A study of equivalence between adaptive learning and loop shaping methods for disturbance compensation

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Abstract

This paper introduces a theoretical analysis of the disturbance compensation methods in positioning systems. There are two major methods used to compensate for disturbances. The first one is based on a loop shaping method and the second one is based on an adaptive learning method. We make a comparison between a resonant filter used as the loop shaping method and an enhanced adaptive feed-forward cancellation (AFC) method used as the adaptive learning method. To compensate disturbances, the resonant filter decreases the gain of the sensitivity function at the disturbance frequency by using the vector locus. The adaptive learning method adjusts the coefficients of the enhanced AFC to compensate for a disturbance. We found that the LTI model of an enhanced AFC is the same as that for a resonant filter when we compared their characteristics. A simulation of the disturbance compensation in a hard disk drive showed that these time responses and frequency responses coincided. Moreover, the results showed advantages of each of the control methods with the exception of the performance. A resonant filter realizes low cost implementation with compare to an enhanced AFC. That is, a resonant filter easier to implement than an enhanced AFC. An enhanced AFC has the utility to realize a high quality function control system simultaneously able to compensate and estimate for disturbances. The results of the evaluation in this study will provide design engineers with guidelines for selecting an optimum control strategy in actual applications.

Key words : Disturbance compensation, Periodic disturbance, Non-periodic disturbance, Adaptive feed-forward cancellation, Resonant filter, Hard disk drive

1. Introduction

It is important to improve positioning accuracy of a positioning system. For example, increasing the recording density of a hard disk drive (HDD) requires improving the head positioning accuracy in the head positioning control system (Yamaguchi et al. 2012). The positioning accuracy is worsened by disturbances in the positioning system. Therefore, disturbance compensation is very important for positioning systems. Various disturbance compensation methods have been developed for previous studies. There are two kinds of methods to compensate for disturbances.

One compensation method is based on the loop shaping method. This method decreases the gain of the sensitivity function of the disturbance frequency using loop shaping techniques to reject any disturbances. The H-infinity method (McFarlane and Glover, 1992; Boukhniifer and Ferreira, 2006) and the disturbance observer (White et al. 2000; Mizuochi

Fig. 1 Block diagram of positioning control systems with resonant filter

et al. 2006) are representative methods used to create the desired loop shaping. In these design methods, we focus on the resonant filter (Atsumi et al. 2007; Messner, 2008) as an example of the loop shaping method. The sensitivity function's gain is the reciprocal of the distance from an open loop characteristic to the critical point $[-1, 0]$ on the Nyquist diagram. The vector locus recedes from the critical point $[-1, 0]$ by using the resonant filter. The resonant filter can compensate for both periodic and non-periodic disturbances.

The other compensation method is based on the adaptive learning method. The parameters of the filter are updated by using the adaptive algorithm in real-time to compensate for any disturbances. The representative method is an adaptive feed-forward cancellation (AFC) (Bodson et al. 1992; Sacks et al. 1993; Zhang et al. 2000; Wu and Tomizuka 2006). The adaptive algorithm adjusts the coefficients of the AFC to compensate for disturbances. Although an AFC is generally developed to compensate for periodic disturbances, one of the features of the AFC was expanded to also compensate for non-periodic disturbances, and this new modification is called the enhanced AFC (Yabui et al. 2013). The enhanced AFC can compensate for both types of disturbances.

We can apply a resonant filter or an enhanced AFC to compensate for both periodic and non-periodic disturbances. When selecting a control theory, developers have to be able to evaluate whether a selected control strategy is the best for specific specifications, and performance comparison studies have been conducted for some control methods to assist in the selection (Moreno and Mejia 2013). There are few studies comparing the adaptive learning method and the loop shaping method for disturbance compensation. We conducted a theoretical analysis and made a comparison between the resonant filter and the enhanced AFC for this paper. As a result, we found that the LTI model of an enhanced AFC is the same as a resonant filter. Moreover, the LTI model helps to show the relationship between the resonant filter's parameters and the enhanced AFC's parameters. A simulation of the disturbance compensation in a hard disk drive showed that these time and frequency responses coincide. That is, these methods are fully consistent with respect to the input-output relation (transfer function). However, critical factors other than performance need to be considered in the development of control systems. With the results here, design engineers can evaluate which control theory is superior (the resonant filter or the enhanced AFC) as the control system to apply in specific applications.

2. Positioning Systems with Resonant Filter

This chapter introduces a resonant filter in a positioning system (Atsumi et al. 2007; Messner, 2008). Figure 1 shows a block diagram of the control system with the resonant filter, where P is the controlled object and C is the stabilizing controller. The transfer function of resonant filter F is described as

$$F_{res}(s) = \kappa_r \frac{s - z_r}{s^2 + 2\zeta_r \omega_r s + \omega_r^2}, \quad (1)$$

where, ζ_r is the damping ratio, and ω_r is the natural frequency that is the target frequency for disturbance compensation. In addition, κ_r is the gain and z_r is zero for the LTI model. The z_r design method is based on the vector locus of an open loop transfer function. The sensitivity function gain should be decreased at the disturbance frequency by using a resonant filter to reject the disturbances. The sensitivity function's gain is the reciprocal of the distance from an open loop characteristic to the critical point $[-1, 0]$ on the Nyquist diagram. Therefore, the vector locus recedes from the critical point $[-1, 0]$ by using the resonant filter. This design concept is a disturbance rejection method that is based on the feedback control theory.

Fig. 2 Block diagram of positioning control systems with enhanced AFC

3. Positioning Systems with Enhanced AFC

This section introduces an enhanced AFC used in a positioning system. One of the features of the AFC was expanded to compensate for non-periodic disturbances. Figure 2 is a block diagram of the control system with the enhanced AFC. The term $u(k)$ is the output of the enhanced AFC described as

$$u(k) = p(k-1) \cos(\sqrt{1-\zeta^2}\omega Tk) + q(k-1) \sin(\sqrt{1-\zeta^2}\omega Tk). \quad (2)$$

The adaptive parameters $p(k)$ and $q(k)$ are updated by the adaptive laws to form the following equations:

$$p(k) = e^{-\zeta\omega Tk} p(k-1) + \lambda e(k) \cos(\sqrt{1-\zeta^2}\omega Tk + \theta), \quad (3)$$

$$q(k) = e^{-\zeta\omega Tk} q(k-1) + \lambda e(k) \sin(\sqrt{1-\zeta^2}\omega Tk + \theta). \quad (4)$$

In these equations, T is the sampling time, ω is the frequency that is the target frequency for disturbance compensation, λ is the learning rate of the algorithm, ζ is the forgetting factor of the algorithm, and θ is the phase parameter of the AFC output. In the recurrence formula of the enhanced AFC, the adaptive algorithm has a damping function used as the forgetting factor, which is $e^{-\zeta\omega Tk}$. If ζ is equal to 0, the adaptive algorithm is equal to the current AFC. The adaptive algorithm adjusts the coefficients $p(k)$ and $q(k)$ to compensate for disturbance $d(t)$. The disturbance $d(t)$ is perfectly cancelled when the estimates of coefficients $p(k)$ and $q(k)$ have nominal values.

4. Theoretical Study of Relationship between Enhanced AFC and Resonance Filter

The resonant filter and enhanced AFC were introduced in the previous chapters. Both methods can similarly compensate for periodic and non-periodic disturbances. In this chapter, the theoretical analysis of these methods is introduced.

4.1. LTI model of enhanced AFC

The adaptive algorithm of enhanced AFC was converted into a LTI model in order to compare it with the LTI model of a resonant filter. We rewrote the adaptive algorithm, Eqs. (3) and (4), into the following equations.

$$p(k) = \sum_{a=1}^k e^{-\zeta\omega T(k-a)} \lambda e(a) \cos(\Omega Ta + \theta), \quad (5)$$

$$q(k) = \sum_{a=1}^k e^{-\zeta\omega T(k-a)} \lambda e(a) \sin(\Omega Ta + \theta), \quad (6)$$

where $\sqrt{1-\zeta^2}\omega = \Omega$. Equations (2), (5), and (6) thus provide

$$u(k) = \sum_{a=0}^k e^{-\zeta\omega T(k-a)} \lambda e(a) \cos(\Omega Ta + \theta) \cos(\Omega Tk) + \sum_{a=0}^k e^{-\zeta\omega T(k-a)} \lambda e(a) \sin(\Omega Ta + \theta) \sin(\Omega Tk). \quad (7)$$

The trigonometric function can be transferred into the following equation.

$$\cos(\Omega Ta + \theta) \cos(\Omega Tk) + \sin(\Omega Ta + \theta) \sin(\Omega Tk) = \cos(\theta) \cos(\Omega T(k-a)) + \sin(\theta) \sin(\Omega T(k-a)). \quad (8)$$

By using a matched Z-transformation and convolution theory, the transfer function from $e(k)$ to $u(k)$ can be described as

$$\begin{aligned} F_{AFC}(z) &= \mathcal{Z}\left[e^{-\zeta\omega T k} \lambda \cos(\theta) \cos(\Omega T k)\right] + \mathcal{Z}\left[e^{-\zeta\omega T k} \lambda \sin(\theta) \sin(\Omega T k)\right] \\ &= \lambda \frac{z^2 \cos(\theta) + e^{-\zeta\omega T} z \cos(\Omega T) \cos(\theta)}{z^2 - 2e^{-\zeta\omega T} z \cos(\Omega T) + e^{-2\zeta\omega T}} + \lambda \frac{e^{-\zeta\omega T} z \sin(\Omega T) \sin(\theta)}{z^2 - 2e^{-\zeta\omega T} z \cos(\Omega T) + e^{-2\zeta\omega T}} \\ &= \lambda \frac{z^2 \cos(\theta) - ze^{-\zeta\omega T} \cos(\Omega T + \theta)}{z^2 - 2ze^{-\zeta\omega T} \cos(\Omega T) + e^{-2\zeta\omega T}}. \end{aligned} \quad (9)$$

Equation (9) is the LTI model of enhanced AFC.

4.2. LTI model of resonant filter in discrete-time

The LTI model of the resonant filter is described using Eq. (1). We transform the continuous-time model into a discrete-time model to compare it with the LTI model of enhanced AFC. First, the LTI model was rewritten as

$$\begin{aligned} F(s) &= \kappa_r \frac{s - z_r}{s^2 + 2\zeta_r \omega_r s + \omega_r^2} \\ &= \kappa_r \frac{s + \zeta_r \omega_r + \alpha_r \Omega_r}{(s + \zeta_r \omega_r)^2 + \Omega_r^2}, \end{aligned} \quad (10)$$

where $-z_r = \zeta_r \omega_r + \alpha_r \Omega_r$ and $\sqrt{1 - \zeta_r^2} \omega_r = \Omega_r$. The matched Z-transformation was applied to Eq. (10) to transfer the continuous-time model. The matched Z-transformation maps, poles, and zeros in the discrete-time model given as,

$$\begin{aligned} F_{res}(z) &= \mathcal{Z}[F(s)], \\ &= \kappa_r \frac{z(z - e^{-\zeta_r \omega_r T} \cos(\Omega_r T))}{z^2 - 2e^{-\zeta_r \omega_r T} z \cos(\Omega_r T) + e^{-2\zeta_r \omega_r T}} + \kappa_r \frac{\alpha_r z e^{-\zeta_r \omega_r T} \sin(\Omega_r T)}{z^2 - 2e^{-\zeta_r \omega_r T} z \cos(\Omega_r T) + e^{-2\zeta_r \omega_r T}}. \end{aligned} \quad (11)$$

Equation (11) represents a discrete-time model of the resonant filter.

4.3. Comparison of enhanced AFC and resonant filter

In this section, we compare the LTI model of enhanced AFC with the LTI model of the resonant filter in discrete-time. Equation (11) can be rewritten as,

$$\begin{aligned} F_{res}(z) &= \lambda \frac{z(z - e^{-\zeta\omega T} \cos(\Omega T)) \cos(\theta)}{z^2 - 2e^{-\zeta\omega T} z \cos(\Omega T) + e^{-2\zeta\omega T}} + \lambda \frac{ze^{-\zeta\omega T} \sin(\Omega T) \sin(\theta)}{z^2 - 2e^{-\zeta\omega T} z \cos(\Omega T) + e^{-2\zeta\omega T}} \\ &= \lambda \frac{z^2 \cos(\theta) - ze^{-\zeta\omega T} \cos(\Omega T + \theta)}{z^2 - 2ze^{-\zeta\omega T} \cos(\Omega T) + e^{-2\zeta\omega T}}, \end{aligned} \quad (12)$$

where

$$\zeta_r = \zeta, \quad (13)$$

$$\begin{aligned} -z_r &= \zeta_r \omega + \alpha_r \Omega \\ &= \zeta_r \omega + \tan(\theta) \Omega, \end{aligned} \quad (14)$$

$$\kappa_r = \lambda \cos(\theta), \quad (15)$$

$$\omega_r = \omega. \quad (16)$$

We found that the discrete-time model of the resonant filter Eq. (12) is consistent with the LTI model of the enhanced AFC Eq. (9). That is, the characteristics of the adaptive learning method are equivalent to the loop shaping method. The LTI model of the enhanced AFC also includes a resonant mode as an internal model (Francis et al. 1975; Messner and Bodson, 1994). Moreover, the LTI model shows the relationship between the resonant filter's parameters and the enhanced AFC's parameters.

In Eq. (13), ζ_r is the damping ratio of the resonant filter. The forgetting factor ζ exponentially discards the old data in the adaptive algorithm. It corresponds to the damping ratio ζ_r that dissipates the mechanical energy of the resonant filter.

In Eq. (14), z_r is a zero of the resonant filter. The phase parameter θ decides the zero of the LTI model. The zero can be designed by using the vector locus to reject the disturbances (Atsumi et al. 2007; Messner, 2008; Okuyama et al.

Fig. 3 Frequency response of mechanical system P from HDD Benchmark ProblemTable 1 Parameters of P from HDD Benchmark Problem

i	Gain of models α_{mi}	Damping ratio ζ_{mi} [%]	Natural frequency ω_{mi} [rad/s]
1	1.00	0	0
2	0.01	0.5	$2 \times \pi \times 3000$
3	-1.00	2	$2 \times \pi \times 4100$
4	0.03	0.1	$2 \times \pi \times 5000$
5	1.00	2	$2 \times \pi \times 8200$
6	-1.00	2	$2 \times \pi \times 12300$
7	1.00	2	$2 \times \pi \times 16400$

2011; Yabui et al. 2012). The vector locus recedes from the critical point $[-1, 0]$. Therefore, the same design method can be applied to both methods.

In Eq. (15), κ_r is the gain of the resonant filter. Therefore, $\lambda \cos(\theta)$ decides the gain of the enhanced AFC. It relates the learning rate λ and phase parameter θ in the adaptive algorithm. The phase parameter θ is designed to achieve the optimal condition in the Nyquist diagram. The learning rate λ should be designed to compensate for the disturbances by the enhanced AFC's output.

5. Verification for Characteristics of Enhanced AFC and Resonant Filter in Simulation

We have designed a head positioning control system using an HDD Benchmark Problem (Yamaguchi et al. 2012) to verify the results of the theoretical analysis. A technical committee consisting of representatives from HDD manufacturers and major universities doing hard disk drive servo research in Japan has developed an open-source HDD benchmark problem to encourage more studies on the head positioning control system.

5.1. Simulation conditions

We conducted simulations on the head positioning to verify the characteristics of enhanced AFC and the resonant filter. We used the track-following control system in benchmark problem Ver. 3.1 for the head-positioning simulation.

In the HDD Benchmark Problem, the mechanical system P is given by the following equation.

$$P_m(s) = K_p \sum_{i=1}^l \frac{\alpha_{mi}}{s^2 + 2\zeta_{mi}\omega_{mi}s + \omega_{mi}^2}, \quad (17)$$

where i is the number of modes under consideration, α_{mi} is the residue of each mode, ω_{mi} and ζ_{mi} are the natural frequency and damping ratio of the resonance, and K_p is the plant gain. In this model, l was seven for a model order of fourteen. The plant gain K_p was 3.74×10^9 , and the values of the other parameters are listed in Table 1. Figure 3 shows the frequency responses of the controlled object. The stabilizing controller C is given by the product of the PI-lead filter C_p and two

Fig. 4 Frequency response of feedback controller C from HDD Benchmark Problem

notch filters C_{n1} and C_{n2} . The PI-lead filter C_p is discretized using a bilinear transformation so the control system could be implemented. The notch filters C_{n1} and C_{n2} are discretized using the bilinear transformation with frequency prewarping.

To enlarge the gain margin to more than 5 dB, we set the phase margin to more than 35° and the open-loop gain 0-dB crossover frequency to 1 kHz, and C_p , C_{n1} , and C_{n2} are set based on the following equations,

$$C_p(s) = \frac{0.0408(s + 2 \times \pi \times 10)(s + 2 \times \pi \times 250)}{s(s + 2\pi \times 4000)}, \quad (18)$$

$$C_{n1}(s) = \frac{s^2 + 2\zeta_{nntc1}\omega_{ntc1}s + \omega_{ntc1}^2}{s^2 + 2\zeta_{dntc1}\omega_{ntc1}s + \omega_{ntc1}^2}, \quad (19)$$

$$C_{n2}(s) = \frac{s^2 + 2\zeta_{nntc2}\omega_{ntc2}s + \omega_{ntc2}^2}{s^2 + 2\zeta_{dntc2}\omega_{ntc2}s + \omega_{ntc2}^2}, \quad (20)$$

where

$$\begin{aligned} \zeta_{nntc1} &= 0.1, \zeta_{dntc1} = 0.5, \omega_{ntc1} = 2 \times \pi \times 4700, \\ \zeta_{nntc2} &= 0.1, \zeta_{dntc2} = 0.8, \omega_{ntc2} = 2 \times \pi \times 8000. \end{aligned} \quad (21)$$

Figure 4 shows the frequency response of C .

Fig. 5 Frequency response of disk flutter model $D(s)$ from HDD Benchmark Problem

Table 2 Model parameters of disk flutter vibration

i	Gain of models a_{di}	Damping ratio ζ_{di} [%]	Natural frequency ω_{di} [rad/s]
1	0.09	1	$2 \times \pi \times 750$
2	0.17	1	$2 \times \pi \times 780$
3	0.20	1	$2 \times \pi \times 900$
4	0.17	1	$2 \times \pi \times 1020$
5	0.06	0.5	$2 \times \pi \times 1080$
6	0.09	1	$2 \times \pi \times 1230$
7	0.06	0.5	$2 \times \pi \times 1800$

Fig. 6 Time response of disturbance signal $d(t)$ caused by disk flutter vibration in position unit

We injected a disturbance into the feedback control system in order to evaluate the disturbance suppression performance of the proposed AFC. We focused on the disk flutter vibration induced positioning error. In the HDD Benchmark Problem, a disturbance caused by disk flutter vibration is defined as the output when white Gaussian noise with a standard deviation = 1 is injected into the disk flutter vibration models. The disk flutter vibration model is expressed by using Eq. (22). Table 2 lists the parameters of these vibration models. Figure 5 shows the frequency response of disk flutter model

Table 3 Design parameters of resonant filters

i	Gain of filters κ_{ri}	Zero z_{ri}	Damping ratio ζ_{ri} [%]	Target frequency ω_{ri} [rad/s]
1	-0.11	4.0×10^3	1	$2 \times \pi \times 750$
2	-0.08	8.7×10^3	1	$2 \times \pi \times 780$
3	0.16	-4.6×10^3	1	$2 \times \pi \times 900$
4	0.13	-5.8×10^3	1	$2 \times \pi \times 1020$
5	-0.00	8.0×10^4	0.5	$2 \times \pi \times 1080$
6	-0.01	5.4×10^3	1	$2 \times \pi \times 1230$
7	0.01	-9.7×10^4	0.5	$2 \times \pi \times 1800$

Table 4 Design parameters of enhanced AFC

i	Learning rate λ_i	Phase θ_i [deg]	Forgetting factor ζ_i [%]	Target frequency ω_i [rad/s]
1	0.15	-103	1	$2 \times \pi \times 750$
2	0.17	-105	1	$2 \times \pi \times 780$
3	0.20	-114	1	$2 \times \pi \times 900$
4	0.17	-114	1	$2 \times \pi \times 1020$
5	0.10	-89	0.5	$2 \times \pi \times 1080$
6	0.12	-142	1	$2 \times \pi \times 1230$
7	0.10	-188	0.5	$2 \times \pi \times 1800$

$D(s)$. Figure 6 shows the disturbance signal, where the signal is injected as disturbance $d(t)$ in Fig. 1.

$$D(s) = \sum_{i=1}^7 \frac{2a_{di}\zeta_{di}\omega_{di}^2}{s^2 + 2\zeta_{di}\omega_{di}s + \omega_{di}^2}. \quad (22)$$

We designed the enhanced AFC and resonant filter to compensate for the disturbance signal. The enhanced AFC is given in Eq. (25), and resonant filter is given in Eq. (26). Table 3 lists the parameters of resonant filters. Table 4 lists the parameters of enhanced AFCs.

$$u(k) = \sum_{i=1}^7 p_i(k-1) \cos(\sqrt{1-\zeta_i^2}\omega_i T k) + \sqrt{1-\zeta_i^2} q_i(k-1) \sin(\omega_i T k), \quad (23)$$

$$p_i(k) = e^{-\zeta_i\omega_i T k} p_i(k-1) + \lambda_i e(k) \cos(\sqrt{1-\zeta_i^2}\omega_i T k + \theta_i), \quad (24)$$

$$q_i(k) = e^{-\zeta_i\omega_i T k} q_i(k-1) + \lambda_i e(k) \sin(\sqrt{1-\zeta_i^2}\omega_i T k + \theta_i). \quad (25)$$

$$F_{res}(s) = \sum_{i=1}^7 \kappa_{ri} \frac{s - z_{ri}}{s^2 + 2\zeta_{ri}\omega_{ri}s + \omega_{ri}^2}. \quad (26)$$

The parameters ω_{ri} , the parameters ζ_{ri} and κ_{ri} were designed so that the disturbances are sufficiently compensated. The enhanced AFCs parameters ω_i , ζ_i , and λ_i were similarly designed. z_{ri} and θ_i were designed to recede from the critical point $[-1, 0]$ on the Nyquist chart. The design method for a resonant filter has been discussed in previous studies (Atsumi et al. 2007; Messner, 2008). The design method of an enhanced AFC has been discussed in our previous studies (Okuyama et al. 2011; Yabui et al. 2012).

Fig. 7 Frequency response of designed filters: blue line is enhanced AFCs and magenta dashed line is resonant filters

Fig. 8 Frequency response of open loop: blue line is open loop with enhanced AFCs and magenta dashed line is open loop with resonant filters

5.2. Simulation results

5.2.1. Comparison study with respect to the performance First, the characteristics of the AFC and resonant filter were verified in the frequency domain. These frequency responses are shown in Fig. 7. The results show that the characteristics of the enhanced AFC are the same as the resonant filters in the frequency domain. Figure 8 shows the frequency responses of the open loop characteristics, and Fig 9 shows the vector locus of the open loop. Figure 10 shows the frequency responses of the sensitivity function. These results show that both methods have the same disturbance compensation performance.

Second, the AFC and resonant filter characteristics were verified in the time domain. The time responses of the position error signal when $d(t)$ was injected are plotted in Fig. 11. We can see that if we compare the position error signal for the cases with enhanced AFC and without the filter, these signals are consistent. Therefore, the performance of the proposed AFC is equivalent to that of the resonant filter. The outputs of the enhanced AFCs and the resonant filters are shown in Fig. 12. It is clear from the figure that these time responses coincide.

Fig. 9 Vector locus of open loop: blue line is vector locus with enhanced AFCs and magenta dashed line is vector locus with resonant filters

Fig. 10 Frequency response of sensitivity function: black line is sensitivity function without filters, blue line is sensitivity function with enhanced AFCs, and magenta dashed line is sensitivity function with resonant filters

Fig. 11 Time response of position error signal $e(k)$: black line is without filters, blue line is enhanced AFCs, and magenta dashed line is with resonant filters

Fig. 12 Time response of filter output $u(k)$: blue line is enhanced AFCs and magenta dashed line is resonant filters

Fig. 13 Block diagram of the resonant filter in a real control system

5.2.2. Comparison study with respect to the implementation Here these method are compared with respect to implementation (volume of calculation and memory resources). Figure 13 outlines the composition of a second order filter like a resonant filter in a control system. The discrete transfer function of the resonant filter is described as

$$F_{res}(z) = \frac{Az^2 + Bz + C}{z^2 - Ez - F}. \quad (27)$$

In Fig.13, there are four addition and five multiplication units, and five parameters as coefficients and two state values are required to execute the functions of the resonant filter. In Fig.13, $x(k)$ and $y(k)$ are the state values of the resonant filter.

Figure 14 shows the outline of an enhanced AFC in a control system. There are five addition, eight multiplication, and three parameters λ , θ , ζ and two adaptive parameters $p(k)$ and $q(k)$ are required to execute an enhanced AFC. Moreover, a sine table or a cosine table must be used to calculate the adaptive algorithm. The implementation cost of an enhanced AFC is larger than a resonant filter's. Therefore, a resonant filter can be implemented to a control system easily.

However, an enhanced AFC can estimate disturbances by using $p(k)$ and $q(k)$. The estimated amplitude of the disturbance d_{afc} is calculated as

$$|d_{afc}(k)| = K \sqrt{p^2(k) + q^2(k)}, \quad (28)$$

where K is a plant gain at the disturbance frequency. The phase of a disturbance is calculated as

$$\arg(d_{afc}(k)) = \arctan \frac{p(k)}{q(k)}. \quad (29)$$

To estimate the disturbance using a resonant filter, an accurate identification of the plant P is needed, because a transfer function from disturbances to resonant filter output includes a plant property and a resonant filter does not have a information of a disturbances like an enhanced AFC. An enhanced AFC has many more functions than a resonant filter.

Fig. 14 Block diagram of the enhanced AFC in a real control system

6. Conclusion

This paper introduced a theoretical analysis for two kinds of major disturbance compensation methods used in positioning systems. We made a comparison between a resonant filter used as the loop shaping method and an enhanced AFC as the adaptive learning method. We found that the AFC was comparable to the resonant filter when we compared their characteristics. The adaptive algorithm of the enhanced AFC has the same function as the feedback control method. The relationship between the parameters of the enhanced AFC and the resonant filter's parameters were determined. A simulation of the disturbance compensation in a hard disk drive showed that these methods are fully consistent with respect to the performance. Moreover, the results showed advantages of each of the control methods with the exception of the performance. A resonant filter realizes low cost implementation with compare to an enhanced AFC. That is, a resonant filter easier to implement than an enhanced AFC. An enhanced AFC has the utility to realize a high quality function control system simultaneously able to compensate and estimate for disturbances. An enhanced AFC has many more functions than a resonant filter. This paper brings out the relationship between adaptive learning method and the loop shaping method for disturbance compensation. The results of the evaluation in this study will provide design engineers with guidelines for selecting an optimum control strategy in actual applications.

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