Effect of Cell Local-Deformation of Foam Material on Compression and Bending Stiffness

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Based on the experimental results obtained in our earlier work, it was reported that aluminum alloy foam demonstrated a significant discrepancy in the elastic modulus measured from the uniaxial compression tests and the flexural modulus determined from the flexural vibration tests and the static bending tests. The work presented in this paper is aimed at investigating the reason of the discrepancy in stiffness by means of theoretical analysis. A detailed analysis on the cell local-deformation behavior under uniaxial loading and bending conditions is conducted to derive the stiffness equations. The derived equations clearly describe how the cell local-deformation affects the stiffness value with regards to the loading mode.

1. Introduction

In recent years, metal foams have become a potential material to be used in many engineering applications due to its novel properties and behavior in combination with the light-weight structure\(^1\). For example, in vehicle industries, the car makers like to use this material as the body component to improve the noise absorption ability while at the same time reducing the weight for energy saving. For that reason, a detailed understanding on the mechanical properties and behavior is obviously required to support the design process.

In our earlier works, the elastic moduli of metal foam, i.e. aluminum alloy foam, were evaluated by means of experimental investigations\(^2,3\). It was found that the elastic moduli determined from the flexural vibration tests and the static bending tests were significantly larger than that measured from the uniaxial compression tests. An analysis based on a simple curve-fitting method suggested that the discrepancy in stiffness is due to the different in cell local-deformation under uniaxial loading and bending conditions.

In the present work, we conduct a theoretical analysis on the elastic modulus of a closed-cell foam material in order to clarify the effect of cell local-deformation on the compression and bending stiffness. The cell model introduced by Gibson and Ashby\(^4\) is utilized to derive the stiffness equation for the uniaxial loading and bending conditions. Then, the result is used to explain the reason of the discrepancy in stiffness and compared with the experimental results for validation.

2. Experiments

2.1. Foam Materials

In the experiments, we used closed-cell aluminum alloy foams (Alporas, Shinko Wire) with four different densities as shown in Table 1. The materials showed a high degree of isotropy with no significant spatial or rotational variation.

2.2. Compression Test

In Young’s modulus, \(E\), were measured from compression tests using a displacement-controlled universal testing machine at room temperature and at a displacement rate of 1 mm/min. The compression load was measured by 5kN load-cell and the specimen displacement was measured by laser displacement sensor. Loading and unloading was performed several times during each test and \(E\) was determined from the gradient of the unloading curves.

2.3. Flexural Vibration Test

Flexural moduli, \(E_f\), were determined by measuring the flexural-natural frequency of vibration of free-hanging specimens in response to impact. Impact was made using an impact hammer and the resulting vibrations were measured using an accelerometer attached to the specimen surface using adhesive glue. The measured natural frequencies were used to calculate the flexural modulus using the Bernoulli-Euler beam theory.

### Table 1 Foam materials

<table>
<thead>
<tr>
<th>Density, kg/m(^3)</th>
<th>Cell diameter, mm</th>
<th>Cell-wall thickness, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>177</td>
<td>3.98 ± 0.09</td>
<td>0.05 – 0.10</td>
</tr>
<tr>
<td>222</td>
<td>4.30 ± 0.11</td>
<td>0.01 – 0.20</td>
</tr>
<tr>
<td>258</td>
<td>2.82 ± 0.10</td>
<td>0.01 – 0.20</td>
</tr>
<tr>
<td>346</td>
<td>2.90 ± 0.07</td>
<td>0.05 – 0.70</td>
</tr>
</tbody>
</table>

2.4. Static Bending Test

Beside flexural vibration tests, 3-point and 4-point bending tests were also conducted to measure the flexural modulus using a self-developed testing apparatus. The testing apparatus was primarily validated to ensure the good reliability measurement (error less than 10%). The flexural modulus was determined by measuring the specimen deflection based on Bernoulli-Euler beam theory.

3. Theory

Gibson and Ashby\(^5\) derived an equation for the effective Young’s modulus of a closed-cell foam subject to uniaxial loading. They explained that when a closed-cell foam is loaded, the bending of the cell-edge and the stretching of the cell-face give the main contribution to the stiffness (the gas pressure effect is assumed to be negligible). The equation is derived as follows. The force \(F\) causes the cell edge to deflect by \(\delta\) (Fig. 1(a)); this makes the work \(F\delta/2\) is done against the restoring force caused by cell-edge bending, \(k\delta^2/2\), and cell-face stretching, \(E_E\delta V/2\). In this formula, \(k\) is the stiffness of the cell-edge \((k \propto E_E L_1^2)\), \(E_E\) is the Young’s modulus of the bulk parent material, \(I\) is the second moment of area \((I \propto t_1^3)\), \(e\) is the strain caused by stretching of a cell-face \((e \propto \delta L)\), and \(V\) is the volume of solid in a cell-face \((V \propto L^2 t_f)\). Finally, we can get

\[
\frac{1}{2} F\delta = \frac{1}{2} E_E \frac{\delta^2}{L^2} + \beta E_E \frac{\delta^2}{L^2} L_1^2 t_f.
\]  
(1)

This equation is applicable for both compression and tension load. When a moment \(M\) is applied to the cell, so it deflects by \(\theta\) (Fig. 1(b)), using the same approach we get

\[
\frac{1}{2} M\theta = \frac{1}{2} E_E \frac{\theta^2}{L^2} + \psi E_E \frac{\theta^2}{L^2} L_1^2 t_f.
\]  
(2)
Considering the boundary condition of the cell-edge is fixed at both ends, thus \( k = 192E_JL \) for uniaxial loading condition and \( k = 16E_JL \) for bending condition, and by detailed calculation on the model (Fig. 1), the constants of \( a, \alpha, \beta, \gamma \) and \( \varphi \) in Eqs. (1) and (2) can be replaced by the values as shown in Eqs. (3) and (4);

\[
2E^*L\delta^2 = \frac{384}{3} \frac{E_JL^2}{L^3} + 8E_s \frac{\delta^2}{L^2} t_f
\]

\[
\frac{1}{E_f} \frac{s^2}{L^4} \theta^2 = 32 \frac{E_sL^2}{L^3} + \frac{4}{3} \frac{E_s}{L^2} \theta t_f,
\]

where, \( 2E^*L\delta \) and \( 1/6 E_fL^3\theta \) are the work done due to the uniaxial deflection \( \delta \) and the radial deflection \( \theta \) of the body; \( E^* \) and \( E_f \) is the effective Young’s modulus and the effective flexural modulus of the cell, respectively. Assuming the cell-edge has a square cross section area \( I = t_f^2/12 \) we find

\[
E^* = \frac{16}{15} \frac{t_f^4}{L^3} + \frac{4}{3} \frac{t_f}{L}
\]

\[
E'_f = \frac{16}{15} \frac{t_f^4}{L^3} + \frac{8}{3} \frac{t_f}{L}
\]

Substituting \((t_f/L)^2 \times (\rho_f/\rho_s)\) and \((t_f/L)^4 \times (\rho_f/\rho_s)^2\) \(\to\) Eqs. (5) and (6) gives the stiffness equations in the form of relative density function \((\rho_f/\rho_s)\) as described by Eqs. (7) and (8),

\[
\frac{E^*}{E_s} = 4A \left( \frac{\rho^*}{\rho_s} \right)^2 + B \left( \frac{\rho^*}{\rho_s} \right)
\]

\[
\frac{E'_f}{E_s} = 4A \left( \frac{\rho^*}{\rho_s} \right)^2 + 2B \left( \frac{\rho^*}{\rho_s} \right)
\]

where \( A \) and \( B \) are constants of proportionality. Hence, as the loading condition of the cell changes, the cell local-deformation also changes; this finally results the different forms of stiffness equation as described by Eqs. (7) and (8).

4. Result and discussion

Equations (7) and (8) show the unique contribution of cell-edge bending effect and cell-face stretching effect on the stiffness where both effects demonstrate a different portion depending on the loading mode. From these equations, we find the cell-edge bending effect exhibits the same contribution (4:4) either in the uniaxial loading or in the bending modes. On the other hand, the contribution of cell-face stretching effect increases by double (1:2) when the load changes to the bending mode. This indicates that the elastic modulus of a closed-cell foam material would become larger under bending condition due to the doubling effect of cell-face stretching.

Figure 2 shows the plot of normalized elastic moduli measured from the experiments to the relative density. It is seen in the figure, the flexural moduli exhibit a significantly larger value compared to the corresponding Young’s moduli. This is actually in a good agreement with what is predicted by the theory. Furthermore, from the Young’s moduli data, we can draw a curve-fitting to determine the empirical value of \( A \) and \( B \) based on Eq. (7). Then, by these values, we can draw another curve for flexural moduli based on Eq. (8). Finally, we find two empirical stiffness equations as follows,

\[
\frac{E^*}{E_s} = 0.3985 \left( \frac{\rho^*}{\rho_s} \right)^2 + 0.0458 \left( \frac{\rho^*}{\rho_s} \right)
\]

\[
\frac{E'_f}{E_s} = 0.3985 \left( \frac{\rho^*}{\rho_s} \right)^2 + 0.0916 \left( \frac{\rho^*}{\rho_s} \right)
\]

From the plots, we find Eqs. (7) and (8) fairly estimate the discrepancy in stiffness found in the experiments as described by Eqs. (9) and (10). Hence, it is confirmed that the different in cell local-deformation does affect the elastic modulus of a closed-cell foam material. In particular, it is also confirmed that the cell-face stretching effect plays a major role in the discrepancy in stiffness. In addition, the gap found between Eq. (10) and the real value of flexural moduli is possibly due to the simplification used in the theory.

5. Conclusion

The effect of cell local-deformation on the compression and bending stiffness was studied by theoretical analysis. The derived equations describe fairly the contribution of cell-edge bending effect and cell-face stretching effect on the stiffness under uniaxial loading and bending conditions. It is confirmed that the discrepancy in stiffness found in the experiments is due to the different in cell local-deformation, where the effect from cell-face stretching playing a prominent role.

References