Uniform $L^p$-stability theory for the Boltzmann equation

Seung-Yeal Ha\textsuperscript{1}, Seoul National University
Mitsuru Yamazaki, University of Tsukuba
Seok-Bae Yun, Seoul National University
\textsuperscript{1} FAX: 92-2-887-4694, Email: syha@smu.ac.kr

In this talk, I will present a new uniform $L^p$-stability theory for the spatially inhomogeneous Boltzmann equation near vacuum via the nonlinear functional approach proposed in [1, 2]. Our stability analysis is based on new nonlinear functionals which are equivalent to the $p$-th power of $L^p$-distance. The $L^1$-nonlinear functionals play the key role of “modulators” which make the accumulative functional be non-increasing in time $t$ along classical solutions.

In this talk, we will present a new uniform $L^p$-stability theory for the spatially inhomogeneous Boltzmann equation near vacuum via the nonlinear functional approach proposed in [1, 2], more precisely, we will obtain the following accumulative $L^p$-type stability estimate: For any positive integer $M \geq 1$,
\[ \sum_{p=1}^{M} ||f(t)-\bar{f}(t)||_p^p \leq G \sum_{p=1}^{M} ||f_0-\bar{f}_0||_p^p, \quad t \geq 0, \]
where $f$ and $\bar{f}$ are classical solutions corresponding to initial data $f_0$ and $\bar{f}_0$ respectively, and we used a simplified notation
\[ ||f(t)-\bar{f}(t)||_p \equiv ||f(t)-\bar{f}(t)||_{L^p_{x,v}}. \]

Our stability analysis is based on a new nonlinear functional $\mathcal{H}^p(t) \equiv \mathcal{H}^p(f(t),\bar{f}(t))$ with the following key properties:

- Equivalence with $p$-th power of $L^p$-distance between $f$ and $\bar{f}$:
  \[ ||f(t)-\bar{f}(t)||_p^p \leq \mathcal{H}^p(t) \leq C_1 ||f(t)-\bar{f}(t)||_p^p, \quad t \geq 0, \]
  where $C_1$ is a positive constant independent of $t$.

- Uniform stability estimate:
  \[ \mathcal{H}^p(t) + C_2 \int_0^t \Lambda^p(f(s))ds \leq \mathcal{H}^p(0), \quad t \geq 0, \]
  where $C_2$ and $C_3$ are positive constants independent of $t$ and $\Lambda^p(f(s))$ is a generalized $(p,1)$-type collision production functional.

The $L^1$-nonlinear functionals in [1, 2] play the key role of “modulators” which make the accumulative functional be non-increasing in time $t$ along classical solutions.

References