Numerical Simulation for Transient Behaviors of Mechanical Sensors Using Conducting Polymers

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The present study attempts to develop a method of the numerical simulation for the transient behaviors of mechanical sensors using conducting polymers. The feature of the transient behaviors is the relaxation and time lag of reaction force and electric potential. The relaxation and time lag are affected by fluid pressure in the porous materials, but the pressure is unknown in the analysis of the transient behaviors. The pressure is estimated by the undrained parameters of Biot poroelastic theory, and the fields of solid stress, fluid pressure, ion concentration and electric potential are fully coupled and numerically analyzed.

Keywords - conducting polymers, transient behaviors, fully coupled numerical simulation, porous materials, undrained parameters

1. Introduction

Conducting polymers are extraordinary plastics able to conduct electricity. They are common in our lives such as batteries, displays and so on. Ionic conducting polymers are classified by the mechanism of ion transport interacting electricity. The mechanical sensors using ionic conducting polymer generate electricity corresponding to the magnitude of the mechanical stimulation. Their behaviors are very much complicated, so numerical are needed analyses for their investigation, design and control.

The most difficulty of the estimation of the sensor behaviors is non-invertible input-output relation in mechanical sensor and actuator of the same structure using ionic conducting polymer. The generated electric potential of the sensor is one thousandths of the supplied electric potential of the actuator of the same structure with respect to the same displacement. Hence, the simulation results of the sensors have not been reported, even though many papers for the simulations results of the actuators.

The responses of the mechanical sensors are characterized as relaxation and hysteresis of reaction force and electric potential. The ionic conducting polymer can be operated as actuators and sensors simultaneously. In other words, the physical phenomena of the actuators and sensors cannot be separately analyzed. The behaviors of the actuators and sensors can be simulated by the same models and parameters.

2. Computational model for conducting polymer sensor

The basic governing equation of a continuum from the Cauchy’s equations of motion is, neglecting acceleration and body force, as follows.

\[
\frac{\partial \sigma_{ij}}{\partial x_j} = 0
\]  

(1)

where, \( \sigma_{ij} \) is the total stress component of a porous element.

The Biot constitutive equation with zero initial stress and fluid pressure is given as follows [10].

\[
\sigma_{ij} = (K' - 2G' + 3) \varepsilon_{ij} - 2G' \varepsilon_{ij} + bp \delta_{ij}
\]

(2)

\[
\varepsilon' = \varepsilon_{ik} \delta_{ik}, \quad b = 1 - \frac{K'}{K}
\]

(3)

where, \( \varepsilon' \) is total stress components, \( \varepsilon' \) is fluid pressure in pores, \( \varepsilon_{ij} \) is total strain components, \( \varepsilon_{ij} \) is total volumetric strain, \( G' \) is drained shear modulus, \( K' \) is drained bulk modulus, \( K' \) is non-porous bulk modulus, \( b \) is Biot coefficient, and \( \delta_{ij} \) is Kronecker delta.

The Darcy law to governing the flow of compressible fluid inside a saturated porous medium is given as [10]

\[
f = \frac{\kappa_s}{\eta} (\nabla p - \rho_f \vec{g})
\]

(4)

where, \( f \) is volume flux vector of fluid, \( \kappa_s \) is hydraulic permeability, \( \eta \) is dynamic viscosity of fluid, \( \rho_f \) is the density of fluid, and \( \vec{g} \) is body force vector.

Rice and Cleary introduced the term of ‘undrained condition’ assuming that time scale is too short to allow the loss or gain of pore fluid in an element by diffusive transport to or from neighborhood element, and proposed the following relations [8].

\[
\Delta p = -B \frac{2 \sigma_{ij}}{3}
\]

(5)

\[
B = \frac{1 + \phi}{K'} + \frac{\phi}{K'} + \frac{1}{K''}
\]

(6)

\[
\nu^s = \frac{3 \nu^d + B(1 - 2 \nu^d) b}{3 - B(1 - 2 \nu^d) b}
\]

(7)

where, \( B \) is Skempton coefficient, \( \phi \) is the Lagrangian porosity referring the ratio of pore space to the overall volume, \( K' \) is bulk modulus of pore fluid, \( \nu^s \) is drained Poisson’s ration \( \nu^d \) is undrained poisson’s ratio, and the others are mentioned previously.

The pressure of the pore fluid interacts with mechanical stress and electric field in both of the actuation and sensation of the ionic conducting polymers. Coupling with the mechanical stress and electric potential fields, the field equation of the pressure is rewritten as follows.

\[
\frac{\kappa_s}{\eta} (V^2 p - \phi FC V^2 V) = \frac{3 (\nu^d - \nu^s)}{2G' B(1 + \nu^d)(1 + \nu^s)} \frac{\partial}{\partial t} \left( \sigma_{ij} + \frac{3}{B} p \right)
\]

(8)

where, \( V \) is electric potential, \( C \) is ion concentration, \( \kappa_s \) is hydraulic permeability, \( \eta \) is dynamic viscosity, and \( F \) is Faraday constant, and the others are mentioned previously.

The Timoshenko beam model is employed as constitutive equations. The present study modifies the undrained Poisson’s ratio obtained from Eq.(7) in the Biot poroelastic theory, and employs the Poisson effects of the pore pressure to the axial direction. The constitutive equations are the followings.

\[
\sigma_{ij}^s = E' \varepsilon'_{ij} - (1 + 2 \nu^d) b p
\]

(9)
\[ v' = \beta e^n \]  
(10)

\[ \sigma_y = \sigma_v = -bp \]  
(11)

\[ \tau'_{yz} = \alpha G' \gamma_{yz} \]  
(12)

where, \( E' \) is drained elastic modulus, \( \psi \) is beam undrained Poisson's ratio, \( \beta \) is correction factor of the Poisson effect, and \( \alpha \) is shear correction factor employed as \( \frac{5}{6} \) in case of rectangular section. The correction of the undrained Poisson's ratio makes the peak of the numerical reaction force reached at the peak of the experimental reaction force.

The generated electric potential in the mechanical sensor using ionic conducting polymers is micro-volts as about one thousandth of the supplied electric potential in the actuator with respect to the same displacement and structure. The very small amount of electricity from the initially neutral condition is caused by the gradient of mobile ion flux as the pore fluid in Nernst-Plank equation, the redistribution of the mobile ion concentration is obtained as follows.

\[ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2} - \varepsilon_e \frac{F}{RT} \frac{\partial V}{\partial z} - \frac{\partial}{\partial C} \left( \zeta C \right) \]  
(13)

where, \( C \) is ion concentration, \( D \) is diffusivity coefficient, \( R \) is the gas constant, \( T \) is absolute temperature.

Next, the well known Poisson equation in electrostatics is employed as

\[ \frac{\partial^2 V}{\partial z^2} = -\frac{\varepsilon_e}{\varepsilon_0} \left( C - C^0 \right) \]  
(14)

where, \( \varepsilon_e \) is electric permittivity, and \( C^0 \) is concentration of immobile ions. Finally, the electric potential is obtained from the above equations as the output of the sensor model.

3. Numerical results

Fig. 1 Time history of tip deflection and Reaction force

Fig. 2 Pressure distribution over thickness at beam root

Fig. 3 Mobile ion concentration over thickness at beam root

4. Conclusion

The computational system for the transient behavior of the mechanical sensors using ionic conducting polymers was newly introduced. The transport phenomena of pore fluid is very important in the transient behaviors of the mechanical sensors, so mechanical properties related with the transport of the pore fluid such as porosity and permeability are key factors for design and control of the mechanical sensors. Furthermore, the relaxation and hysteresis of the mechanical reaction force and electric potential is explained by the pressure diffusion as the typical phenomena in fluid-saturated porous media. In addition, the transient behavior of porous beams is significantly sensitive to the Poisson effect.

References

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