Energy trapping of thickness-twist waves in an inhomogeneous piezoelectric plate with an imperfect joint

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In this paper, we study the energy trapping phenomenon of thickness-twist waves propagating through an imperfect joint between two semi-infinite piezoelectric plates of crystals with 6mm symmetry. Considering the imperfect joint by a spring-type relation which covers both mechanical imperfect continuity condition and electrical imperfect continuity condition, we obtain an exact solution from the three-dimensional equations of piezoelectricity. The solution can be reduced to some well-known results when the spring-type parameters for the imperfect joint take some special values. The effects of the imperfect joint on the energy trapping phenomenon are graphically shown. The results are of fundamental significance to the design of resonators and other piezoelectric devices when imperfect joints are considered.

1. Introduction

Thickness-twist vibration modes of crystal plates are often used as the operating modes for resonators and acoustic wave sensors. When the six-fold axis of a 6mm crystal is parallel to the major surface of a plate, thickness-twist waves can propagate in an unbounded plate. The thickness-twist mode also exists in an inhomogeneous piezoelectric plate, such as a plate in which the central portion is different from the rest portion, a plate with a joint between two semi-infinite piezoelectric plates, and a plate consisting of multi-sectioned piezoelectric materials, and so on. In these studies, the interface between different materials is assumed to be prefect or ideal bonding. For various reasons, such as thin interphase, chemical action, and/or interface damage, imperfect bonding sometimes appears at such bi-material interfaces. Jinn et al. and Fan et al. investigated the Love-type waves propagating in an elastic layer/piezoelectric substrate with an imperfect interface, respectively. Fan et al. also analyzed the SH interfacial waves propagating along the imperfect interface between two piezoelectric half spaces. For the thickness-twist waves propagating in an inhomogeneous piezoelectric plate with an imperfect joint, however, to the best of our knowledge such work has not been done yet.

In this paper, motivated by the work, we study the effect of imperfect bonding on the energy trapping phenomenon of thickness-twist waves in an inhomogeneous piezoelectric plate. The imperfect bonding is modeled by a spring-type relation. The results are of fundamental importance to the design of resonators and other devices when imperfect joints are considered.

2. Problem statement and solution

Consider an inhomogeneous piezoelectric plate of uniform thickness 2h, as shown in Fig.1. The left portion x_1 < 0 is made of one piezoelectric material, and the right portion x_1 > 0 is made of another one. The six-fold axis (or the poling direction of ceramics) is along x_2 determined by the right-hand rule from x_1 and x_2. The bonding of the two materials is assumed to be imperfect. The plate is unelectroded and free of tractions at the major surfaces x_2 = ±h. Due to the relatively high symmetry of 6mm crystals, the structure allows simple and exact modes with only one anti-plane displacement u_1, i.e., thickness-twist waves. We need to analyze the two halves of the structure separately, and then apply the imperfect continuity conditions at the joint.

For the left half, let u and \phi denote separately the mechanical displacement and electric potential. Then the thickness-twist waves are described by

\[ u_i = 0, \quad \phi_i = 0, \quad u_3 = u(x_1, x_2, t), \quad \phi = \phi(x_1, x_2, t), \]

which are governed by

\[ \nabla^2 u + \rho \nabla^2 \phi = \rho u_t, \]

\[ \nabla^2 u - e \nabla^2 \phi = 0 \]

where c = c_{46}, \epsilon = \epsilon_{15} and \epsilon = \epsilon_{11} are the relevant elastic, piezoelectric and dielectric constants. \rho is the mass density. \nabla^2 is the two-dimensional Laplacian. A function \psi can be introduced through \phi = \psi + u/\epsilon, then the governing equations for u and \psi are decoupled as

\[ \nabla^2 \psi = \rho u_t, \]

\[ \nabla^2 \psi = 0 \]

where \alpha = \epsilon_{46}/\epsilon is the piezoelectrically-stiffen elastic constant.

For the right half, similarly, let u' and \psi' denote separately the mechanical displacement and electric potential. Hereafter, symbols with prime indicate the field quantities and material constants for the piezoelectric material occupying x_1 > 0.) Then the governing equations are

\[ \nabla^2 u' + \rho' u'_t = 0, \]

\[ \nabla^2 \psi' = 0 \]

where \psi' = \psi' - u'/\epsilon is an introduced function similar to \psi.

The major surfaces of the plate are unelectroded and free of tractions so that we have the following boundary conditions

\[ T_{x_2} = 0, \quad D_z = 0, \quad \psi = 0. \]

For the case of perfect bonding at the joint between the two piezoelectric materials, the continuity of u, T_{x_2}, \phi, and D_z need to be imposed. In this paper, however, the joint is treated as an imperfect bonding for which we use the following spring-type relation

\[ x_1 = 0; \quad T_{x_2} = T_{x_1} = K (u' - u), \]

\[ D_z = D'_z = \Gamma (\phi' - \phi) \]

where K (in unit of N/m^2) is the effective interface elastic stiffness parameter and \Gamma (in unit of C/Vm^2) is the electrical...
imperfection parameter. The two parameters are simultaneously used to define how well the two materials are bonded.

Consider incident waves coming from $-\infty$ and propagating along the positive direction of $x_1$. The waves may propagate through the joint and also may get reflected at the joint. Following Yang et al\cite{5-6}, the solutions to (3) and (5)\cite{1} can be classified into waves symmetric or antisymmetric in $x_2$. For the waves transmitted into the right half of the piezoelectric plate, similarly, the solutions to (4) and (5)\cite{2} can be also classified into symmetric and antisymmetric fields. Using the piezoelectric constitutive equations, we can obtain easily the expressions for the stress and electric displacement components. Substitution of the corresponding stress and electric displacement components into (6) gives rise to a linear, homogeneous algebraic equation set for five unknown constants. Solving the system of equations yields the final solution. The detailed solution procedure, which is suppressed here due to the limitation of pages, can be found in the work by Yang et al\cite{5-6}.

3. Numerical example

In this Section, we graphically show the effects of the imperfect joint on the thickness-twist wave propagation by a numerical example. We adopt PZT5 and PZT6B for the left and right portions of the piezoelectric plate respectively. In the numerical example, $a_{th}$ or $a_{tw}$ is the cut-off frequency of thickness-twist waves obtained from the dispersion relations for these waves in an unbounded plate. In particular, $m = 0$ is called a face-shear wave, which is not considered. Here, we consider the case $m = 2$, and the plate thickness is set to $h = 1$ mm.

Figure 2 shows the distribution of displacement across the joint when the incident wave frequency $\omega$ satisfies $a_{th} < \omega < a_{tw}$. The black curve in Fig. 2 corresponds to the case of perfect bonding when $\omega = 7.235$ MHz and agrees well with the result in Ref. 6). The red line in Fig. 2 corresponds to the case of imperfect bonding and resembles the black line closely except for the displacement discontinuity at the joint. Whether the joint is imperfect or not, the transmitted wave decays rapidly and monotonically away from the joint along $x_1$ direction. This phenomenon is closely related to the energy-trapping feature of thickness-twist waves\cite{7}.

![Figure 2](image2.png)  
**Figure 2** Displacement distribution across the joint.

Figure 3 shows the distribution of displacement across the joint when the incident wave frequency satisfies $\omega > a_{tw}$. The black curve in Fig. 3 corresponds to the case of perfect bonding when $\omega = 7.474$ MHz and have the same amplitude in both the incident and transmission halves, which means that there is no energy-trapping feature. The red line in Fig. 3 corresponds to the case of imperfect bonding and resembles the black line closely except for the displacement discontinuity at the imperfect joint. It can also be seen that the imperfect joint absorbs some energy of the incident wave since the amplitude of the transmitted wave becomes smaller than that of the incident wave.

![Figure 3](image3.png)  
**Figure 3** The same as Fig. 2 but for the case $\omega > a_{tw}$.

4. Conclusions

In this paper, we studied the thickness-twist wave propagation in an inhomogeneous piezoelectric plate with an imperfect joint. The effects of the imperfect bonding on the energy trapping phenomenon are graphically shown by a PZT-6B/PZT-5 system. When the incident wave frequency satisfies $a_{th} < \omega < a_{tw}$, the energy-trapping phenomenon exists disregarding the bonding status. While the incident wave frequency satisfies $\omega > a_{tw}$, the energy-trapping phenomena disappears but some energy of the incident wave is absorbed due to the imperfect bonding.

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