Dynamic friction modelling without drift and its application in the simulation of a valve controlled hydraulic cylinder system

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Abstract
The frictional modelling literature is reviewed, and it is demonstrated that unrealistic drift results when the shape coefficient is 1.0 for the LuGre and the integral friction models. Drift will not occur but other dynamic friction characteristics can’t be represented when the shape coefficient is 0. Based on the above, the LuGre friction model and the Integral friction model are improved. The velocity-friction characteristic, the stick-slip and the cycling caused by friction and the drift are compared in simulation. The results show that the improved friction model well reflects realistic friction dynamic characteristics and avoids drift. Finally, the improved friction model is used in a nonlinear mathematic model of a valve controlled hydraulic cylinder system. The cylinder’s motion at low velocity is simulated and the related experimental results are presented. The results show that the improved friction model gives realistic low velocity motion of the cylinder.

Key words: Friction model, Stick-slip, Limit cycles, Drift, Valve controlled cylinder

1. Introduction

Friction is inevitable in mechanical systems. The nonlinear behaviour caused by friction has an adverse influence on the ultra-low velocity and high precision position control of servo mechanisms and hydraulic systems. Friction severely affects the stability of the control system designed to obtain very small steady-state error and could lead to limit cycles and stick-slip, and affect the frequency response bandwidth of the closed loop system (Friedland and Park, 2009, Dupont and Dunlap, 1995).

It is important to establish an accurate friction model for both understanding the friction phenomenon and compensating for friction. Until now, a lot of research on friction modelling has been undertaken (Satoshi, et al., 2008, Daniel, et al., 2009, Olsson, et al., 1998, Bonisignore, et al., 1999 ). Many experiments on friction show that there exist two friction regimes (Lampaert, et al., 2003): the pre-sliding regime and the sliding regime. In the pre-sliding regime the friction force appears to be a function of relative micro-displacement (elastic deformation and plastic deformation), and its characteristics are similar to a nonlinear spring. As the displacement becomes larger, the “spring” suddenly ruptures leading to the relative motion between two contact surfaces, and the sliding regime begins. In the sliding regime the friction force appears to be a function of relative velocity.

Up to now, many friction models have been proposed, and they can be classified into two categories: static friction models and dynamic friction models. Among static friction models the representatives are the Coulomb plus viscous friction model (Shang, et al., 2010) and the exponential friction model (Marton, et al., 2011), these models can’t predict dynamic friction. Though the seven parameters model (Armstrong, et al., 1994, Berger, 2002) can reflect the friction’s static and dynamic characteristics, but it has no explicit physical content for the model’s parameters , and moreover it contains redundant parameters. The dynamic friction models include: the Dahl model (Dahl, 1968, Astrom, et al., 2008, Ferretti, et al., 2004a), the LuGre model (Canudas, et al., 1995), the Elastic-Plastic model (Dupont, et al., 2000,.
Dupont, et al., 2002), the Integral friction model (Ferretti, et al., 2004b), the Leuven model (Swevers, et al., 2000), and the GMS model (Ruderman and Bertram, 2011).

2. The development of dynamic friction modelling

2.1 Dahl friction model

The Dahl model, which was developed in the late 1950s, is a dynamic model with one state, and is widely used to simulate aerospace systems (Dahl, 1968, Astrom and Wit., 2008). The Dahl dynamic model essentially describes the friction’s pre-sliding regime, and in this regime the friction force is the function of relative micro-displacement between two friction surfaces. The mathematical expression of the Dahl model is as follows (Dahl, 1968).

\[
\frac{df}{dx} = \sigma_0 (1 - \frac{f_c}{f_c}) \text{sgn}(v)^\alpha
\]  

Where \( f_c \) is the friction force, \( x \) is the relative micro-displacement between two friction surfaces, \( \sigma_0 \) is the stiffness coefficient, \( f_c \) is the Coulomb friction force, \( v \) is the relative velocity of two friction surfaces, \( \alpha \) is the coefficient determining the shape of the curve between friction force and relative micro-displacement and is always larger than zero.

Let \( v = \frac{dx}{dt} \), \( z = \frac{f_c}{\sigma_0} \), and the friction force can be expressed as equation (2) when \( \alpha = 1 \).

\[
\frac{dz}{dt} = v - \left| v \right| \frac{\sigma_0 z}{f_c} 
\]

\[
f_c = \sigma_0 z
\]

2.2 LuGre friction model

The Dahl dynamic model does not take into account the friction’s sliding regime. In the sliding regime the lubricant film plays a dominant role, and it is appropriate to describe the friction force as a function of relative velocity between two friction surfaces. In the case that the velocity is very low, the lubricant film isn’t formed fully, and the friction force would decreased when the relative velocity increases, as per the Stribeck phenomenon. The Stribeck phenomenon can be described by the following equation:

\[
f_s = \text{sgn}(v) g(v)\]

\[
g(v) = f_c + (f_s - f_c) \exp\left[-\left(\frac{v}{v_s}\right)^{\delta_s}\right]
\]

Where \( f_s \) is the maximum static friction force, \( f_c \) is the Coulomb friction force, \( v \) is the relative velocity of two friction surfaces, \( v_s \) is the velocity at the turning point of the Stribeck curve, and \( \delta_s \) is the corrective coefficient of curve.

The literature (Canudas and Olsson, 1995) integrates the Dahl model (2) and the Stribeck equation (3) to derive the following LuGre friction model:

\[
\frac{dz}{dt} = v - \left| v \right| \frac{\sigma_0 z}{g(v)}
\]

\[
g(v) = f_c + (f_s - f_c) \exp\left[-\left(\frac{v}{v_s}\right)^{\delta_s}\right]
\]
\[ f_1 = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \]  

(4c)

Where \( f_1 \) is the total friction force, \( f_c \) is the Coulomb friction force, \( f_s \) is the maximum static friction force, \( v \) is the relative velocity of two friction surfaces, \( v_t \) is the velocity at the turning point of Stribeck curve, \( \sigma_0 \) is the equivalent stiffness coefficient between the friction force and the relative displacement of two friction surfaces when the relative velocity’s direction changes, \( \sigma_1 \) is the micro-viscous friction coefficient, and \( \sigma_2 \) is the viscous friction coefficient.

### 2.3 Elastic-plastic friction model

The LuGre model produces steady-state drift when a tiny external vibratory stimulation is applied (Dupont, et al., 2000), whereas in practice there isn’t relative motion between two the friction surfaces because the vibratory stimulation is smaller than the maximum breakout friction. In the same case, though steady-state drift doesn’t arise in the Karnopp friction model, micro-displacement which should arise does not. Based on the above problem, (Dupont, et al., 2000) and Dupont, et al., 2002) propose the elastic-plastic friction model as a development of the LuGre friction model.

\[ \frac{dz}{dt} = v - \alpha(z, v) \frac{\sigma_0 z}{g(v)} \]  

(5a)

\[ g(v) = f_c + (f_s - f_c) \exp\left[-\left(v / v_t\right)^2\right] \]  

(5b)

\[ f_1 = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v \]  

(5c)

\[ \alpha(z, v) = \begin{cases} 
0, & \text{if } |z| \leq z_{in} \text{ or } \text{sgn}(v) \neq \text{sgn}(z) \\
1, & \text{if } |z| \geq z_{in}(v) \\
\frac{1}{2} \sin\left(\pi - \frac{z - \left(z_{ss} + z_{in}\right)}{z_{ss} - z_{in}}\right) + \frac{1}{2}, & \text{else}
\end{cases} \]  

(5d)

Where \( f_1 \), \( f_c \), \( f_s \), \( v \), \( v_t \), \( \sigma_0 \), \( \sigma_1 \), \( \sigma_2 \) are the same as the parameters in the LuGre friction model shown in section 2.2.

\[ 0 < z_{in} < z_{in}(v) \]  

(6)

\[ z_{in}(v) = \left|\frac{g(v)}{\sigma_0}\right|_{\max} = \frac{f_c}{\sigma_0} \]  

(7)

where \( z_{in} \) represents the range of friction state variable \( z \) when friction is characterized by linear damping and a linear spring, and \( z_{in} \) represents the range of friction state variable \( z \) when the friction contact surface is in the pre-sliding regime in which there are only elastic deformation and plastic deformation.

### 2.4 The Integral friction Model

Integral friction proposes an integral friction model (Ferretti, et al., 2004b), and the simulation results show that the model is consistent with the LuGre model in terms of reflecting stick-slip, limit cycles, and so on, while being
computationally more efficient and avoiding non-physical drift through letting \( \alpha = 0 \) in the Dahl model. The integral friction model is given in the following.

\[
f_f = f_p + f_v + f_z
\]  

(8a)

\[
\frac{df_f}{dx} = \sigma_0 (1 - f_v f_v) v^a
g(v) = f_v (f_v - f_v) \left\{ \exp\left[-\left(\|v\|/v_c\right)^a\right] - 1\right\} + \sigma_2 v
\]  

(8b)

(8c)

\[
f_v = \sigma_1 (1 - \text{sgn}(v)) \frac{f_p}{f_v} v
\]  

(8d)

Where \( f_p \) is the term which relates friction to micro-displacement in the pre-sliding regime, \( f_v \) is the term which relates friction to velocity in the sliding regime, and \( f_z \) is the micro-viscous friction term when friction transitions from the pre-sliding regime to the sliding regime, and the other parameters are the same as the parameters in the LuGre model shown in section 2.2.

2.5 Leuven friction model

In (Swevers, et al., 2000), the pre-sliding regime friction force is modelled as a hysteresis function of relative micro-displacement, with nonlocal memory. Considering this factor, it corrects the LuGre model and derives the Leuven friction model.

\[
\frac{dz}{dt} = \nu \left(1 - \text{sgn}(f_v(z)) \left| \frac{f_v(z)}{g(v)} \right| \right)
\]  

(9a)

\[
g(v) = f_v + (f_v - f_v) \exp[-(\|v\|/v_c)^k]
\]  

(9b)

\[
f_v = \sigma_1 (1 - \text{sgn}(v)) \frac{f_p}{f_v} \nu
\]  

(9c)

Where \( f_v(z) \) is the hysteresis function with non-local memory, and it is the point symmetrical and strictly increasing function with the input state variable \( z \). \( f_v(z) \) can be found by experimental identification (Swevers, et al., 2000) or by theoretical modelling (Bjorklund, et al., 1997). \( \delta_1 \) is similar to the shape coefficient \( \alpha \) in the Dahl model (1). The other parameters are the same as the parameters in the LuGre model shown in section 2.2.

2.6 GMS (Generalized Maxwell-Slip) friction model

The literature (Lampaert, et al., 2003, Ruderman, et al., 2011) takes into account advantages and disadvantages of the elastic-plastic friction model and the Leuven friction model and proposes the GMS friction model. It can not only reflect the non-local memory hysteresis loops and the Striebeck phenomenon in the pre-sliding regime, but also avoid the same steady drift as the elastic-plastic model.

\[
f_f = \sum_{i=1}^{N} f_i + \sigma_2 v
\]  

(10a)

\[
\frac{df_f}{dt} = \begin{cases} k_i \nu & \text{if } f_i \leq \alpha g(v) \\ \text{sgn}(v)C(\alpha - \frac{f_i}{g(v)}) & \text{else} \end{cases}
\]  

(10b)

\[
g(v) = f_v + (f_v - f_v) \exp[-(\|v\|/v_c)^k]
\]  

(10c)
Where $N$ is the number of friction model units, $k_i$ is the contact stiffness, $\alpha_i$ is the split coefficient of the friction model, $C$ is the coefficient concerning the velocity $v$, $f_i$ is the friction force unit, $\sigma_i$ is the viscous friction coefficient, and $f_f$ is the total friction force. The other parameters are the same as the parameters in the LuGre model shown in section 2.2.

3. Improvement of the LuGre friction model and the Integral friction model

Though the elastic-plastic model, the Leuven model and the GMS model can accurately reflect the friction’s dynamic characteristics, they are complex. And so it is very difficult to utilize them in closed loop control system analysis and design. Most of the literature about friction for control purpose has adopted the LuGre friction model (Karim, et al., 2009, Schindele, et al., 2009, Reza, et al., 2013). The literature (Ferretti, et al., 2004a) indicates by simulation that the Integral friction model has the same characteristics as the LuGre friction model, and also it is computationally more efficient.

The Integral friction model can avoid unrealistic drift when $\alpha=0$ (Ferretti, et al., 2004a), while the LuGre friction model and the Integral friction model can’t accurately reflect the friction’s dynamic characteristics when the parameter $\alpha=0$ in the friction model. The difference in the friction force as a function of velocity is shown in figure 1 and figure 2 when $\alpha=0$ and $\alpha=1$, and it is found that the friction model can fully reflect the friction’s dynamic characteristics when $\alpha=1$, but can’t reflect the friction’s dynamic characteristics when $\alpha=0$.

Through the above summary of the literature, the following conclusion can be reached: (1) the drift of the friction model mainly occurs in the pre-sliding regime in which there is no obvious relative motion in reality, and furthermore, the drift mainly occurs in the elastic deformation stage of the pre-sliding regime; (2) the basis of the LuGre friction model and Integral friction model is the Dahl friction model, and the parameter $\alpha$ of the Dahl friction model is the coefficient determining the shape of curve between friction force and relative micro-displacement in the pre-sliding regime; (3) when $\alpha=0$, drift can be avoided, and when $\alpha=1$, the friction’s dynamic characteristics can be well reflected.

Therefore the following improvement of the LuGre and the Integral friction models is derived. The friction process is divided into the two stages. The first stage is the elastic deformation, in which the friction force is not larger than the coulomb friction force $c_f$, and by letting $\alpha=0$ in this stage drift is avoided. The second stage is the plastic deformation and sliding friction, and by letting $\alpha=1$ in this stage ensures the friction’s dynamic characteristics are modelled correctly.

3.1 The improved LuGre friction model

\[
\frac{dz}{dt} = v(1 - \frac{\sigma z}{g(v)} \text{sgn}(v))^\alpha
\]  

(11a)

\[
g(v) = f_c + (f_s - f_c) \exp[-(v / v_s)^6]
\]  

(11b)

\[
f_t = \sigma c z + \sigma_i \frac{dz}{dt} + \sigma_i v
\]  

(11c)

\[
\alpha = \begin{cases} 
0 & |z| \leq z_{e} \\
1 & |z| > z_{e}
\end{cases}
\]  

(11d)

where $z_{e} = f_s / \sigma_s$, $f_c$ is the coulomb friction force, $\sigma_s$ is the stiffness coefficient. The term $z_{e}$ represents the range of friction state variable $z$ when the friction contact surface is in the pre-sliding regime in which there is only elastic deformation.

When the shape coefficient $\alpha$ switches from 0 to 1, $\alpha$ will not chatter. The reason is that whenever $\alpha=0$ or $\alpha=1$, the state variable $z$ is an increasing function when $|z| < (f_s + 0.37(f_s - f_c)) / \sigma_s$. It means that $|z|$ will not decrease along with time near $z_{e}$, and $\alpha$ doesn’t switch from 1 to 0 again.
3.2 The improved Integral friction model

\[ f_t = f_p + f_c + f_z \]  
(12a)

\[ \frac{df_c}{dx} = \sigma_0 (1 - \frac{f_c}{f_s}) \text{sgn}(v) \]  
(12b)

\[ f_c = \text{sgn}(v) (f_c - f_z) \left\{ \exp\left[\left(\frac{|v|}{v_s}\right)^\frac{1}{n}\right] - 1 \right\} + \sigma_2 v \]  
(12c)

\[ f_z = \sigma_1 (1 - \text{sgn}(v) \frac{f_c}{f_s}) v \]  
(12d)

\[ \alpha = \begin{cases} 
0 & |f_c| \leq f_c \\
1 & |f_c| > f_c 
\end{cases} \]  
(12e)

Where \( f_p \) is the term which relates friction to micro-displacement in the pre-sliding regime, \( f_c \) is the coulomb friction force, and \( |f_c| \leq f_c \) means that the friction is in the elastic deformation stage and \( |f_c| > f_c \) means that the friction is in the plastic deformation and sliding regime. It is the same as the above improved LuGre friction model that the shape coefficient \( \alpha \) doesn’t chatter.

4. Analysis of simulation experiment

Simulation results (Ferretti, et al., 2004a) already indicate the consistency of the LuGre model and the Integral friction model, and when \( \alpha=0 \) both can avoid drift, while through simulation it is found that the model can’t reflect stick-slip and limit cycles caused by friction nonlinearity when \( \alpha=0 \). For brevity, in the following only results for the Integral friction model and the improved Integral friction model are given, and the simulation parameters which are the same as (Ferretti, et al., 2004a) are shown in table 1.

<table>
<thead>
<tr>
<th>( \sigma_0 )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( f_c )</th>
<th>( f_s )</th>
<th>( v_s )</th>
<th>( \delta_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^5 \text{ N/m}</td>
<td>\sqrt{10^5} \text{ Ns/m}</td>
<td>0.4 \text{ Ns/m}</td>
<td>1 \text{ N}</td>
<td>1.5 \text{ N}</td>
<td>0.004 \text{ m/s}</td>
<td>2</td>
</tr>
</tbody>
</table>

4.1 Simulation of the relation between the friction force and velocity

Let \( \nu=0.001t \), define the integrator that its absolute tolerance is equal to \( 1 \times 10^{-6} \), external reset is none, initial condition source is internal, initial condition is zero, and there is not limit output, not ignore limit and reset when linearization. The simulation can adopt variable simulating step or fixed simulating step. When adopting variable simulating step, it is needed to add a two-order low filter whose cut off frequency is 10KHz before the derivate of \( z \) goes into the equation (11a). When simulating the improved LuGre model, the solver is ode45 (Dormand-Prince), and the relative tolerance is the default value — 1e-3. If the fixed simulating step is adopted for the two improved friction models, let the simulating step equal to 0.0001 and the solver is ode3 (Bogacki-Shampine). Separately solving the improved Integral friction model, the Integral friction model \((\alpha=1)\) and the Integral friction model \((\alpha=0)\) gives the curves between the friction force and the relative velocity shown in Figure 1 and Figure 4. In fact, the solver’s parameters can be adjusted according to the requirement of the dynamic modelling precision. For example, in the simulation of the relation between the friction force and velocity, when the relative tolerance of 1e-6 or 1e-8 is set, the simulation can also be done. Different accurate solutions are corresponding to the different solvers and simulation parameters.
The simulation result shows that the friction characteristics well reflected by the improved Integral friction model and the Integral friction model with $\alpha=1$ are very similar, and the friction characteristics are not well reflected by the Integral friction model with $\alpha=0$. For the improved Integral friction model, the shape coefficient $\alpha$ is shown in Fig.2, in which $\alpha$ switches at 0.14s when $f_p = f_c$, and the friction force $f_p$ determining the switching of $\alpha$ is shown in Fig.3.

4.2 Simulation of Stick-slip and Limit Cycles

The simulation of stick-slip adopts the model shown in Figure 5, where the spring stiffness $k=2\text{N}\cdot\text{m}^{-1}$, the moving speed of the spring’s end is $\bar{v}=0.1\text{ m/s}$, $m$ is unit mass, and other simulation parameters are shown in table 1. The stick-slip simulation results are shown in Figure 6 and Figure 7. The dynamic model for simulation is as follows:

$$k\int_0^t (\bar{v} - \dot{x})dt - f_f = m\ddot{x}$$

(13)
Figure 6. Stick-slip simulation experiment
(dashed line: Integral friction model with $\alpha=1$, solid line: improved integral friction model)

Figure 7. Stick-slip simulation experiment (Integral friction model with $\alpha=0$)

Figure 8 and Figure 9 show the simulation of a Proportional-Integral-Derivative closed loop position control of the unit mass $m$ (taking out the spring) with friction. The figures show this control system’s step response, where the input step reference signal $\bar{x}=1$m, the output signal is the displacement of unit mass, and the parameters of the PID controller are $k_p=3$ N·m$^{-1}$, $k_i=4$ N·m$^{-1}$·s$^{-1}$, $k_d=6$ N·s·m$^{-1}$. The dynamic model for simulation is as follows:

$$-k_p(x-\bar{x})-k_i\int (x-\bar{x})dt - k_d\dot{x} - f_f = m\ddot{x}$$  \hspace{1cm} (14)

Figure 8. Hunting simulation experiment
(dashed line: Integral friction model with $\alpha=1$, solid line: improved integral friction model)
The simulation results show that both the improved Integral friction model and the Integral friction model with $\alpha=1$ can reflect stick-slip and limit cycles caused by friction, while the Integral friction model with $\alpha=0$ can’t reflect these phenomena.

4.3 Simulation of non-physical drift

With a horizontal vibratory stimulation on the unit mass $m$ in Figure 5, and when the maximum of the vibratory stimulation’s amplitude doesn’t exceed the maximum of breakout friction (the maximum static friction force), in reality the unit mass wouldn’t exhibit micro-motion, but when adopting the LuGre friction model or the Integral friction model with $\alpha=1$ in the control system, the simulation result shows that there is a steady drift of the unit mass. In the simulation, the input signal is the imposed external force on the mass, and the output signal is the displacement of unit mass. The imposed external force is $u(t) = a + b \sin(\omega t)$, where $a=0.5$ N, $b=0.25$ N, $\omega=6\pi/5$ rad·s$^{-1}$. So the maximum external force is 0.75N, and it is smaller than the maximum static friction force $f_s$ shown in Table 1. Other parameters are as shown in table 1.

The displacements of the unit mass for different friction models are shown in Figure 10 and Figure 11. It can been seen that the Integral friction model with $\alpha=1$ produces steady drift as shown in Figure 11, and this drift would not occur in reality because the stimulating force is less than the maximum static friction force. Figure 10 shows that both the Integral friction model with $\alpha=0$ and the improved Integral friction model don’t produce the steady drift as shown in Figure 11.

From the simulation results, it can been seen that the improved Integral friction model can reflect the stick-slip and limit cycle phenomena caused by friction, and at the same time avoid drift. The authors have found that the improved LuGre model has the same characteristics as the improved Integral friction model, but results are not included here.
5. Application of the improved integral friction model in the simulation of a valve controlled hydraulic cylinder system

Friction nonlinearity in valve controlled hydraulic cylinders degrades the position tracking precision, and can result in limited cycles, stick-slip, and reduces the frequency response bandwidth of the closed loop system (Wang, 2010, Huang and Wang, 2002). Building a model which can reflect the real dynamic friction characteristic is very useful for simulation analysis and friction compensation control.

A valve controlled cylinder system is shown in Figure 12. When the friction force is not considered, the plant’s nonlinear state space model is given by equation (15) (Yang, et al., 2006), where the state variable \( x_1 \) is the displacement of the cylinder \( y \), \( x_2 \) is the velocity of the cylinder, \( x_3 \) is the pressure of the cylinder rodless chamber, \( x_4 \) is the pressure of the cylinder rod chamber, and other parameters is shown in table 2.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{A_1}{M} x_1 - \frac{B}{M} x_2 - \frac{A_2}{M} x_4 - \frac{F_e}{M} \\
\dot{x}_3 &= g_1(x) u - \frac{A_1}{V_1} x_2 - \frac{C_1}{V_1} (x_1 - x_3) - \frac{C_2}{V_2} x_3 \\
\dot{x}_4 &= g_2(x) u + \frac{A_2}{V_2} x_2 - \frac{C_1}{V_1} (x_1 - x_3) - \frac{C_2}{V_2} x_3
\end{align*}
\] (15)

Where

\[ g_1(x) = \text{sgn}((1 + \text{sgn}(u)) p_\alpha / 2 - \text{sgn}(u) x_1) \times \frac{C_2 w_1}{V_1} \sqrt{\left(\frac{2}{p_\alpha ((1 + \text{sgn}(u)) p_\alpha / 2 - \text{sgn}(u) x_1)}\right)} \]
\[ g_2(x) = -\operatorname{sgn}((1 - \operatorname{sgn}(u))p_x/2 + \operatorname{sgn}(u)x_i) \times \frac{C_i}{V_i} \sqrt{\frac{2 \rho}{\rho}} \left| (1 - \operatorname{sgn}(u))p_x/2 + \operatorname{sgn}(u)x_i \right| \]

When considering friction, the second equation of state space model (15) should be changed into the following equation.

\[ \dot{x}_2 = \frac{A_p}{M} x_1 - \frac{B}{M} x_2 - \frac{A_s}{M} x_3 - \frac{F_x}{M} - \frac{f_f}{M} \operatorname{sgn}(x_2) \] (16)

Where \( f_f \) is the friction force and can be found through the improved Integral friction model.

Firstly, the Coulomb friction force \( f_c \) and the maximum static friction force \( f_s \) of the improved Integral friction model need to be determined experimentally. Lay the cylinder horizontally, and give the servo valve a small opening signal. The piston starts to move at a very small constant velocity. The inertial force caused by the piston rod can be ignored due to the small mass of the piston rod and its very small acceleration. Measure the pressure of the rodless chamber and the rod chamber, and the friction force of the cylinder can be derived by the force balance formula \( f_f = p_i A_i - p_s A_s \). The curves between the friction force and the velocity is shown in Figure 13. The dynamic friction

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Value</th>
<th>Parameter name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valve opening area gradient</td>
<td>( w_3/m )</td>
<td>Bulk modulus</td>
<td>( K/\text{MPa} )</td>
</tr>
<tr>
<td>Valve opening area gradient</td>
<td>( w_5/m )</td>
<td>Viscous damping coefficient</td>
<td>( B/(\text{N} \cdot \text{s} \cdot \text{m}^{-1}) )</td>
</tr>
<tr>
<td>Flow coefficient</td>
<td>( C_d )</td>
<td>Mass of the piston rod</td>
<td>( M/\text{Kg} )</td>
</tr>
<tr>
<td>Area of rodless chamber</td>
<td>( A_1/\text{m}^2 )</td>
<td>Density</td>
<td>( \rho/(\text{kg} \cdot \text{m}^{-3}) )</td>
</tr>
<tr>
<td>Area of rod chamber</td>
<td>( A_2/\text{m}^2 )</td>
<td>Coefficient of internal leakage</td>
<td>( C_i/(\text{m} \cdot \text{s}^{-1} \cdot \text{Pa}^{-1}) )</td>
</tr>
<tr>
<td>Initial volume of rodless chamber</td>
<td>( V_{10}/\text{m}^3 )</td>
<td>Coefficient of external leakage</td>
<td>( C_e/(\text{m} \cdot \text{s}^{-1} \cdot \text{Pa}^{-1}) )</td>
</tr>
<tr>
<td>Initial volume of rod chamber</td>
<td>( V_{20}/\text{m}^3 )</td>
<td>Pressure of power source</td>
<td>( p_s/\text{MPa} )</td>
</tr>
</tbody>
</table>
characteristic is obvious in figure 13, and it can be found in figure 13 that $f_c \approx 100\text{N}$, $f_s \approx 145\text{N}$ and $v_0 = 3 \times 10^{-2} \text{m} \cdot \text{s}^{-1}$, and these parameters are adopted in the friction model.

Secondly, the equivalent stiffness coefficient $\sigma_0$, the micro-viscous friction coefficient $\sigma_1$ and the viscous friction coefficient $\sigma_2$ need to be determined. Because the viscous friction force has been considered in the second equation of the state space model (15), let $\sigma_2 = 0$. The values of $\sigma_0$, $\sigma_1$ and $\delta_s$ are from the reference (Ferretti, et al., 2004): $\sigma_0 = 105 \text{N} \cdot \text{m}^{-1}$, $\sigma_1 = \sqrt{10^5} \text{N} \cdot \text{s} \cdot \text{m}^{-1}$ and $\delta_s = 2$. For the closed loop controller, let the commanded displacement of the piston rod be $r = 0.025 \sin(6.28t) \text{ m}$, and the proportional position controller is given by $u = 0.04(r - y)$. The output displacement of the piston rod is shown in Figure 14 and Figure 15. The corresponding experiment is done on the actual valve controlled cylinder system, and the displacement of the cylinder is shown in Figure 15.

![Figure 14. Displacement of the cylinder with friction model](image1)

![Figure 15. Zoom in on the zone A of Fig.13](image2)

![Figure 16. The hunting displacement of the piston rod near zero velocity](image3)

Figure 14 and Figure 15 show that the state space model including the improved friction model can reflect the limit cycles when the direction of the piston rod movement changes and its velocity is very small.

Finally, the drift is simulated on the valve controlled cylinder system with the Integral friction model and with the improved Integral friction model. The simulation model is shown in Figure 17, and a very small hunting stimulation is exerted on the piston rod through a simple closed loop proportional force control. Let the command force be $F_e = 40 + 20\sin(6\pi/5) \text{ N}$, and the actual force exerted on the piston rod is very close to $F_e$, its maximum value of 60N is less than the Coulomb friction force (100N) and the maximum static friction force (145N). Because of the sign function and absolute value function in the state space equation (15), there are some high-frequency components in the feedback signal, so a two-order filter with 20 rad/s cut off frequency is added as shown in Figure17. The proportional coefficient is $K_p = 0.005$. 

Figure 17. Simulation of the valve controlled hydraulic cylinder system with external vibration force

Figure 18. Displacement of the piston rod with the integral friction model

Figure 18 shows the displacement of the piston rod when the friction part of equation (16) adopts the Integral friction model. This shows that the displacement drifts up over time. In fact, the displacement shouldn’t drift up because the force exerted on the piston rod is less than the Coulomb friction force and the maximum static friction force, and so the mathematic model is not correct under this situation.

Figure 19. Displacement of the piston rod with the improved integral friction model

When adopting the improved friction model, the displacement of the piston rod is shown in Figure 19, and no drift occurs.

6. Conclusions

The development process of a dynamic friction model has been systemically summarized, resulting in a friction model which is simple and well reflects the dynamic behaviour of friction in reality.

It was found that the LuGre and Integral friction models can well reflect the friction dynamic characteristics when the coefficient $\alpha$ determining the shape of curve between friction force and relative micro-displacement equals to 1.0, but drift occurs; when $\alpha=0$, there is no drift in the model, but the friction dynamic characteristics are not well reflected.
Therefore in this work the value of $\alpha$ is varied, so that $\alpha=0$ to avoid drift when friction is in the pre-sliding regime (elastic deformation), and $\alpha=1$ to reflect sliding friction when friction is in the plastic deformation stage and the sliding regime. The corresponding improved LuGre and Integral friction models were developed, and the simulation results show that the improved friction models can well reflect the friction dynamic characteristics and don’t produce drift.

Finally, the improved LuGre friction model was applied to the modelling of a nonlinear valve controlled hydraulic cylinder system, and the related simulation experiments were done. The results show that the model of the hydraulic system including the improved Integral friction model exhibits realistic low velocity friction dynamic characteristics and at the same time doesn’t produce drift.

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References


