Influence of nonlinear spring behavior of friction on dynamic characteristics of a rolling guideway

Yasunori SAKAI*, Tomohisa TANAKA **, Hayato YOSHIOKA *, Jiang ZHU*, Masaomi TSUTSUMI*** and Yoshio SAITO*

*Department of Mechanical and Control Engineering, Tokyo Institute of Technology
2-12-1 Ookayama, Meguro-ku, Tokyo 152-8550, Japan
E-mail: yasu.sakai0927@gmail.com

**Department of Micro-Nano System Engineering, Nagoya University
Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan

***Tokyo University of Agriculture and Technology
3-8-1 Harumi-cho, Fuchu-shi, Tokyo 183-8538, Japan

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Abstract
The friction of rolling guideways in the prerolling region displays hysteretic behavior known as nonlinear spring behavior (NSB). NSB deteriorates motion accuracy and causes vibration in the feed direction. Therefore, the influences of NSB on the dynamic characteristics of rolling guideways should be clarified. This paper describes the influence of NSB on the dynamic characteristics of a rolling guideway. A simple friction model is constructed based on the Masing rule. Because the proposed friction model is described with only three parameters, the factor that inherently affects the dynamic characteristics can be clearly identified. To clarify the influence of NSB, the impulse response, frequency response, and steady state motion are analyzed by numerical analysis. According to the results, the dynamic characteristics in the feed direction depend only on the change rate of friction in the prerolling region, which is introduced to the friction model as the shape factor, n. The stiffness and damping are high when the change rate of friction is high in the prerolling region. The frequency response function is force-dependent, and its tendency is varied by n. The frequency response function includes harmonic and super harmonic resonances. When the carriage is excited with a frequency lower than that of the super harmonic resonance, a displacement spike (quadrant glitch) is observed. Additionally, the nonlinearity cannot be ignored when the carriage is excited with a frequency lower than the harmonic resonance. Finally, an experiment with a roller guideway is conducted to prove the validity of the analysis. The resonance frequency and compliance at the harmonic resonance measured by the experiment accurately conform to the analytical results.

Key words: Rolling guideway, Nonlinear spring behavior, Masing rule, Hysteresis characteristic, Nonlinear vibration, Friction, Prerolling, Damping, Machine tool, Quadrant glitch

1. Introduction
Rolling guideways are widely used in machine tools because they can achieve high speed and precise motion at low cost. The rolling elements circulating inside of a carriage elastically support loads. The elastic deformation of rolling elements and raceways causes rigid-body vibration of a carriage in five degrees of freedom, excluding the feed direction (Ohta and Hayashi, 2000). Additionally, the circulation of rolling elements causes the rolling element passage vibration owing to the variations in stiffness (Matsumoto, 2004). Moreover, the nonlinear elastic behavior of friction causes vibration in the feed direction. This elastic behavior of friction is known as nonlinear spring behavior (NSB) (Otsuka and Masuda, 1998) (Futami et al., 1990) (Koizumi and Kuroda, 1990). These vibrations deteriorate the motion accuracy of a feed drive equipped with rolling guideways.

NSB particularly causes the force dependency of the dynamic characteristics of a rolling guideway in the feed and pitch directions (Sakai et al., 2014). The cutting force varies periodically over a wide range during the machining process. Thus, the dynamic characteristics of a feed drive equipped with a rolling guideway vary throughout the machining
process. This variation in dynamic characteristics deteriorates the motion accuracy and influences the regenerative chatter vibration stability condition. A quantitative prediction about the dynamic characteristics, considering the influence of NSB, can contribute to achieving accurate feed motion by friction compensation and to analyzing the regenerative chatter vibration stability by predicting the dynamic stiffness of feed drives. Additionally, it can promote the dynamical machine design by means of computer aided engineering (CAE). Thus, the influences of NSB on the dynamic characteristics and motion should be clarified.

An earlier study discovered that the characteristics of NSB of a ball guideway vary depending on preload, hardness of raceways, and ball diameter (Al-Bender and Symens, 2005a) (Sato et al., 2007). Furthermore, resonance is caused by NSB, and its frequency and amplitude vary depending on the excitation force amplitude (Al-Bender et al., 2004). However, the influences of NSB on the dynamic characteristics of a rolling guideway, particularly on its damping, have not yet been clarified.

The purpose of this study is to use numerical analysis for clarifying the influences of NSB on the dynamic characteristics in the feed direction of a rolling guideway. We construct a friction model to determine the factor that inherently influences the dynamic characteristics, and analyze the impulse response, frequency response, and steady-state motion of the rolling guideway for investigating the influences of NSB. Finally, the experiments using a roller guideway are conducted to validate the numerical analysis. As the results, the validation of the analytical was indicated because the frequency and compliance at the harmonic resonance obtained by the experiments had good agreement with the numerical calculation results.

2. Friction and analytical models of rolling guideway

2.1 Nonlinear spring behavior of friction

Figure 1 shows the relationship between the friction force, \( F \), and the displacement, \( x \), of the carriage. The friction force increases with increased displacement after movement begins. If the friction force saturated at the steady-state friction force, \( F_s \), it would be constant until achieving a motion reversal point, \( A \). After reversing the motion direction at point \( A \), the friction force changes along the outer hysteresis curve, \( ABA \). If the motion reversed at an arbitrary point, \( a \), on the route of the outer hysteresis curve, the friction force would change along the inner hysteresis curve, \( aba \), starting at point \( a \). When the inner hysteresis curve is closed, the friction force again changes along \( ABA \). This hysteresis rule is known as hysteresis characteristics with nonlocal memory (Al-Bender, 2010). NSB is caused by the elastic deformation of asperities on the contact surface (Jonson, 1955), micro slip (Shinno et al., 1983), and elastic hysteresis loss (Yoshida, 1961). In this paper, curve \( OA \) is designated the virgin loading curve, the displacement region in which the friction depends on displacement is called the prerolling region, and the displacement region in which the friction force becomes constant is designated the rolling region. Additionally, the boundary displacement between the prerolling and rolling regions is called the starting rolling displacement.

Now, let us consider the sinusoidal excitation force, \( P(t) = P_0\sin\omega t \), acting on the carriage, where \( P_0 \) is the amplitude and \( \omega \) is the excitation angular frequency. Figure 2 shows the relationship between the sinusoidal excitation force, \( P \), and
the displacement, \( x \). If the force amplitude, \( P_0 \), is larger than the steady state friction force, \( F_s \), the displacement changes greatly, such as the arc within \( P > F_s \). This suggests that the rolling element begins rolling motion. \( F_s \) can be determined as \( P \) at the motion reversal point (Tanaka et al., 2007).

In the next section, a friction model that can describe NSB of the friction and hysteresis behavior with nonlocal memory will be proposed.

### 2.2 Friction model based on Masing rule

The proposed friction model based on the Masing rule can describe NSB of friction and hysteretic behavior with nonlocal memory. The Masing rule simply generates the hysteresis curve from the virgin loading curve. Thus, it is widely used for foundation models in earthquake response analysis (Yoshida et al., 2003) and for hysteresis models (Muravskii, 2005).

The Masing rule is formulated as follows:

\[
F = \begin{cases} 
  f(x) & (x \geq 0) \\
  -f(-x) & (x < 0)
\end{cases} \tag{1}
\]

\[
F = \begin{cases} 
  F_s + \lambda f \left( \frac{x-x_i}{\lambda} \right) & \left( \frac{x-x_i}{\lambda} \geq 0 \right) \\
  F_s - \lambda f \left( \frac{x-x_i}{\lambda} \right) & \left( \frac{x-x_i}{\lambda} < 0 \right)
\end{cases} \tag{2}
\]

where \( f(x) \) is the virgin loading curve; \( F_s \) and \( x_i \) are the friction force and displacement at an arbitrary motion reversal point, respectively. The hysteresis curve is determined by the geometrically similar curve of the virgin loading curve with similarity ratio \( \lambda = 2 \). If \( \lambda \neq 2 \), the hysteresis curve is not axisymmetric.

The calculation procedures of the friction force are explained in Fig. 3. First, the friction force is calculated by Eq. (1) until reaching motion reversal point A. After reversing the motion direction at point A, the friction force is calculated by Eq. (2), substituting the displacement and friction force at point A for \( x_i \) and \( F_s \). If the motion reversed at point B on the path of the outer hysteresis curve or at motion reversal point C on the inner hysteresis curve, the friction force can be calculated by Eq. (2), substituting the displacement and friction force at points B or C for \( x_i \) and \( F_s \). The friction force after closing the inner hysteresis curve is calculated by Eq. (2), again substituting the displacement and friction at point A for \( x_i \) and \( f_s \). The hysteresis characteristics with nonlocal memory can be described by this method of calculation. If \( x \) becomes larger than the maximum value of displacement at previous motion reversal points, the friction force is calculated by Eq. (1).

This model has fewer parameters than previous friction models, such as the bristle model (Olsson et al., 1998) and the generalized Maxwell slip (GMS) model (Piatkowski, 2014). The friction model based on the Masing rule can clarify the influences of NSB on the dynamic characteristics by means of simplifying the friction behavior. Although Al-Bender calculates the friction force using the Masing rule with an exponential function and an irrational function as the virgin loading curve (Al-Bender and Symens, 2005b), it does not consider both the rolling region and the starting rolling

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**Fig. 3** The relation between friction force \( F \) and displacement \( x \) calculated by Masing rule

**Fig. 4** The proposed virgin loading curve \( f \) for different shape factor \( n \)
displacement, $x_s$. The resonance, however, caused by NSB depends on $x_s$ (Sakai et al., 2014). Thus, the starting rolling displacement and steady state friction force in the rolling region should be introduced into the friction model.

The virgin loading curve proposed in this study has only three parameters: steady state friction force, $F_s$, starting rolling displacement, $x_s$, and shape factor, $n$. The shape factor, $n$, represents the change rate of friction in the prerolling region and is the most important parameter in this study. These parameters can determine the characteristics of NSB. The proposed virgin loading curves for different $n$ values are shown in Fig. 4 and given by Eq. (3).

$$f(x) = \begin{cases} A(x + Bx^n) = f_1(x) & (x \leq x_s) \\ F_s = f_2(x) & (x > x_s) \end{cases}$$

where $A$ and $B$ are constants determined by the continuity conditions as follows:

$$f_1(x_s) = f_2(x_s) \quad (4)$$

$$\frac{df_1(x)}{dx} \bigg|_{x = x_s} = \frac{df_2(x)}{dx} \bigg|_{x = x_s} \quad (5)$$

Because the two functions, $f_1$ and $f_2$, of the virgin loading curve are connected to satisfy the continuous conditions, the virgin loading curve is of continuous class $C^1$. Eventually, the virgin loading curve is described as in Eq. (6).

$$f(x) = \begin{cases} \frac{n}{(n-1)x_s} \left[ 1 - \frac{1}{n} \left( \frac{x}{x_s} \right)^{n-1} \right] F_s & (x \leq x_s) \\ F_s & (x > x_s) \end{cases}$$

where the shape factor is $n \neq 1$. Because $n = 1$ means that the friction does not have hysteretic behavior, it is uncommon in the friction characteristics of a rolling guideway. The virgin loading curves shown in Fig. 4 are calculated by using Eq. (6).

Figure 5 shows the analytical model of a rolling guideway for calculating the dynamic characteristics in the feed direction. The equation of motion is as follows:

$$m\ddot{x} = -F(x) + P(t)$$

where $m$ is mass of a carriage; $t$ is time; $[\cdot] = d/dt$; $P(t)$ is the excitation force acting on a carriage; and $F(x)$ is the friction force calculated from Eqs. (1), (2), and (6).

![Analytical model of rolling guideway](image)

**Fig. 5** The analytical model of the rolling guideway for calculating the dynamic characteristics in the feed direction in consideration of the nonlinear spring behavior of friction

### 3. Influences of NSB on dynamic characteristics of rolling guideway

This chapter clarifies the influences of NSB on the dynamic characteristics of a rolling guideway in the feed direction. When the speed of feed drives is increased, the inertial force acting on the rolling guideway also increases. Thus, the impulse response of the carriage should be evaluated. Additionally, the frequency response of the rolling guideway is a key consideration for developing highly accurate feed drives by using high-performance friction compensators. Thus, the impulse and frequency responses of the rolling guideway are investigated to clarify the influence of NSB.

### 3.1 Influences of NSB on impulse response

Considering the impulse force, $P(t) = P_{imp}$, acting on the carriage for very short time, $\Delta t$, the relation between impulse
and momentum is written as Eq. (8).

\[ P_{imp} \Delta t = m \dot{x}_{imp} \]  

(8)

where \( \dot{x}_{imp} \) is the velocity caused by the impulse force. From Eqs. (7) and (8), the equation of motion is expressed as Eq. (9).

\[ m \ddot{x} = -F(x), \quad \dot{x}(0) = x_{imp} = \frac{P_{imp} \Delta t}{m} \]  

(9)

By introducing \( K_s = F_s/x_s \), \( \omega^2 = K_s/m \), \( u = x/x_s \), \( \gamma_{imp} = P_{imp}/F_s \), \( \tau = \omega \Delta t \), and \( [\cdot]' = d/d\tau \) to Eq. (9), the nondimensional equation of motion is written as follows:

\[ u' = -\frac{F(u)}{F_s}, \quad u'(0) = u_{imp}' = \gamma_{imp} \Delta t = I \]  

(10)

where \( I \) is the nondimensional impulse; \( u_{imp}' \) is the nondimensional velocity; \( F(u)/F_s \) is the nondimensional friction force. \( F(u)/F_s \) is obtained by the nondimensional virgin loading curve, \( f(u)/F_s \), described by Eq. (11).

\[ f(u)/F_s = \begin{cases} n/(n-1) & (u \leq 1) \\ 1 & (u > 1) \end{cases} \]  

(11)

According to Eq. (11), \( f(u)/F_s \) and \( F(u)/F_s \) can be determined only by \( n \). This means that the nondimensional dynamic characteristics in the feed direction of the rolling guideway depend only on the change rate of friction in the prerolling region as long this friction model is used.

Figure 6(a) shows the relationship between \( F(u)/F_s \) and the sinusoidal displacement, \( u_0 \), with different amplitudes, \( u_0 \). The area enclosed with the hysteresis curve increases when \( u_0 \) is large. Figure 6(b) shows the calculation results for different \( n \). As shown in Fig. 6(b), the change rate of friction in the prerolling region is large when \( n \) is small. In particular, \( n \approx 0 \) indicates Coulomb friction and \( n = \infty \) indicates the piecewise linear spring characteristic.

The impulse response is calculated from Eq. (10) by means of the fourth-order Runge-Kutta method with step size \( d\tau = \pi/10000 \). Even if \( d\tau \) was smaller than \( \pi/10000 \), the calculation result would be same for \( d\tau = \pi/10000 \).

Figure 7 shows the impulse response of the rolling guideway in the feed direction calculated with impulse \( I = 0.1 \) and 100. The results for different \( n \) are shown in Fig. 7. The vibration caused by NSB is clear in the impulse response. The period of vibration shortens with decreased response amplitude. When \( n \) is large, the damping decreases. In the case of \( I = 100 \) shown in Fig. 7(b), the impulse responses for various shape factor \( n \) agree with each other until achieving the maximum displacement (\( \tau < 100 \)). By contrast, the impulse responses for various \( n \) differ from after achieving the maximum displacement (\( \tau \geq 100 \)). It can be said that the impulse response is not influenced by \( n \) until achieving the maximum displacement.

Next, the impulse response is evaluated by maximum displacement, \( u_m \), and restoration displacement, \( u_r \), which are
defined as shown in Fig. 8. Figure 9 shows the relation between impulse $I$ and both $u_m$ and $u_r$. The results for different $n$ are shown in Fig. 9. According to these results, $u_m$ increases with increased $I$. If $n$ is small, $u_m$ increases within $I < 2$. After the maximum displacement exceeds the starting rolling displacement $x_s (u_m > 1)$ as shown in Fig. 9(a), $u_m$ does not influenced by $n$, and $u_r$ becomes constant against the impulse $I$. Additionally, $u_r$ is small when $n$ is small.

When the change rate of friction in the prerolling region is large, $u_m$ increases and $u_r$ decreases. This indicates that the carriage can be moved with small impulse force, $P_{imp}$, when the change rate of the friction in the prerolling region is large. Because $u_r$ becomes constant within approximately $u_m > 1.3$ ($I < 2$), the elastic strain energy stored in the elastic contacts is limited by its dependence on the change rate of friction in the prerolling region.

The frequency response is important for designing friction compensators and predicting the dynamic stiffness of machine tools. The frequency response of linear systems can be obtained by Fourier transform. However, it is difficult to obtain the frequency response of nonlinear systems because mode orthogonality cannot be satisfied. Hence, we obtain the frequency response of rolling guideways from the steady-state response, and will discuss the influence of NSB on the frequency response in the feed direction in the following section.

### 3.2 Influence of NSB on frequency response

The influence of NSB on the frequency response of the rolling guideway in the feed direction is investigated. Considering the sinusoidal excitation force, $P(t) = P_0 \sin \omega t$, acting on the carriage, the equation of motion can be expressed as Eq. (12).

$$m \ddot{x} = -F(x) + P_0 \sin \omega t$$

By introducing the nondimensional parameters $\gamma_0 = P_0/F_s$, $\tau = \omega t$, and $\beta = \omega / \omega_s$ to Eq. (12), the nondimensional equation of motion is written as Eq. (13).

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**Fig. 7**  Influence of the shape factor $n$ on the impulse response of the rolling guideway

**Fig. 8**  The definitions of the maximum displacement $u_m$ and the restoration displacement $u_r$

**Fig. 9**  The relationship between the non-dimensional impulse $I$ and the evaluation indices $u_m$ and $u_r$
As shown in Eq. (13), the motion of the rolling guideway is dominated by the nondimensional excitation force amplitude, $\gamma_0$, nondimensional excitation frequency, $\beta$, and nondimensional friction force, $F/F_s$. $F/F_s$ is a function of shape factor $n$. Eventually, the motion of the rolling guideway is dominated by $\gamma_0$, $\beta$, and $n$. Equation (13) is solved by means of the fourth-order Runge-Kutta method with step size $d\tau = \pi/10000$. The frequency response function (FRF) $G(\beta)$ is calculated by Eq. (14).

$$G(\beta) = \frac{u_0}{\gamma_0} = \frac{(u_{\text{max}} - u_{\text{min}})}{2\gamma_0}$$

where $u_0$ is the displacement amplitude in the steady-state response, $u_{\text{max}}$ and $u_{\text{min}}$ are the maximum and minimum displacement in the steady-state response, respectively.

![Graph](image1)

**Fig. 10** The frequency response functions in the feed direction with the non-dimensional excitation force amplitude $\gamma_0 = 0.6$ for different shape factor $n$

![Graph](image2)

**Fig. 11** The phase plane at frequency of A

![Graph](image3)

**Fig. 12** The phase plane at frequency of B

![Graph](image4)

**Fig. 13** The phase plane at frequency of C
Figure 10 shows the FRFs with nondimensional excitation force amplitude $\gamma_0 = 0.6$ for $n = 0.5$ and 1.5. As shown in Fig. 10, the shapes of the FRFs and resonance frequencies are changed by $n$. Thus, NSB influences the frequency response. The FRF displays predominant and small resonance peaks. The small resonance peaks are super harmonic resonances, which are typical phenomena of nonlinear vibration systems caused by the nonlinearity of the restoring force (Kim et al., 2005). These resonances are not the stick-slip phenomenon caused by the negative damping coming from the difference between the static and dynamic friction forces because the difference of the static and dynamic frictions is not be considered in the proposed friction model. In the following, the predominant resonance peak is designated harmonic resonance and the other resonances are designated super harmonic resonances.

The steady-state response for a particular excitation frequency is investigated to clarify the influence of the excitation frequency on steady-state motion in the feed direction. Figures 11 to 13 show the phase planes of the limit cycle for $n = 0.5$ and 1.5 at frequencies A, B, and C shown in Fig. 10, respectively. Frequency A is lower than that of the super harmonic resonance of a neighboring harmonic resonance. Frequency B is between the harmonic resonance and its neighboring super harmonic resonance. Finally, frequency C is higher than that of the harmonic resonance. Because the carriage is not retained in the feed direction by the spring, steady-state motion does not occur symmetrically about the coordinate origin of displacement.

According to the results, velocity reduction and recovery (VRAR) occurs after reversing the motion direction when the carriage is excited by sinusoidal force with frequency A, as shown in Fig. 11. VRAR occurs in a small moving distance when $n$ is small. When the carriage is excited with frequency B, as shown in Fig. 12, VRAR does not occur after reversing the motion direction. The shapes of the phase plane, however, are distorted. On the other hand, the phase plane becomes circular when the carriage is excited with frequency C, as shown in Fig. 13. The nonlinearity caused by NSB can be ignored when the carriage is excited by the force with higher frequency than that of the harmonic resonance because the phase plane can be regarded as circular.

Next, the time histories of steady-state motion are examined to investigate the influences of excitation and friction conditions. Figures 14 and 15 show the nondimensional acceleration, $u''$, nondimensional velocity, $u'$, nondimensional displacement, $u$, and nondimensional friction force, $F/F_s$, for $n = 0.5$ and 1.5 at frequencies A and B, respectively.

Comparing Figs. 14 and 15, the variation of acceleration direction (VOAD) unrelated to sinusoidal displacement motion with the excitation frequency and VRAR are clear when excited with frequency A. Hence, the VOAD causes VRAR. The displacement spike is clear in Fig. 14, but not in Fig. 15. Thus, the displacement spike is caused by the
VOAD. Furthermore, the displacement spike sharpens when the VOAD is caused discontinuously. On the other hand, it loosens when the VOAD is caused continuously. The VOAD is affected by NSB of friction, and becomes discontinuous when $n$ is small.

The displacement spike is known as the quadrant glitch, which is one reason for the deteriorating motion accuracy of feed drives. An earlier study concluded that the quadrant glitch is generated owing to the motion delay caused by the difference between static and dynamic friction forces (Kakino et al., 1990). According to the results of this study, the displacement spike (quadrant glitch) is generated without modeling the difference between static and dynamic friction forces. This means that the quadrant glitch is caused not only by the difference between static and dynamic friction but also by NSB of friction. Moreover, the quadrant glitch occurs even when velocity becomes zero, as shown in Fig. 14. An earlier study concluded that the quadrant glitch is generated by a reduction in acceleration (Sato and Tsutsumi, 2005). These conclusions correspond to our results that the quadrant glitch is caused by the VOAD.

To investigate the nonlinearity, the frequency components of the steady-state response at frequency A are analyzed by FFT analysis. Figure 16 shows the power spectrum densities (PSDs) of the excitation force, $\gamma_0 \sin \tau$, displacement $u$, and $F/F_s$. As shown in Fig. 16, the friction force and displacement display the odd harmonics of the excitation frequency. Because the carriage is excited by these odd harmonics of the friction force, the displacement has the same odd harmonics. The odd harmonics of the friction force are caused by NSB. The PSDs of the odd harmonics are high when $n$ is small. Namely, the nonlinearity of the dynamic characteristics is high for small $n$ because the change rate of friction in the prerolling region is large. In general, the PSD of vibrational components decays with increasing the frequency. Thus the influence of the odd harmonics on the steady-state response decreases when the excitation frequency increases. Figures 17 and 18 show the frequency components of the excitation and friction forces and the displacement at the frequencies B and C as shown in Fig. 10, respectively. The odd harmonics are observed in the friction force and displacement. However, the influence of the odd harmonics gradually decreases with increased excitation frequency.

The dynamic characteristics of the rolling guideway have force dependency caused by NSB. The influence of the change rate of friction in the prerolling region on force dependency is examined in detail as follows.

Figure 19 shows the FRFs for $n = 0.5$ and 1.5. The results for different nondimensional excitation force amplitudes, $\gamma_0$, are also included in this figure. As shown in Fig. 19, the resonance frequency decreases with increasing $\gamma_0$. Furthermore, the resonances do not occur when $\gamma_0$ increases. Compliance at the frequencies of the super harmonic and harmonic resonances is high with large values of $n$. When $n$ is small, not only nonlinearity but also damping increase.

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Fig. 16: Power spectrum density at the frequency of A
(a) Excitation frequency $\beta = 0.54$, $n = 0.5$
(b) Excitation frequency $\beta = 0.45$, $n = 1.5$

Fig. 17: Power spectrum density at the frequency of B
(a) Excitation frequency $\beta = 0.90$, $n = 0.5$
(b) Excitation frequency $\beta = 0.71$, $n = 1.5$

Fig. 18: Power spectrum density at the frequency of C
(a) Excitation frequency $\beta = 1.65$, $n = 0.5$
(b) Excitation frequency $\beta = 1.40$, $n = 1.5$

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For this reason, the compliance at the resonances is not high when \( n \) is small.

Because NSB influences the force dependency, the relationship between \( \gamma_0 \) and the harmonic resonance is obtained for clarifying the influence of NSB on the harmonic resonance. Figure 20 shows the relationship between \( \gamma_0 \) and the frequency and compliance at the harmonic resonance. The results for different \( n \) are also included in this figure. As shown in Fig. 20(a), the resonance frequency decreases with increasing \( \gamma_0 \). When \( n \) is small, the resonance frequency is high and its decrement against \( \gamma_0 \) is large. If the change rate of friction in the prerolling region is large, the stiffness and nonlinearity increase.

The resonances do not disappear when \( P_0 = F_s(\gamma_0 = 1) \). The resonance does not disappear immediately after the excitation force exceeds the steady-state friction force. The carriage requires a certain moving length to dissipate the elastic strain energy caused by NSB of friction as the frictional work of the steady-state friction force. Thus, it is expected that the resonances caused by NSB do not disappear until the restored elastic strain energy equals the frictional work of the steady-state friction force. This result is important for predicting the conditions for resonance occurrence. As shown in Fig. 20(b), compliance at the harmonic resonance is small for all \( \gamma_0 \) with small \( n \). This indicates that damping is large when the change rate of friction is large. Additionally, the compliance increases monotonically with increasing \( \gamma_0 \) at small value of \( n \). On the other hand, the compliance decreases once with increased excitation force amplitude before increasing with large value of \( n \). This difference in change tendency of the compliance against the excitation force is caused by the difference in change rate of the restoring and damping forces generated by NSB against the excitation force.

Tanaka et al. attempted to predict the dynamic characteristics of the ball guideway in the feed direction by using a

![Fig. 19](image1.png)

**Fig. 19** The force amplitude dependency of the frequency response caused by the nonlinear spring behavior

![Fig. 20](image2.png)

**Fig. 20** Relationship between the non-dimensional excitation force amplitude \( \gamma_0 \) and the resonance frequency and compliance at the harmonic resonance
linear single degree of freedom (SDOF) system, the parameters of which are identified from the relationship between friction force and displacement measured in the quasi-static condition (Tanaka et al., 2007). Additionally, the study concluded that the predicted result of damping has large errors if the damping parameter is not corrected. As mentioned in this study, the influence of nonlinearity cannot be ignored if the excitation frequency is lower than the harmonic resonance. Thus, the prediction error attributable to nonlinearity increases when a linear SDOF system is used for the prediction. It is necessary to construct a nonlinear SDOF system in consideration of the influence of super harmonic resonances.

The influences of rolling element shape, lubrication conditions, and preload on the friction characteristics and dynamic characteristics will be clarified in future work. Additionally, an equivalent nonlinear SDOF system for predicting the dynamic characteristics in the feed direction is under consideration by using complex springs.

In the next section, the experiment using a commercial roller guideway is described to validate the analytical solution obtained from the abovementioned method.

4. Experiments with roller guideway
4.1 Experimental setup and method

Figure 21 shows the experimental setup used in this study. A roller guideway (size number: #35) with medium preload was used for the experiment. The experiments were conducted without lubricants to exclude the viscous damping of lubricants. All parts of the roller guideway were cleaned with alcohol and dried adequately to ensure nonlubricated condition. The rail was fixed on the base of the feed drive mechanism with bolts. Because the base was fixed on a stone surface plate supported by air dampers, vibration from the floor could be isolated. The natural frequency of the stone surface plate supporting the system was less than 10 Hz. A steel plate (mass: 1.2 kg) was fixed on the top surface of the carriage with bolts.

Both acceleration and displacement of the carriage in the feed direction were measured simultaneously with a piezoelectric accelerometer and an eddy current displacement sensor, respectively. The accelerometer was fixed on the side surface of the steel plate with wax. The carriage was excited in the feed direction by a vibration generator through a driving rod. The driving rod only transferred the excitation force in the feed direction because its stiffness excluding the feed direction is negligibly small. Both ends of the driving rod were fixed with screws to the vibration generator and a piezoelectric force transducer, respectively. The excitation force was measured by the force transducer fixed on the side surface of the steel plate by a screw. Output signals were monitored and processed with the FFT analyzer. The FRF was recorded with a personal computer. The excitation frequency was changed from 0.5 Hz to 500 Hz and its amplitude was changed from 0.1 N to 2.0 N. The amplitude and frequency of the excitation force can be adjusted by a function generator.

4.2 Experimental results and comparison with analysis

First, steady-state friction force, $F_s$ was identified. Figure 22 shows the relationship between excitation force, $P$ and displacement, $x$ of the carriage with sinusoidal force excitation. The excitation frequency was set to 10 Hz. The results
for different force amplitude are included in the same figure. As shown in Fig. 22, the relationship between excitation force and displacement is similar to that in Fig. 2. Hence, \( F_s \) is identified as the excitation force at the motion reversal point shown in Fig. 22. As a result, \( F_s = 1.1 \) N, which is clear in Fig. 22. This value is used for calculating the nondimensional excitation force amplitude \( \gamma_0 \).

The FRF was measured by means of a stepped sine excitation, of which the sweep velocity was approximately 5 Hz/s. The frequency resolution was set to 1.25 Hz. Figure 23 shows the FRF of the roller guideway in the feed direction. The results for different force amplitudes are shown in Fig. 23. As shown in Fig. 23, the resonance phenomenon caused by NSB is clear in the FRF. When the excitation force amplitude was large, resonance was not observed. This tendency corresponds to the FRF for \( n = 1.5 \), as shown in Fig. 19(b). The phase delay from 180 degree was approximately \(-42\) degrees when the excitation frequency is higher than the resonance frequency. A similar phase delay was experimentally observed in a previous study on the dynamic characteristics of ball bearings (Yoon and Trumper, 2014).

Figure 24 shows the relationship between the excitation force amplitude and the frequency and compliance at the harmonic resonance. The excitation force amplitude, \( P_0 \), was divided by the identified steady-state friction force \( F_s = 1.1 \) N to obtain \( \gamma_0 \). Additionally, the frequency and compliance at harmonic resonance were divided by its values for 0.1 N (\( \gamma_0 \approx 0.1 \)) to obtain the change rate of resonance frequency and compliance. As shown in Fig. 24, the resonance frequency decreases with increasing \( \gamma_0 \). The compliance decreases once with increased \( \gamma_0 \) before increasing. The resonance disappears within approximately \( \gamma_0 > 1.3 \).

The experimental results are compared with the analytical results to prove the validity of the analysis. The analytical
results for \( n = 1.5 \) shown in Fig. 20 are also shown in Fig. 24. The value of \( n \) is identified to agree with the experimental results. The frequency and compliance at the harmonic resonance are divided by their value for \( \gamma_0 = 0.1 \) to obtain the change rate. As shown in Fig. 24, the analytical results for \( n = 1.5 \) conform well to the experimental results. The comparison results prove that the proposed simple analytical model can qualitatively predict the force dependency of the dynamic characteristics of a rolling guideway in the feed direction.

In a previous study, the compliance at the harmonic resonance of the ball guideway (not the roller guideway) increased monotonically with increased excitation force amplitude (Otsuka and Masuda, 1998). In contrast, the compliance at the harmonic resonance of the roller guideway decreases once with increased excitation force amplitude before increasing. These results indicate that NSB of the ball guideway differs from that of the roller guideway because the contact conditions differ between rolling elements and raceways. Moreover, the difference in dynamic characteristics between roller guideways and ball guideways can be described by the proposed friction model. A large and small \( n \) can be applied for roller and ball guideways, respectively.

For quantitatively identifying dynamic characteristics of a rolling guideway in the feed direction, \( x_s \) and \( n \) must be identified by the relationship between friction force and displacement measured in the quasi-static condition. The precise measurements of \( x_s \) and \( n \) are concerns for future study.

5. Conclusion

This paper describes the influence of the NSB of friction on the dynamic characteristics of a rolling guideway in the feed direction by means of the numerical analysis. The simple friction model is proposed for identifying the factor that inherently influences the dynamic characteristics. The influences of the NSB on the impulse response, frequency response, and steady-state motion are investigated. Finally, the validity of the analysis is proved by an experiment with a roller guideway. The results are summarized as follows:

1. The dynamic characteristics in the feed direction are inherently affected by the change rate of friction in the prerolling region, which is defined in this study as the shape factor, \( n \). The change rate of friction is large when \( n \) is small.
2. The impulse response is evaluated by the maximum displacement, \( u_{m} \), and the restoration displacement, \( u_{r} \). When \( n \) is small, \( u_{r} \) becomes small. The \( u_{m} \) is not affected by NSB when the impulse force is large. Additionally, \( u_{r} \) becomes constant with large impulse force because the storable elastic strain energy is limited.
3. Super harmonic resonances are caused by NSB. The steady-state motion is highly nonlinear when the excitation frequency is lower than the harmonic resonance frequency. Furthermore, a displacement spike attributable to variations in the acceleration direction occurs when the excitation frequency is lower than the super harmonic resonance frequency. The nonlinearity is higher with smaller \( n \).
4. The dynamic characteristics in the feed direction display force amplitude dependency. This tendency changes with \( n \). Within \( n > 1 \), compliance at the resonance decreases once with increased force amplitude then increases.
5. The analytical results of the relationship between the excitation force and both frequency and compliance at the harmonic resonance agreed well with the experimental results.

According to these results, both stiffness and damping in the feed direction can be improved by increasing the change rate of friction in the prerolling region. The shape factor, \( n \), should be controlled by any tribological method, such as surface coating or multifunctional surface finishing. It is possible that the proposed analytical method can explain the difference in dynamic characteristics between ball and roller guideways. Finally, this method can be applied to other machine elements such as sliding guideways, rolling bearings, and ball screws because these elements are affected by NSB.

Nomenclature

\[
\begin{align*}
A, B & = \text{Constants} \\
d\tau & = \text{nondimensional time step} \\
f & = \text{virgin loading curve} \\
f_1 & = \text{proposed virgin loading curve in the prerolling region} \\
f_2 & = \text{proposed virgin loading curve in the rolling region}
\end{align*}
\]
\[ F = \text{friction force} \]
\[ F_j = \text{friction force at the motion reversal point } j \ (j = A, B, C, \text{ and } D) \]
\[ F_r = \text{friction force at the arbitrary motion reversal point} \]
\[ F_s = \text{steady-state friction force} \]
\[ G = \text{nondimensional compliance, } u_0/\gamma_0 \]
\[ I = \text{nondimensional impulse} \]
\[ K_s = \text{stiffness at the starting rolling displacement, } F_s/x_s \]
\[ m = \text{mass of a carriage} \]
\[ n = \text{change rate of the friction in the prerolling region} \]
\[ P = \text{excitation force acting on the carriage} \]
\[ P_{\text{imp}} = \text{impulse force} \]
\[ P_0 = \text{sinusoidal excitation force amplitude} \]
\[ t = \text{time} \]
\[ u = \text{nondimensional displacement, } x/x_s \]
\[ u_m = \text{maximum nondimensional displacement in the impulse response} \]
\[ u_{\text{max}} = \text{maximum nondimensional displacement in the steady-state response} \]
\[ u_{\text{min}} = \text{minimum non-dimensional displacement in the steady-state response} \]
\[ u_t = \text{restoration nondimensional displacement} \]
\[ u_{\text{imp}} = \text{nondimensional displacement amplitude in the steady-state response} \]
\[ \gamma = \text{nondimensional velocity caused by the nondimensional impulse force} \]
\[ x_j = \text{displacement at the motion reversal point } j \ (j = A, B, C, \text{ and } D) \]
\[ x_r = \text{displacement at the arbitrary motion reversal point} \]
\[ x_s = \text{starting rolling displacement} \]
\[ \dot{x}_{\text{imp}} = \text{velocity caused by the impulse force} \]
\[ \beta = \text{nondimensional frequency, } \omega/\omega_s \]
\[ \Delta t = \text{very short time} \]
\[ \Delta \tau = \text{very short nondimensional time} \]
\[ \gamma_{\text{imp}} = \text{nondimensional impulse force, } P_{\text{imp}}/F_s \]
\[ \gamma_0 = \text{nondimensional amplitude of the sinusoidal excitation force} \]
\[ \lambda = \text{similarity ratio} \]
\[ \omega = \text{sinusoidal excitation force frequency} \]
\[ \omega_s = \text{natural angular frequency at the starting rolling displacement, } K_s/m \]
\[ \tau = \text{nondimensional time, } \omega t \text{ for the section 3.1, } \omega \tau \text{ for the section 3.2} \]
\[ [\cdot] = \text{differential operator with respect to time } t, \ d/dt \]
\[ [\cdot] = \text{differential operator with respect to nondimensional time } \tau, \ d/d\tau \]

References


Johnson, K., L., Surface interaction between elastically loaded bodies under tangential forces, Proceedings of the Royal