High–order involute modified noncircular bevel gears

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Abstract
Noncircular bevel gears (NBGs) is a kind of spatial transmission mechanism which can be used to transmit the motion and power between two intersecting axes with a variable transmission ratio and according to a suitable motion program. Given that the pitch curve and tooth profile curve of NBGs are spherical curve rather than plane curve, the research methods on NBGs is complicated than bevel gears as well as plane noncircular gears. In this paper, the pitch curve equations of NBGs are obtained for any order and in any configuration during their pure rolling based on the spherical polar coordinate system. The relationship of least teeth number avoiding undercutting and the pitch curve curvature of NBGs is analyzed. Being directed against high–order involute NBGs drive, a kind of varying–coefficient–profile–shift–modification method has been presented. The equations of the modified addendum and dedendum curves are implemented in MATLAB. The algorithm for generating tooth profile of NBGs by the hypothetical involute modified shaper cutter under UG platform is proposed and some significant examples are included. The 3D models and prototype of a pair of conjugate high–order involute modified NBGs are demonstrated to verify the correctness of this modification method.

Key words: Noncircular bevel gears, High–order, Involute, Modification, Generating

1. Introduction

Noncircular bevel gears (NBGs) is a kind of spatial transmission mechanism which can be used to transmit the motion and power between two intersecting axes with a variable transmission ratio and according to a suitable motion program. It has the advantages of compactness, smooth motion, and accurate transmission. This kind of gears are highly specialized, their application being found in some special instances such as the limited slip differential of off–road vehicles, the pump, among others, where the variable transmission ratio between intersecting axes is required (Xia, et al., 2008). Given that the pitch curve and the tooth profile curve of NBGs are spherical curve rather than plane curve, the equations of pitch curve and tooth profile curve can be deduced in spherical polar coordinate (Figliolini and Angeles, 2004), (Lin and Nie, 2013), (Shi, et al., 2013). In addition, research methods on plane noncircular gears are also used to the study of NBGs (Ollson, 1959), (Jia, et al., 2003a), (Zhao, et al., 2008).

The traditional research methods can’t satisfy the demand such as fluctuation range of high speed ratio, few number of teeth, dynamic transmission with large module. It will cause not only undercutting but also folding the gear surface near the tooth root. Folding the gear surface means the mesh points near the root surface will no longer move in one direction because of the increasing fluctuation range of transmission ratio, instead, the points will turn back, even two times or more. This phenomenon will increase the abrasion then affect the life of gears.

The NBGs used in the limited slip differential invented by Wang, et al. (1990a), (2001). Folding surface is avoided by making it coincide strictly, which means some part of the tooth face is allowed to mesh more than one time during a period. The tooth profile curve is designed seperately: the part under the pitch curve is the analytic curve; the above part satisfies the transmission characteristics by solving the mate gear’s above pitch curve part with meshing equation (Wang, et al., 1990b). Based on the mapping algorithm with reserved geodesic curvature between a sphere and a plane, the pitch curve of NBGs can be regarded as an extension of the back cone method for circular bevel gearing to noncircular ones (Jia, 2003b). This design method can get the approximate solutions of the pitch curve as well as the
tooth profile curve, and the tooth profile curve of the NBGs applied in limited slip differential is non-involute curve by shifting, rotating, jointing and fine-tuning (Jia, et al., 2003c), (Jia, et al., 2003d).

In this paper, the varying-coefficient-profile-shift-modification method is used to avoid undercutting and ensuring the root part of tooth face will not participate in the meshing process. Using a hypothetical involute modified shaper cutter, the tooth profile of the NBGs is obtained. To start with, the pitch curve equations are deduced for high-order NBGs during their pure rolling motion. Then design a modified coefficient function following the rule of cosine. The equations of the modified addendum and dedendum curves are implemented in MATLAB. The algorithm for generating tooth profile of NBGs by the hypothetical involute modified shaper cutter under UG platform is proposed and some significant examples are included. The 3D models and prototype of a pair of conjugate high-order involute modified NBGs are demonstrated to verify the correctness of this modification method.

2. The pitch curve of high-order NBGs

The elliptical bevel gear is described as below: the sum of the distance between any point \( P \) on the spherical surface to any other two fixed points (\( C \) and \( D \)) are constant \( \lambda \). Referring to Fig. 1. Suppose the spherical radius is \( R = 1 \), \( \theta_1 \) is the included angle between \( \overrightarrow{PC} \) and \( \overrightarrow{CD} \), \( \overrightarrow{CD} f = \phi \). The pitch curve equation for driving gear is

\[
\phi_1 = \arctan \left( \frac{\cos \phi - \cos \lambda}{\sin \lambda \sin \phi \cos \theta_1} \right) \tag{1}
\]

Where

\( \phi_1 \) denotes the polar angel between the radius vector and the positive \( Z \)-axis

A pair of conjugate elliptical bevel gear mesh at point \( P \), as shown in Fig. 2. \( \phi_0 = \phi_1 + \varphi_2 \) is the sum of spherical distance between point \( P \) to the two rotating axis of the driving gear and the driven gear. The transmission ratio is

\[
i_{12} = \frac{\sin \varphi_0}{\cos \phi - \cos \lambda} \frac{\sin \lambda \sin \phi \cos \theta_1}{\cos \phi_0} \tag{2}
\]

High-order NBGs whose order is greater than 1 is the development of elliptical bevel gear. If the perigon angle \( \theta_i \) turns to \( n_1 \cdot \theta_i \), the pitch curve equation of high-order NBGs is
\[
\varphi_i = \arctan \frac{\cos \phi - \cos \lambda}{\sin \lambda - \sin \phi \cdot \cos(n_i \cdot \theta_i)}
\]  

(3)

The pitch curve of high-order NBGs is closed. And the variation period of \( \varphi_i \) from 0 to \( 2\pi \) is \( n_i \), if it is required that the variation period of \( \varphi_2 \) from 0 to \( 2\pi \) is \( n_2 \), the condition to be closed for pitch curve of the driven gear would be expressed by

\[
\frac{2\pi}{n_2} = \int_0^{2\pi} \frac{A_i}{B_i + C_i - D_i - E_i \cdot \cos(n_i \cdot \theta_i)} \, d\theta_i
\]  

(4)

Where

\[
A_i = \cos \phi - \cos \lambda
\]

\[
B_i = \sin \varphi_0 \cdot \sin \lambda
\]

\[
C_i = \cos \varphi_0 \cdot \cos \lambda
\]

\[
D_i = \cos \varphi_0 \cdot \cos \phi
\]

\[
E_i = \sin \varphi_0 \cdot \sin \phi
\]

Then

\[
2n_1^2 \cdot F_i + 2n_2^2 \cdot G_i - J_i \cdot \cos \lambda + K_i \cdot \cos \phi = 0
\]  

(5)

Where

\[
F_i = \cos^2 \varphi_0 \cdot \cos \lambda
\]

\[
G_i = \cos \varphi_0 \cdot \sin \varphi_0 \cdot \sin \lambda
\]

\[
J_i = n_1^2 + n_2^2
\]

\[
K_i = n_2^2 - n_1^2
\]

The pitch curve equation of the driven gear is

\[
\begin{aligned}
\theta_2 &= \frac{1}{n_2} \int_0^{2\pi} \frac{A_i}{B_i + C_i - D_i - E_i \cdot \cos(n_i \cdot \theta_i)} \, d\theta_i \\
\varphi_2 &= \varphi_0 - \arctan \frac{\cos \phi - \cos \lambda}{\sin \lambda - \sin \phi \cdot \cos(n_i \cdot \theta_i)}
\end{aligned}
\]  

(6)

Where
3. The high–order modified NBGs

3.1 The addendum and dedendum

As shown in Fig. 3, the $Z$–axis and spherical surface $O_R$ intersect at point $A$. The numbers 1, 2, and 3 represent the dedendum curve, pitch curve, and addendum curve, respectively. $\overrightarrow{BC}$ is the normal arc of the pitch curve at point $P$. Orthodromes $\overrightarrow{AC}$, $\overrightarrow{AP}$, and $\overrightarrow{AB}$ are created over point $A$, with $\beta$, $\phi$, and $\gamma$ as their central angles over points $C$, $P$, and $B$, respectively. $\zeta$ denotes the included angle of tangent lines of orthodromes $\overrightarrow{AP}$ and $\overrightarrow{BP}$ at point $P$, also known as the included angle of $\overrightarrow{AP}$ and $\overrightarrow{CP}$; $\delta$ denotes the tangent azimuth angle of the pitch curve at point $P$. When $\delta < \pi / 2$, then $\zeta = \delta + \pi / 2$ and $\xi = \pi / 2 - \delta$; when $\delta > \pi / 2$, then $\zeta = 3\pi / 2 - \delta$ and $\xi = \delta - \pi / 2$; $\alpha_a$ is the addendum angle, and $\alpha_{\alpha_a} = h_a^* \cdot \alpha_a$; $\alpha_j$ is the dedendum angle, and $\alpha_j = (h_j^* + c^*) \cdot \alpha_a$; $h_a^*$ is the addendum factor, and $h_j^* = 1$; $c^*$ is the clearance factor, and $c^* = 0.25$; and $\alpha_{\alpha_m}$ is the modular angle, which is defined as the degree of the central angle sector extended from the conic part divided by $\pi$. Such a conic part is derived from the pitch surface, which is at the same side of the tooth surface for the adjacent teeth. $\Delta \theta_\gamma$ is the included angle of $\overrightarrow{AP}$ and $\overrightarrow{AB}$, known as relative perigon of the pitch curve and the addendum curve at an equidistant point. $\Delta \theta_\theta$ is the included angle of $\overrightarrow{AP}$ and $\overrightarrow{AC}$, known as relative perigon of the pitch curve and the dedendum curve at an equidistant point.

The equations of addendum and dedendum curve are given by

\[
\cos(n_i \cdot \theta_i) = \frac{H_i \cdot \cos^2 \left(\frac{n_i \cdot \theta_i}{2}\right) - L_i \cdot \sin^2 \left(\frac{n_i \cdot \theta_i}{2}\right)}{H_i \cdot \cos^2 \left(\frac{n_i \cdot \theta_i}{2}\right) + L_i \cdot \sin^2 \left(\frac{n_i \cdot \theta_i}{2}\right)}
\]

\[
H_i = B_i + C_i - D_i + E_i
\]

\[
L_i = B_i + C_i - D_i - E_i
\]
\[ \begin{align*}
\gamma &= \arccos(\cos \phi \cdot \cos \alpha_x + \sin \phi \cdot \sin \alpha_x \cdot \cos \zeta) \\
\Delta \theta_x &= \arccos\left(\frac{\cos \alpha_x - \cos \phi \cdot \cos \gamma}{\sin \phi \cdot \sin \gamma}\right) \\
\beta &= \arccos(\cos \phi \cdot \cos \alpha_y + \sin \phi \cdot \sin \alpha_y \cdot \cos \zeta) \\
\Delta \theta_y &= \arccos\left(\frac{\cos \alpha_y - \cos \phi \cdot \cos \beta}{\sin \phi \cdot \sin \beta}\right)
\end{align*} \tag{7}
\]

\[ \begin{align*}
\theta_x &= \begin{cases} 
\theta - \Delta \theta_x, & 0 \leq \theta \leq \pi \\
\theta + \Delta \theta_x, & \pi \leq \theta \leq 2\pi 
\end{cases} \\
\theta_y &= \begin{cases} 
\theta + \Delta \theta_y, & 0 \leq \theta \leq \pi \\
\theta - \Delta \theta_y, & \pi \leq \theta \leq 2\pi 
\end{cases}
\end{align*} \tag{8}
\]

Where

3.2 The least teeth number avoiding undercutting

The pitch curves of NBGs are pure rolling while the intersection angle of their rotate axis remains the same. If the order of NBGs is high and the teeth number is low, undercutting may appear in the NBGs. Actually, on the pitch curve of the NBGs, the smaller the curvature radius is, more serious the undercutting is.

As shown in Fig. 4. The extreme case of undercutting is: the intersecting point $E$ is the addendum circle of the shaper cutter and the normal of the tooth profile, point $E$ is also the tangent point between the normal of the tooth profile and the evolute of NBGs. When the intersecting point position of the addendum circle of the shaper cutter and the normal of the tooth profile is beyond $E$, the undercutting occurs.

According the the spherical triangle cosine theorem, $\widehat{PE}$ can be expressed by

\[ \widehat{PE} = \arccos\left[\cos \varphi_n \cdot \cos \varphi_m + \sin \varphi_n \cdot \sin \varphi_m \cdot \cos(\theta_i - \theta_j)\right] \tag{9} \]

Where
\[ \phi_b \] denotes the polar angle of the shaper cutter’s base circle

\[ \phi_a \] denotes the polar angle of the shaper cutter’s addendum circle, and \[ \phi_a = \phi_b + \alpha_a \]

\[
\theta_1 = \arccos \left( \frac{\sin \phi_b}{\sin \phi_a} \right)
\]

\[
\theta_2 = \arccos \left( \frac{\sin \phi_b}{\sin \phi_a} \right)
\]

In order to void undercutting
\[
\overline{PE} \leq \arcsin \left( \sin \rho_{\text{max}} \cdot \sin \alpha_a \right)
\]

Where

\[
\rho_{\text{max}} = \min \left\{ \rho, \theta \in (0, 2\pi) \right\}, \text{ denotes the polar angle of the NBGs’ minimum curvature radius and}
\]

\[
\tan \rho = \frac{\left[ \sin^2 \varphi + \varphi' \left( \theta \right) \right]^2}{\cos \varphi \cdot \sin^2 \varphi + 2 \cos \varphi \cdot \varphi'' \left( \theta \right) - \sin \varphi \cdot \varphi'' \left( \theta \right)}
\]

\[ \alpha_a \] denotes the tooth profile angle on shaper cutter’s pitch circle

According to Eq. (9) and Eq. (10), the maximum \[ \alpha_a \] avoid undercutting can be deduced.

Suppose \[ \alpha_a = h_a^+ \cdot \alpha_a \], the least teeth number avoid undercutting can by expressed by

\[
z \geq \frac{h_a^+}{\pi \cdot \alpha_a} \int_0^{2\pi} \sqrt{\sin^2 \varphi + \varphi'' \left( \theta \right) \, d\theta}
\]

3.3 The modified coefficient function

The same modification will cause different effect at different part of the tooth: the smaller the curvature radius is, the better the effort will get. In order to achieve well-distributed modification, one should define the modified coefficient due to the curvature radius of the location. The smaller part of the curvature radius of the driving gear pitch curve is corresponding to the larger part of the curvature radius of the driven gear pitch curve when they are pure rolling. Therefore, positive modification which is taken on the smaller curvature radius part may increase the thickness of tooth and eliminate undercutting, the larger curvature radius part is undertook negative modification correspondingly. In addition, the modification coefficient of the largest curvature radius must be the opposite number of the smallest curvature radius.

The modified function following the rule of cosine can be expressed as

\[
\Delta y = \Delta y_{\text{max}} \cdot \cos \left[ \frac{2\pi \cdot S_y}{S_z / n} \right]
\]
Where,

\[ \Delta y_{\text{max}} \] denotes the modification coefficient

\[ S_2 \] denotes the length of the pitch curve

\( n \) denotes the order numbers

\[ S_\theta \] denotes the corresponding arc length of the pitch curve in scope \[ [0, \theta] \]

The modification function of the driven gear is \(-\Delta y\) in the conjugate joints.

The addendum angle is

\[
\alpha_a = h_s^* \cdot \alpha_m + h_s^* \cdot \alpha_m \cdot \Delta y
\]  \hspace{1cm} (13)

The dedendum angle is

\[
\alpha_f = (h_s^* + c^*) \cdot \alpha_m - (h_s^* + c^*) \cdot \alpha_m \cdot \Delta y
\]  \hspace{1cm} (14)

Along with Eq. (7) and Eq. (8), the equations of modified addendum and dedendum curve of NBGs can be derived.

4. The tooth profile curve of NBGs

The shaper cutter is introduced to produce the tooth profile curve of NBGs. If the pitch surface of the shaper cutter rolls around the pitch surface of NBGs from a certain position, a pair of conjugate gear tooth profile curves can be obtained. The tooth profile curve equations of NBGs are deduced in previous paper by author and the correctness of the equations has been verified (Shi K, et al., 2013).

The algorithm for generating tooth profile of NBGs by shaper cutter under UG platform is followed.

Step1. Position the shaper cutter and ensure the pitch curves of the shaper cutter and NBGs are tangent.

Step2. Adjust the rotation angle of the shaper cutter and ensure the sign point on the shaper cutter coincides with the tangent point.

Step3. The shaper cutter revolves on its own axis to realize the pure rolling.

Step4. Cut the gear blanks.

4.1 The positioning of the shaper cutter

The shaper cutter and the gear blanks are pure rolling on the spherical. As shown in Fig. 5. The initial direction vector is \( \vec{t}_I = (x_1, y_1, z_1) \), the position direction vector is \( \vec{t}_2 = (x_2, y_2, z_2) \), the rotation angle \( \chi \) and the rotation axis \( \vec{p}_{\text{precl}} \) can be expressed by

\[
\chi = \arccos \frac{\vec{t}_I \cdot \vec{t}_2}{|\vec{t}_I| \cdot |\vec{t}_2|} = \frac{x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_2^2}}
\]  \hspace{1cm} (15)

And

\[
\vec{p}_{\text{precl}} = \vec{t}_I \times \vec{t}_2 = (y_1 z_2 - y_2 z_1, x_2 z_1 - x_1 z_2, x_1 y_2 - x_2 y_1)
\]  \hspace{1cm} (16)

The track of the shaper cutter is the equal space line of the pitch curve in spherical surface. It can be derived by Eq. (7) as the addendum curve. Then, the shaper cutter is positioned and the pitch curves of the shaper cutter and NBGs are tangent.
4.2 The adjustment angle of the shaper cutter

The sign point means the tangent point between the shaper cutter and the pitch curve of NBGs at the initial positon. The sign point doesn’t coincide with the tangent point besides then.

As shown in Fig. 6, $E(0,0,1)$ is the intersection point of the rotation axis of the shaper cutter and the spherical surface. $F(0,\sin \omega, \cos \omega)$ is the sign point. Where $\omega$ is the cone angle. The sign vector $\vec{t} = (0,\sin \omega, \cos \omega - 1)$.

The included angle of $\vec{prec}1$ and $\vec{r}$ is $\theta_1$. $\vec{t}_2$ is the position vector which is attained by $\vec{t}$ revolves on $\vec{prec}1$ by $\theta_4$. The coordinate of sign vector $\vec{t}_2$ in coordinate system $xyz$ is

$$\vec{t}_2 = R(z, \theta_4) \cdot R(y, \theta_4) \cdot \vec{t}_1 = \begin{pmatrix} \cos \theta_4 & -\sin \theta_4 & 0 \\ \sin \theta_4 & \cos \theta_4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta_4 & 0 & \sin \theta_4 \\ 0 & 1 & 0 \\ -\sin \theta_4 & 0 & \cos \theta_4 \end{pmatrix} \cdot \vec{t}_1$$

As shown in Fig. 7,

$$\vec{GP} = (\sin \varphi \cdot \sin \theta - \sin \gamma \cdot \sin \theta_1, \cos \varphi - \cos \gamma, \sin \varphi \cdot \cos \theta - \sin \gamma \cdot \cos \theta_1)$$

The included angle between $\vec{GP}$ and $\vec{t}_2$ is

$$\varepsilon_1 = \frac{\vec{GP} \cdot \vec{t}_2}{|\vec{GP}| \cdot |\vec{t}_2|}$$

The adjustment angle of the shaper cutter is

$$\varepsilon_2 = 2arcsin \frac{\sin c / 2}{\cos \omega / 2}$$

Where

$0 \leq \theta \leq \pi$, the adjustment angle is clockwise

$\pi \leq \theta \leq 2\pi$, the adjustment angle is anticlockwise
4.3 Revolve on its own axis

The length of the pitch curve of NBGs is given by

\[ S_i = \int_0^\theta \sqrt{\sin^2 \varphi + \varphi'^2(\theta)} \, d\theta \]  \hspace{1cm} (21)

The angle of the shaper cutter revolves on its own axis is

\[ \theta_s = \frac{S_2}{\sin \omega} \]  \hspace{1cm} (22)

5. Example

The design method and the algorithm have been implemented in MATLAB and UG to run significant examples, as shown in Fig. 8–Fig. 11.

The driving gear is 3-order NBGs while the driven gear is 4-order NBGs for the parameter has given \( \lambda = 74^\circ \), \( \varphi_0 = 90^\circ \). The pitch curve, addendum and dedendum curve equations of the driving gear are obtained as well as the driven gear. The curvature radius of the pitch curve is deduced, and the least teeth number avoid undercutting is 38.

In order to ensure the complete tooth shape, the number of teeth should meet the following condition

\[ \frac{S}{S_i} = \frac{Z}{Z_i} \]  \hspace{1cm} (23)

\[ S \] denotes the pitch curve length of the shaper cutter and \( S = 2\pi \cdot \sin \omega \)

\[ S_i \] denotes the pitch curve length of the NBGs and \( S_i = \int_0^{2\pi} \sqrt{\sin^2 \varphi + \varphi'^2(\theta)} \, d\theta \)

\[ Z \] denotes the teeth number of the shaper cutter

\[ Z_i \] denotes the teeth number of driving gear and driven gear
\( \omega \) denotes the cone angle of the shaper cutter.

The 3D models of 3-order and 4-order NBGs are shown in Fig. 8.

![3D models of NBGs](image)

(a) \( n_1 = 3 \), \( Z_1 = 39 \)

(b) \( n_2 = 4 \), \( Z_2 = 52 \)

Fig. 8 The 3D models of NBGs

The modified addendum and dedendum curves of NBGs has been deduced and implemented in **MATLAB**, as shown in Fig. 9.

![Modified curves](image)

(a) \( n_1 = 3 \), \( \Delta y_{\text{max}} = 0.4 \)

(b) \( n_2 = 4 \), \( \Delta y_{\text{max}} = 0.4 \)

1–Addendum curve 2–Pitch curve 3–Dedendum curve
Fig. 9 Pitch curve, addendum and dedendum curves of high–order modified NBGs

The 3D models of modified high–order NBGs are implemented, as shown in Fig. 10. We can see clearly that the tooth thickness distribution and the undercutting are improved, the varying–coefficient–profile–shift–modification method for high–order involute NBGs is successful.

![Image of 3D models showing improved performance](image)

Fig. 10 The 3D models of high–order involute modified NBGs

A pair of high–order involute modified NBGs has been generated by rapid prototyping, they can mesh smoothly, as shown in Fig. 11.

![Image of prototype showing smooth meshing](image)

Fig. 11 The prototype of high–order involute modified NBGs
6. Conclusions

A general design method for high–order involute modified NBGs has been proposed. In particular, the pitch curve equations of NBGs are obtained for any order and in any configuration during their pure rolling motion. The relationship of least teeth number avoiding undercutting and the pitch curve curvature of NBGs is analysed.

Being directed against high–order involute NBGs drive, a kind of varying–coefficient–profile–shift–modification method has been presented. The equations of the modified addendum and dedendum curves are implemented in MATLAB.

Furthermore, the algorithm for generating tooth profile of NBGs by shaper cutter under UG platform is proposed and significant examples are included. The 3D models and prototype of a pair of conjugate high–order involute modified NBGs are demonstrated to verify the correctness of this modification method.

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References

**Notation**

- $c^*$: clearance factor
- $h_a^*$: addendum factor
- $i_{12}$: transmission ratio and $i_{12} = f(\theta_i)$
- $n_i$: order numbers of NBGs
- $\overrightarrow{\text{prec1}}$: rotation axis of the shaper cutter
- $\overrightarrow{i}$: sign vector
- $\overrightarrow{t_1}$: initial direction vector
- $\overrightarrow{t_2}$: position direction vector
- $\Delta y$: modified function
- $\Delta y_{\text{max}}$: modification coefficient
- $A$: intersection point of the $Z$-axis and spherical surface $O_R$
- $B$: point on spherical surface
- $C$: point on spherical surface
- $D$: point on spherical surface

**NBGs** Noncircular Bevel Gears

- $O$: sphere center
- $O_R$: sphere with radius $R$
- $P$: point on spherical curve
- $R$: spherical radius
- $S$: length of the shaper cutter
- $S_i$: length of the pitch curve of the driving gear
- $Z$: number of teeth of the shaper cutter
- $Z_i$: number of teeth
- $S_z$: length of the pitch curve of the driven gear
- $S_\theta$: length of the pitch curve in scope $[0, \theta]$
\( \alpha_a \) addendum angle
\( \alpha_f \) dedendum angle
\( \alpha_m \) modular angle
\( \alpha_t \) tooth profile angle on shaper cutter’s pitch circle

\( \beta \) central angle of \( \widehat{AC} \)
\( \gamma \) central angle of \( \widehat{AB} \)
\( \rho \) polar angle of the NBGs’ curvature radius
\( \chi \) rotation angle of shaper cutter
\( \delta \) tangent azimuth angle
\( \zeta \) included angle of tangent lines of orthodromes \( \widehat{AP} \) and \( \widehat{BP} \)
\( \xi \) included angle of \( \widehat{AP} \) and \( \widehat{CP} \)
\( \zeta \) included angle of tangential line of orthodrome \( \widehat{AC} \) and normal line of arc around \( C \)
\( \theta_i \) perigon angle
\( \theta_s \) included angle of \( \text{precl} \) and \( y \)
\( \theta_4 \) angle \( \text{precl} \) revolves on \( \text{precl} \) axis
\( \theta_5 \) angle of the shaper cutter revolves on its own axis
\( \theta_6 \) perigon of dedendum curve
\( \theta_7 \) perigon of addendum curve
\( \Delta \theta_3 \) included angle of \( \widehat{AP} \) and \( \widehat{AC} \)
\( \Delta \theta_4 \) included angle of \( \widehat{AP} \) and \( \widehat{AB} \)
\( \lambda \) Constant
\( \phi_i \) polar angle between the radius vector and the positive \( Z \)-axis
\( \phi_0 \) included angle between \( Z_1 \)-axis and \( Z_2 \)-axis
\( \phi_n \) polar angle of the shaper cutter’s pitch circle
\( \phi_b \) polar angle of the shaper cutter’s base circle
\( \phi_m \) polar angle of the shaper cutter’s addendum circle
\( \omega \) cone angle of the shaper cutter
$\varepsilon_1$ included angle between $GP$ and $t_2$

$\varepsilon_2$ adjustment angle of the shaper cutter

$\phi$ central angle of focal length

Subscript

$i = 1, 2...$ denotes the driving or driven gears, segerately.