Reliability analysis of spherical function generating mechanisms

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Abstract
Spherical mechanisms are widely used in engineering applications. Minimizing their motion error is critical for high performance of such mechanisms. The motion error is significantly affected by uncertainties in spherical mechanisms. The impact of the uncertainties on the motion error, however, is rarely considered in mechanism analysis and synthesis. In this work, we quantify the effects of randomness in the major mechanism dimension variables on the motion error through kinematic reliability with a probabilistic approach. Two types of reliability, the point kinematic reliability and the interval kinematic reliability, are discussed. For the former the First Order Second Moment (FOSM) method is employed while for the latter a multiple-extreme-value method is proposed to maintain high accuracy. The Monte Carlo simulation (MCS) is performed as a benchmark for the accuracy comparison. The effectiveness of the proposed method is demonstrated with two examples.

Keywords: Reliability, Time-dependent reliability, Spherical mechanisms, Function generators

1. Introduction

A spherical mechanism is different from a general spatial mechanism (Albert and Federico, 1999). The joint axes of the spherical mechanism all intersect at the same point (the center of the sphere), and any point on a moving body is confined to move within a spherical surface, and all the spherical surfaces of movement are concentric. Spherical mechanisms are particularly suitable for situations with varying spatial poses and high rotating speeds. For its special flexible locus, spherical mechanisms are widely used in engineering applications, such as in universal joints, robot wrists (Gupta, 1986), infinity fan structures (Pierre, et al., 2001), surgical robots (Lum, et al., 2004), and Agile Eyes (http://robot.gmc.ulaval.ca/en/research/).

One of the applications of a spherical mechanism is to serve as a function generator. The synthesis of such a function generating mechanism has been extensively studied, and many analytical (Rasim and Özgür, 2005, Zimmerman, 1967, Chiang, 1988, Cervantes-Sánchez, et al., 2009a, 2009b) and optimization synthesis methods (Liu and Angeles, 1992, Gosselin and Angeles, 1989, Wang, et al., 2004) have been proposed. In general, the two types of synthesis methods can effectively reduce the motion error in a deterministic sense, in which the motion error is viewed as the difference between the actual motion output and the desired motion output without considering any uncertainty. Without the involvement of uncertainty, the motion error is called the design error or structural error.

In reality, uncertainties are inherent in mechanisms (Du, et al. 2009), such as those in random dimensions (tolerances), random clearances at joints, and random deformations of structural components. The ignorance or mistreatment of the uncertainties during mechanism synthesis may result in significant motion errors. Traditionally, we can use a deterministic synthesis method to design a spherical mechanism with zero structural error at a several precision points. This could be only true if the mechanism was made without any manufacturing imprecisions and the materials and operation environment of the mechanism were perfectly deterministic and predictable. In reality, the motion error is always stochastic due to aforementioned uncertainties. In addition to the structural error, therefore, there exists the other kind of error – the random error. It is then desirable to use probabilistic approaches in the analysis and synthesis of spherical mechanisms when the random error is significant.

In spite of the abovementioned mechanism reliability methods, there is still a research need for the reliability of spherical function generating mechanisms. The reasons are as follow: (1) Most of mechanism reliability methods are for planar mechanisms. (2) Reliability approaches to spherical function generating mechanisms are currently lacking. (3) The major reliability methods are only for the point reliability at a specific instant of time while the reliability of the spherical function generating mechanism is defined over a time interval where the mechanism is required to perform their desired motion.

In this work, we extend the planar mechanism reliability methodologies to spherical function generating mechanisms. We will discuss two types of kinematic reliability, including point kinematic reliability and time-dependent (interval) kinematic reliability (Zhang and Du, 2010). The former reliability is defined at a specific instance of time and can provide local information at a specific point in the motion interval of a mechanism. This type of reliability is called point reliability. The point kinematic reliability is commonly used in the literature of mechanism reliability analysis and synthesis. It is usually calculated by the First Order Second Moment (FOSM) method and Monte Carlo Simulation (MCS) (Du, et al. 2009, Bhatti, 1989, Chakraborty, 1975, Sergeyev, 1974, Choubey and Rao, 1982, Baumgarten and Werf, 1985, Mallik and Dhande, 1987, Shi, 1997, Choi, et al., 1998, Shi, 2005, Liu and Wang, 1994, Rao and Bhatti, 2001, Bhatti and Rao, 1988, Kim, et al., 2010). Recently, the time-dependent (interval) kinematic reliability is also proposed (Zhang and Du, 2010). It is defined over a time interval and can provide complete information over the entire motion range of interest. Herein, the time interval represents the range of the motion input where the desired function is defined. The first passage method with the Poisson approximation can be used to estimate the interval kinematic reliability. But its accuracy may not be good. In this work, we propose a joint extreme point method to study the interval kinematic reliability of spherical mechanisms.

This paper is organized as follows: In Section 2, basic concepts and formulas of the point and interval kinematic reliabilities are reviewed. In Section 3, the kinematic analysis of spherical four-bar function generator mechanisms is presented, and the probabilistic analysis model of the mechanism motion error is developed. Two numerical examples are presented in Section 4 followed by conclusions in Section 5.

2. Kinematic reliability of Mechanisms

Suppose a functional relationship \( y = f(x) \) is to be achieved by a mechanism with a mapping function \( \psi = \psi(\theta) \) where \( \psi \) is the motion output and \( \theta \) is the motion input. In general, the actual motion output always deviates from the desired motion output except at several precision points. This deviation is called the motion error and is the difference between the actual motion output \( \psi \) and the desired motion output \( \psi_d \). The motion error at \( \theta \) is given by

\[
g(\mathbf{X}, \theta) = \psi(\mathbf{X}, \theta) - \psi_d(\theta)
\]

where \( \mathbf{X} = (X_1, \ldots, X_n) \) is an \( n \)-dimensional vector, which consists of the dimension variables of the mechanism.

2.1 Point kinematic reliability

If \( \mathbf{X} = (X_1, \ldots, X_n) \) are random variables, the motion error is also random. Using the concept of reliability, we can assess the motion error in a probabilistic sense. For a function generating mechanism, the point kinematic reliability \( R(\theta) \) at \( \theta \) is the probability that the motion error \( g(\mathbf{X}, \theta) \) is less than a specified value \( \varepsilon \). The design of the mechanism is acceptable if the following condition holds:

\[
g |(\mathbf{X}, \theta)| = |\psi(\mathbf{X}, \theta) - \psi_d(\theta)| \leq \varepsilon
\]

Then the point reliability \( R(\theta) \) at \( \theta \) is defined by the following probability

\[
R(\theta) = \Pr \{ |g(\mathbf{X}, \theta)| \leq \varepsilon \} = \Pr \{ -\varepsilon \leq g(\mathbf{X}, \theta) \leq \varepsilon \}
\]
where \( \Pr \{ \cdot \} \) stands for a probability.

The point probability of failure is then given by

\[
p_f(\theta) = \Pr \{ |g(X, \theta)| > \varepsilon \} = \Pr \{ g(X, \theta) > \varepsilon \cup g(X, \theta) < -\varepsilon \} \tag{4}
\]

The above reliability definition implies that the mechanism is in the working condition if \( |g(X, \theta)| \leq \varepsilon \) and is in the failure condition otherwise. Then \( R(\theta) \) indicates the likelihood that the mechanism works properly at \( \theta \) regardless whether it is failed or not before that.

Suppose the elements of \( \mathbf{X} \) are independent and their means and standard deviations are \( \mu_x = (\mu_1, \ldots, \mu_n) \) and \( \sigma_x = (\sigma_1, \ldots, \sigma_n) \), respectively. The standard deviations \( \sigma_x \) are much smaller than the means \( \mu_x \) because the tolerances of dimensions are much smaller than \( \mu_x = (\mu_1, \ldots, \mu_n) \). As a result, with good accuracy, we can linearize the error function \( g(X, \theta) \) at \( \mu_x \) (Zhang and Du, 2010).

\[
g(X, \theta) \approx \tilde{g}(X, \theta) = g(\mu_x, \theta) + \sum_{i=1}^{n} \frac{\partial g(X, \theta)}{\partial X_i} \bigg|_{\mu_x} (X_i - \mu_i) \tag{5}
\]

It is commonly assumed that a dimension variable follows a normal distribution \( X_i \sim N(\mu_i, \sigma_i^2) \) \( (i = 1, \ldots, n) \). For simple derivations, we transform \( X_i \) transformed into its standard counterpart \( U_i \) with

\[
X_i = \mu_i + \sigma_i U_i \tag{6}
\]

where \( U_i \sim N(0,1^2) \).

Then \( \tilde{g}(X, \theta) \) becomes

\[
\tilde{g}(U, \theta) = b_0(\theta) + \sum_{i=1}^{n} b_i(\theta) U_i \tag{7}
\]

where \( U = (U_1, \ldots, U_n) \), \( b_0(\theta) = g(\mu_x, \theta) \), and \( b_i(\theta) = \frac{\partial g(X, \theta)}{\partial X_i} \bigg|_{\mu_x} \cdot \sigma_i \).

Eq. (7) shows that the motion error is the sum of the structural error and random error. \( b_0(\theta) \) is deterministic and is actually the structural error \( \varepsilon_{\text{struct}} \). The random error is given by \( \sum_{i=1}^{n} b_i(\theta) U_i \), which is governed by random variables \( U \) or \( \mathbf{X} \).

As \( \tilde{g}(U, \theta) \) is a linear with respect to \( U \), \( \tilde{g}(U, \theta) \) is normally distributed. The mean value of \( \tilde{g}(U, \theta) \) is given by

\[
\mu_{\tilde{g}}(\theta) = b_0(\theta) \tag{8}
\]

which is the structural error \( \varepsilon_{\text{struct}} \).

The standard deviation of \( \tilde{g}(U, \theta) \) is given by

\[
\sigma_{\tilde{g}}(\theta) = \left[ \sum_{i=1}^{n} b_i^2(\theta) \right]^{0.5} \tag{9}
\]

which is the standard deviation of the random error.

With the FOSM, the point reliability defined in Eq. (3) is computed by

\[
R(\theta) = \Phi \left( \frac{\varepsilon - \mu_{\tilde{g}}(\theta)}{\sigma_{\tilde{g}}(\theta)} \right) - \Phi \left( \frac{-\varepsilon - \mu_{\tilde{g}}(\theta)}{\sigma_{\tilde{g}}(\theta)} \right) \tag{10}
\]

and the point probability of failure defined in Eq. (4) is then given by

\[
p_f(\theta) \approx 1 - \Phi \left( \frac{\varepsilon - \mu_{\tilde{g}}(\theta)}{\sigma_{\tilde{g}}(\theta)} \right) + \Phi \left( \frac{-\varepsilon - \mu_{\tilde{g}}(\theta)}{\sigma_{\tilde{g}}(\theta)} \right) \tag{11}
\]

where \( \Phi(\cdot) \) is the cumulative distribution function (CDF) of a standard normal variable.
2.2 Time-dependent (interval) kinematic reliability

Recently, interval kinematic reliability was introduced (Zhang and Du, 2010). It is the probability that the motion error is less than the specified allowance $\varepsilon$ over an interval of motion input $[\theta_0, \theta_f]$ and is defined by
\[
R(\theta_0, \theta_f) = \Pr \{|g(X, \theta)| \leq \varepsilon, \forall \theta \in [\theta_0, \theta_f]\} = \Pr \{-\varepsilon \leq g(X, \theta) \leq \varepsilon, \forall \theta \in [\theta_0, \theta_f]\}
\] (12)

The corresponding interval probability of failure is given by
\[
p_f(\theta_0, \theta_f) = \Pr \{|g(X, \theta)| > \varepsilon, \exists \theta \in [\theta_0, \theta_f]\} = \Pr \{g(X, \theta) > \varepsilon \cup g(X, \theta) < -\varepsilon, \exists \theta \in [\theta_0, \theta_f]\}
\] (13)

Both $\mu_g(\theta)$ and $\sigma_g(\theta)$ are dependent on $\theta$ and are hence time dependent. $\hat{g}(U, \theta)$ is therefore a non-stationary Gaussian process. The mean value first-passage (MVFP) method (Zhang and Du, 2010) can solve the interval kinematic reliability defined in Eq. (13). The method is based on FOSM and the first-passage method with the Poisson approximation (Cornell, 1969, Lutes and Sarkani, 2009). Since the Poisson approximation neglects the statistical dependence between the events of upcrossing the failure threshold, MVFP may not accurate for some problems. In the next subsection, we discuss the new method that improves the accuracy of the interval kinematic reliability.

2.3 Joint extreme point method

For function generating mechanisms, there are only several precision points, and the number of local extreme value points of the motion error is finite. We can use these points to approximate the motion error and therefore the interval probability of failure. Since the motion failure is governed by the extreme values, if we find the joint probability density functions (PDF) of those local extreme points, we can easily calculate the interval kinematic reliability. We then call the method the joint extreme point (JEP) method.

Suppose the local extreme points locate at $\theta_i$ ($i = 1, \ldots, m$). The interval reliability analysis is then given by
\[
R(\theta_0, \theta_f) = \Pr \{|g(X, \theta)| \leq \varepsilon, \forall \theta \in [\theta_0, \theta_f]\} = \Pr \left\{ \bigcap_{i=1}^{m} |g(X, \theta_i)| \leq \varepsilon, \forall \theta_i \in [\theta_0, \theta_f] \right\}
\] (14)

and
\[
p_f(\theta_0, \theta_f) = \Pr \{|g(X, \theta)| > \varepsilon, \exists \theta \in [\theta_0, \theta_f]\} = \Pr \left\{ \bigcup_{i=1}^{m} |g(X, \theta_i)| > \varepsilon, \exists \theta_i \in [\theta_0, \theta_f] \right\}
\] (15)

Letting $E_\varepsilon(\theta_i) = \{ |g(X, \theta_i)| \leq \varepsilon \}$, we rewrite Eq. (14) as
\[
R(\theta_0, \theta_f) = \Pr \left\{ \bigcap_{i=1}^{m} E_\varepsilon(\theta_i) \right\}
\] (16)

We should also include the two end points $\theta_0$ and $\theta_f$ of the time interval, and then
\[
R(\theta_0, \theta_f) = \Pr \left\{ \bigcap_{i=1}^{l} E_\varepsilon(\theta_i) \right\}
\] (17)

where $l = m + 2$.

Thus the interval reliability can be estimated by a multivariate normal distribution with mean $\mu$ and covariance $\Sigma$, or $\Phi_l(\varepsilon, \mu, \Sigma)$, where
\[
\mu = (\mu_g(X, \theta_1), \ldots, \mu_g(X, \theta_m))
\] (18)

\[
\Sigma = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1l} \\
\sigma_{21} & \sigma_{22} & \cdots & \sigma_{2l} \\
& \sigma_{32} & \cdots & \sigma_{3l} \\
& & \ddots & \cdots \\
& & & \sigma_{ll}
\end{bmatrix}
\] (19)
where $\sigma_{ij}$ is the covariance between $g(X, \theta_i)$ and $g(X, \theta_j)$.

The covariance matrix $\Sigma$ should be definite positive. If the requirement is not satisfied, not all the time instants are needed, and some of them should be eliminated. Since the maximum number of the time instants should equal to the rank of $\Sigma$, letting the rank be $r$, we need to eliminate $I - r$ time instants. According to the suggestion in (Du, 2013), we delete those time instants where the point probabilities are smallest and keep $r$ instants $\theta_i'$, where $i = 1, 2, \ldots, r$.

We will use the initial point $\theta_i'$ of the local extreme $g(X, \theta_i')$, where $i = 1, 2, \ldots, r$.

The parameters the multivariate normal distribution are $\mu'$ and $\Sigma'$, where

$$\mu'_i = b_0(\theta_i') \quad (20)$$

and

$$\sigma'_{ij} = b_i(\theta_i')b_j(\theta_j') \quad (21)$$

The multivariate normal CDF $\Phi_r(\epsilon', \mu', \Sigma')$ for the interval reliability can be calculated by the following integral with a numerical algorithm.

$$R(\theta_0, \theta_f) = \int_{-\epsilon'}^{\epsilon'} \frac{1}{(2\pi)^{r/2} |\Sigma'|} \exp\left(-\frac{(X - \mu')\Sigma'^{-1}(X - \mu')^T}{2}\right) dX \quad (22)$$

To use the above approach, we need to find the extreme points of the motion error. In general, it is difficult to search for all the possible extreme values of a nonlinear function with an optimization method. To obtain the extreme points, we at first obtain the derivative of the motion error with respect to time, $\dot{g}(X, \theta) = \frac{\partial g(X, \theta)}{\partial \theta}$. If $\dot{g}(X, \theta_i') = 0$, $\theta_i'$ may be a local extreme point of $g(X, \theta)$. Since the analytical solution of $\dot{g}(X, \theta) = 0$ is in general not available, a numerical method is should be employed. The procedure is given below.

Step 1. Discretize the interval $[\theta_0, \theta_h]$ into $p$ small equal intervals, where $\theta_0 \leq \theta_h \leq \theta_f$.

Step 2. Calculate the derivative $\dot{g}(X, \theta)$ of the motion error $g(X, \theta)$ at each of the discrete points on $[\theta_0, \theta_h]$.

Step 3. Generate three arrays $IP$, $LB$ and $UB$, where $IP$ is used store the initial point of the local extreme values, and $LB$ and $UB$ are used to store the upper and lower bounds of the small intervals, respectively. Let $S = \dot{g}(X, \theta_k)\dot{g}(X, \theta_{k+1})$, and then from $k = 1$ to $p$ calculate $S$. If $S \leq 0$ then set $IP_i = 0.5(\theta_k + \theta_{k+1})$.

Step 4. From $i = 1$ to $m$, use the initial point $IP_i$ find the root $\theta_i$ of $\dot{g}(X, \theta) = 0$ on the small interval $[\theta_k, \theta_{k+1}]$ by numerical method.

3. **Kinematic analysis of spherical mechanism**

3.1 **Four-bar function generating spherical mechanism**

A four-bar spherical mechanism is shown in Fig.1. The four links of the mechanism are connected by four revolute joints $A$, $B$, $C$ and $D$, whose rotation axes intersect in one common point $O$. This point is called the spherical center. $AD$ is the fixed linkage, $AB$ and $CD$ are the input and output linkages, and $BC$ is the coupler. The input angle is $\theta$, and the output angle $\psi$. The dimensions are given by the angle between adjacent joint axes, $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$.

![Fig. 1 Four-bar spherical mechanisms](image-url)
The relationship between the input and output angles can be easily derived. According to (Cervantes-Sánchez, et al., 2009a, 2009b)
\[ \mathbf{P} \cdot \mathbf{Z} = 0 \]  
(23)
where the dot stands for an inner product, and
\[ \mathbf{P} = [P_1, P_2, P_3, P_4, P_5] \]  
(24)
where \( P_1 = \sin \alpha_1 \sin \alpha_3 \), \( P_2 = -\sin \alpha_1 \sin \alpha_3 \cos \alpha_4 \), \( P_3 = \cos \alpha_1 \sin \alpha_3 \sin \alpha_4 \), \( P_4 = \sin \alpha_1 \cos \alpha_3 \sin \alpha_4 \), and \( P_5 = \cos \alpha_1 \cos \alpha_3 \cos \alpha_4 - \cos \alpha_2 \);
\[ \mathbf{Z} = [Z_1, Z_2, Z_3, Z_4, Z_5] \]  
(25)
in which \( Z_1 = \sin \theta \sin \psi(\mathbf{X}, \theta) \), \( Z_2 = \cos \theta \cos \psi(\mathbf{X}, \theta) \), \( Z_3 = \cos \psi(\mathbf{X}, \theta) \), \( Z_4 = \cos \theta \), and \( Z_5 = 1 \).

Solving Eq. (23) yields a closed-form solution
\[ \psi(\mathbf{X}, \theta) = 2 \arctan \left( \frac{-A \pm \sqrt{A^2 + B^2 - C^2}}{C - B} \right) \]  
(26)
where \( A = P_1 \sin \theta \), \( B = P_2 \cos \theta + P_3 \), and \( C = P_4 \cos \theta + P_5 \).

### 3.2 Probabilistic model of motion error

In Section 2, the motion error function is defined by Eq. (1). Its mean and standard deviations are computed by Eqs. (8) and (9). To obtain the analytical expressions of \( \mu_g(\theta) \) and \( \sigma_g(\theta) \), we need to derive \( b_0(\theta) \) and \( b(\theta) \).

According to the definition of \( b_0(\theta) \) in Eq. (7) and the motion error function in Eq. (1), \( b_0(\theta) \) is given by

\[ b_0(\theta) = \psi(\mu, \theta) - \psi_d(\theta) \]  
(27)

\( \mu_g(\theta) \) is then
\[ \mu_g(\theta) = \psi(\mu, \theta) - \psi_d(\theta) \]  
(28)

We now linearize the error function. From Eq. (23), we obtain

\[ \frac{\partial \mathbf{P}}{\partial \mathbf{X}} \bigg|_{\mu} \cdot \mathbf{Z} + \mathbf{P} \cdot \frac{\partial \mathbf{Z}}{\partial \psi(\mathbf{X}, \theta)} \bigg|_{\mu} \delta \psi(\mathbf{X}, \theta) = 0 \]  
(29)

where \( \mathbf{X} = (\alpha_1, \ldots, \alpha_4) \).

Rearranging Eq. (29), we obtain

\[ \delta \psi(\mathbf{X}, \theta) = -\frac{\partial \mathbf{P}}{\partial \mathbf{X}} \bigg|_{\mu} \cdot \mathbf{Z} \cdot \frac{\partial \mathbf{Z}}{\partial \psi(\mathbf{X}, \theta)} \bigg|_{\mu} \cdot \delta \mathbf{X} \]  
(30)

According to Eqs. (7) and (30), vectors \( \mathbf{b}(\theta) \) are obtained as follows:

\[ \mathbf{b}(\theta) = -\frac{1}{w} \mathbf{Q} \cdot \sigma \mathbf{X} \]  
(31)

where \( \mathbf{Q} = (Q_1, \ldots, Q_4) \) and \( Q_i = \left( \frac{\partial \mathbf{P}}{\partial \alpha_i} \right) \cdot \mathbf{Z} \bigg|_{\mu} \), \( w = \mathbf{P} \cdot \frac{\partial \mathbf{Z}}{\partial \psi(\mathbf{X}, \theta)} \bigg|_{\mu} \).

\( \frac{\partial \mathbf{P}}{\partial \alpha_1} \) and \( \frac{\partial \mathbf{Z}}{\partial \psi(\mathbf{X}, \theta)} \) are given by

\[ \frac{\partial \mathbf{P}}{\partial \alpha_1} = \left( \cos \alpha_1 \sin \alpha_3, -\cos \alpha_1 \sin \alpha_3 \cos \alpha_4, -\sin \alpha_1 \sin \alpha_3 \sin \alpha_4, \cos \alpha_1 \cos \alpha_3 \sin \alpha_4, -\sin \alpha_1 \cos \alpha_3 \cos \alpha_4 \right) \]  
(32)
\[
\frac{\partial P}{\partial \alpha_2} = (0, 0, 0, 0, \sin \alpha_2)
\]
(33)
\[
\frac{\partial P}{\partial \alpha_3} = (\sin \alpha_1 \cos \alpha_3, -\sin \alpha_1 \cos \alpha_3 \cos \alpha_4, \cos \alpha_1 \cos \alpha_3 \sin \alpha_4, -\sin \alpha_1 \sin \alpha_3 \sin \alpha_4, -\cos \alpha_1 \sin \alpha_3 \cos \alpha_4)
\]
(34)
\[
\frac{\partial P}{\partial \alpha_4} = (0 \sin \alpha_1 \sin \alpha_3, \cos \alpha_1 \sin \alpha_3 \cos \alpha_4, \sin \alpha_1 \cos \alpha_3 \cos \alpha_4, -\cos \alpha_1 \cos \alpha_3 \sin \alpha_4)
\]
(35)
\[
\frac{\partial Z}{\partial \psi(X, \theta)} = (\sin \theta \cos \psi(X, \theta), -\cos \theta \sin \psi(X, \theta), -\sin \psi(X, \theta), 0, 0)
\]
(36)

4. Numerical examples

In this section, two desired functions given in (Cervantes-Sánchez, et al., 2009a) are employed to demonstrate the proposed method. The desired functions and mechanism parameters are listed in Table 1 and Table 2.

Replacing \( \theta \) with \( \theta_0 + \theta \) in Eq. (23), we could use the proposed method to solve the kinematic reliability. To evaluate the accuracy, Monte Carlo simulation (MCS) was also used with a sample size of \( 10^6 \). The results are plotted in Figs. 2 and 3 where \( \phi = 180 - \theta, PR, \) and \( IR_{MVFP} \) and \( IR_{JEP} \) stand for the point reliability, interval reliability from the mean value first-passage method, and interval reliability from the joint extreme point method, respectively. \( MCS_{Point} \) and \( MCS_{Time} \) stand for the point reliability and interval reliability from MCS, respectively.

<table>
<thead>
<tr>
<th>Function</th>
<th>( y = \sin x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale</td>
<td>( x^o = \left(\frac{90}{58}\right) (180 - \theta^o) )</td>
</tr>
<tr>
<td>Relationships</td>
<td>( \psi^o = 180 - (60\psi + 87.105) )</td>
</tr>
<tr>
<td>Range of ( \theta )</td>
<td>( \theta \in (180^o, 130^o) )</td>
</tr>
<tr>
<td>Random variable</td>
<td>( \mu_X(^o) )</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>( \mu_1=132.78 )</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>( \mu_2=96.99 )</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>( \mu_3=164.34 )</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>( \mu_4=131.82 )</td>
</tr>
<tr>
<td>Deterministic variable ( \theta_0 )</td>
<td>-61.2925 (^o)</td>
</tr>
<tr>
<td>Allowable error ( \varepsilon )</td>
<td>0.5 (^o)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2 Parameters of tangent mechanism</th>
</tr>
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<tbody>
<tr>
<td>Function</td>
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<tr>
<td>Scale</td>
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<tr>
<td>Relationships</td>
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<td>( \alpha_1 )</td>
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<tr>
<td>( \alpha_4 )</td>
</tr>
<tr>
<td>Deterministic variable ( \theta_0 )</td>
</tr>
<tr>
<td>Allowable error ( \varepsilon )</td>
</tr>
</tbody>
</table>
The results show the following:

(1) The point probability of failure could increase or decrease with time while the interval probability of failure always increases with time. The former only reflects the probability of not realizing the intended function at specific points over the time interval, and the latter indicates such a probability over the entire time interval.

(2) The interval probability of failure is always larger than or equal to the point probability of failure. As shown in Figs. 2 and 3, both probabilities are equal at the initial time and the interval probability of failure is much larger than the point probability of failure at the end of the time interval. When the point probability of failure increases, the interval probability of failure also increases. When the point probability of failure decreases or remains constant, the interval probability of failure becomes constant over time.

(3) The solutions from the proposed method are very close to those of MCS, and the proposed method is therefore
accurate. The probability of failure from the proposed method is slightly smaller than those of MCS.

(4) The mean value first-passage method is inaccurate.

5. Conclusions

A new interval reliability method, the joint extreme point (JEP) method is proposed for spherical mechanisms in order to improve the accuracy of the existing mean value first-passage (MVFP) method. JEP at first linearizes the motion error and then finds the local extreme values of the motion error. Then the motion error over the time interval is approximated by the finite number of the extreme values. Since the extreme values follow a multivariate normal distribution, the interval reliability can be found by integrating the joint probability density of the extreme values. As shown in the example with a sin function and a tangent function, JEP is accurate referenced with the Monte Carlo simulation. Compared with MVFP, JEP is also more accurate. The reason is that the assumption of independent upcrossings by MVFP may not hold for spherical mechanisms. JEP takes the dependence between the extreme values into consideration and therefore produces higher accuracy.

The results also show that the interval kinematic reliability is more critical than the point kinematic reliability. The reason is that the former provides complete information about the likelihood of the desired function being realized over the entire motion range of interest, while the latter only gives the instantaneous likelihood at a specific time instant.

As shown in the example, the interval probability of failure from JEP is slightly smaller than that from Monte Carlo simulation. This error primarily comes from two sources. The more significant one is the use of all the local extreme values. Ideally, only one extreme value, or the global maximum absolute motion error over the time interval, should be used. However, it is difficult or even impossible to find such a global extreme value given the randomness in the dimension variables. Since the extreme values used by JEP include the global extreme value, which dominates the failure event for spherical mechanisms, the error of JEP is small and acceptable. The other error source is the linearization of the motion error. The accuracy of the linearization is good because the randomness in the dimension variables is small.

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