Implicit Finite Element Implementation of an Anisotropic Soil Model

1. Introduction

Implicit numerical algorithm using return-mapping method has been proven to provide an excellent performance when integrating a nonlinear isotropic elastoplasticity; i.e., a pressure-dependent model, in particular, where only a few scalar equations are required to formulate whole governing equations (Aravas, 1987). The simplicity lies in the fact that return directions to yield surface are coaxial with updated stresses in principle stress space. Accordingly, an explicit form of a consistent tangent modulus is complicated and relatively cumbersome due to the complexity of anisotropy; therefore, the method loses a performance and appears impractical to initial inversion of material stiffness tensor. However, the similar procedure is not conveniently applicable to an anisotropic model mainly because return directions to anisotropic yield surface are not coaxial with updated state of stresses. Luccioni et al. (2000) employed a return-mapping technique to an anisotropic Bear-Clay model and concluded that the formulation of governing equations under a return-mapping scheme is complicated and relatively cumbersome due to the complexity of anisotropy; therefore, the method loses a performance and appears impractical to initial boundary value problems.

In this study, a return-mapping regularization applicable to anisotropic models was developed following a typical procedure but a newly-developed process corresponding to invariant-based tensor basis was applied to solve a concerned limitation. An implementation of implicit finite element method and numerical illustration were presented to demonstrate a computational performance under the proposed procedure. The mathematical technique may suggest a solution or extend a performance to other similar anisotropic plasticity models.

2. Anisotropic plasticity

The anisotropic soil plasticity proposed by Sekiguchi and Ohta (1977) is adopted in the study. A stress-strain-strength response of model behaves anisotropically due to the existence of the joint invariant between current stresses and stress history induced by the initial yield stress. The yield function expressed in terms of stress invariants, hardening stress parameter and their related tensorial notations are summarized in Box 1.

3. Rate constitutive equations

In general, return-mapping methods are simply founded by a set of five equations expressed below,

\[ \mathbf{\sigma}' = \mathbf{c} : (\mathbf{e} - \mathbf{e}^p) \]  
(1)

Flow rule (associative case): 
\[ \dot{\mathbf{e}}^p = \mathbf{f}'(\mathbf{n}') \]  
(2)

Evolution law of hardening: 
\[ \dot{I}_{ct} = \frac{I_{ct}}{MD} \]  
(3)

Yield function: 
\[ f(\mathbf{\sigma}', \mathbf{\sigma}'_c) = f(I_1, J_2, I_3) \]  
(4)

Kuhn-Tucker complementarity condition: 
\[ \gamma \geq 0, \quad f' = 0, \quad \psi' = 0 \]  
(5)

where elastic stiffness tensor: 
\[ \mathbf{c}^e = K(1+2\nu) \]  
(6)

Bulk and shear moduli: 
\[ K = \frac{1}{3\kappa} (1 + \nu), \quad G = \frac{3(1-2\nu)}{2(1+\nu)} \kappa \]  
(7)

Basic material parameters for SO (the Sekiguchi-Ohta) model are described by \( M \) (critical state parameter), \( D \) (coefficient of dilatancy), \( K_0 \) (coefficient of earth pressure at rest), \( \epsilon \) (void ratio), \( \nu' \) (effective Poisson’s ratio), \( \kappa \) (swelling index) and \( p_c' \) (pre-consolidation stress).

Box 1 Summarized expressions of Sekiguchi-Ohta plasticity

4. Finite element implementation

The flow chart for global and local Newton iterations used in non-linear FEM is shown in Fig. 1. Global solution scheme is shown in Fig. 2. Fig. 3 demonstrates the interaction between functions in FEM procedure.
6. Numerical illustration

The performance of the proposed procedure is evaluated through numerical simulations of drained bi-axial (plane strain) compression tests up to half of over-burden pre-consolidation pressure. Soft clays parameters are shown in Fig. 5. Isoparametric rectangular element with 4-Gauss points is employed. A tolerance is set to $10^{-5}$ for both global and local iterations. Fig. 6 shows the calculation results with varying increments of 1, 5, 10, 20, and 50. By 50 increments, the solution does not change significantly and hence the exact solution by the algorithm is achieved. It is found that the resulting solutions can reach a convergence with considerably accuracy even by a relatively large strain.

![Fig. 5 Simulation of drained bi-axial compression](image)

**Fig. 5** Simulation of drained bi-axial compression

**Fig. 6 Results of element no.4 with varying increments**

![Fig. 6](image)

References