responses of $F_{\text{acc}}$ and $F_{\text{pos}}$. The time response of the $d(t)$ at the head position was given as the result shown in Fig. 21. In this case, $d(t)$ is mainly caused by the disk flutter after track seeking. The solid-line in Fig. 22 shows the output signal from the proposed enhanced AFC, and the solid-line in Fig. 23 shows the positioning error signal around the target track. The proposed enhanced AFC can also compensate for the position error signal immediately after track seeking. These results showed that the convergence characteristic of the enhanced AFCs can be improved by using the variable gains. Therefore, the proposed control system can compensate for the amplitude variation of the RRO and disk flutter induced positioning error in the head positioning systems of HDDs.

5. Conclusion

We proposed an enhanced AFC with variable gains in the head positioning control of an HDD to improve the convergence characteristic of the enhanced AFC. The control system included the enhanced AFCs with variable gains that are optimized according to the head position. The control system can compensate for any amplitude variations of the RROs and disk flutter induced positioning errors. The convergence characteristic of the enhanced AFC improved the settling responses in the head positioning system of HDDs. The track seeking simulation results of HDDs demonstrated that the proposed method compensated for the
RROs and disk flutter induced positioning errors immediately after the track seeking control.

References


(12) Shen, I. Y. and Kim, H., “A Linearized Theory on Ground-Based Vibration Response of
Fig. 22 Output signal $u(k)$ from enhanced AFC: ID ($x = 0.02$) to OD ($x = 0$)

Fig. 23 Position error signal $e(k)$: ID ($x = 0.02$) to OD ($x = 0$)


