A Practical Procedure for Vertical Two-dimensional Hydrothermal Simulation by Finite Element Method

By

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Abstract: A procedure for the numerical simulation of a two-dimensional, single phase geothermal hot water convection system was developed. Galerkin's finite element method with triangular elements was applied. The procedure was implemented by FORTRAN and was verified by execution tests. The program was applied to simulation of the natural evolution of a geothermal system to its final steady state. The procedure is based on general conservation equations having accumulation terms, and thus can avoid troubles on convergence which might occur in case the steady state is postulated in the program. The procedure may be applied to the development of simulators effectively used for understanding the physical processes in liquid dominated hydrothermal systems.

1. Introduction

Numerical simulation in reservoir engineering study has been recognized as a quite useful technique for the evaluation of geothermal resources. It is usually available even if the behavior of a geothermal reservoir is difficult to be treated by analytical methods.

The study covers wide range of reservoir models, from macroscopic natural convection models to microscopic reservoir behavior models. Recently, efforts have been made to apply numerical simulators to geothermal areas. There are some examples of the applications to the modeling of natural state of geothermal systems. BODVARSSON *et al.* (1984) or INGEBRITSEN *et al.* (1985) are among them. They used three-dimensional, two-phase simulator for two-phase reservoirs.

As for geothermal systems in Japan, there are many cases in which the system can be analyzed by treating the macroscopic hydrother-

mal movement of single phase hot water. In those cases, single phase simulator is suitable to be used effectively rather than multi-phase simulator. If there are sufficient data on the system, three-dimensional analysis is desirable, but it is unusual. In general cases, three-dimensional analysis makes the problem complicated unnecessarily, while the spatial precision is forced to be low for computational time. In the examples mentioned above, the models are two-dimensional vertical sections, though the simulators are capable of treating three-dimensional models. Vertical two-dimensional models are more applicable than areal models, to the study of physical processes, because the effect of gravity has an important role in hydrothermal convection.

Therefore, a single phase, vertical two-dimensional simulator which is easy to be developed and used is required at an early stage for the analysis of the natural state of a geothermal system.

Capabilities for good approximations of the reservoir are required for the simulator. The

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capabilities would be those of treating arbitrary reservoir shapes, steady and transient flows, rock properties of individual blocks of the reservoir, adequate fluid properties and boundary conditions. On the other hand, easiness for data preparation and calculation efficiency may be practically very important. Developing a model of a geothermal system requires repetition of simulation executions with different input data.

In order to develop such a simulator, finite element method (FEM) seems to be the best discretization procedure. There are three methods which are usually used for the discretization of governing equations. They are FEM, finite difference method (FDM), and integrated finite difference method (IFDM). Since the movement of hot water is considerably affected by the shape of the reservoir, FEM and IFDM, which are capable of treating arbitrary reservoir shape, are preferable. Input data for FEM may be prepared more easily than for IFDM, for the specification of relations among elements in the latter method is rather complicated.

MERCER and FAUST (1975) presented a procedure for two-dimensional (areal), two-phase reservoir by GALERKIN'S FEM. The procedure can be modified into a procedure for our purpose. HANAOKA (1980) presented a detailed procedure for a vertical two-dimensional reservoir with triangular elements. It treated only steady state of the reservoir. The purpose of this paper is to describe a procedure by FEM capable of treating both transient and steady state of a vertical two-dimensional reservoir, by introducing a discretization procedure to the basic conservation equations (FAUST and MERCER, 1979) based on the procedures mentioned above. The simulator developed by this procedure is capable of handling various flow phenomena. It has advantages inherent in FEM, such as the capabilities for treating arbitrary shape of sections, easiness for treating boundary conditions or heterogeneity and anisotropy of permeability.

Also, problems in the study of the relation between steady state flow of a natural geothermal system and transient flow may be discussed using this procedure.

2. Basic equations and discretization

For developing natural convection models, finite element models of liquid dominated systems are suitable. HANAOKA (1980) developed a procedure applying GALERKIN'S FEM to liquid hydrothermal system. But it employed steady state assumption, neglecting accumulation terms in conservation equations. MERCER and FAUST (1975) described the approximation of the time derivative using finite difference techniques. Combining these, a simple and efficient discretization procedure can be derived.

FAUST and MERCER (1979) described general basic equations for the conservation of mass and energy of two phase steam and water flow. Assuming that steam is absent, the equations for liquid system become

$$\begin{aligned} \frac{\partial (\phi \rho)}{\partial t} - \nabla \cdot \left[\frac{k \rho}{\mu} \cdot (\nabla p - \rho g \nabla D) \right] \\ - q_m = 0 \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial}{\partial t} [\phi \rho h + (1 - \phi) \rho_s h_s] \\ - \nabla \cdot \left[\frac{k \rho h}{\mu} \cdot (\nabla p - \rho g \nabla D) \right] \\ - \nabla \cdot \left[\lambda \left(\frac{\partial T}{\partial p} \right)_h \nabla p + \lambda \left(\frac{\partial T}{\partial h} \right)_p \nabla h \right] \\ - q_h = 0 \end{aligned} \quad (2)$$

The meanings of variables in the equations of this paper are explained in the nomenclature.

The approximation below is appropriate for most of single phase liquid systems.

$$\begin{aligned} \lambda \left(\frac{\partial T}{\partial p} \right)_h \nabla p + \lambda \left(\frac{\partial T}{\partial h} \right)_p \nabla h \\ = \frac{\lambda}{C} \nabla h \end{aligned} \quad (3),$$

for temperature is assumed to depend only on enthalpy. Assuming that the fluid density depends only on temperature (and so enthalpy)

and porosity is uniform over an element and constant, the accumulation term in the mass balance becomes

$$\frac{\partial(\phi \rho)}{\partial t} = \phi \left(\frac{\partial \rho}{\partial h} \right)_p \frac{\partial h}{\partial t} \quad (4).$$

If the enthalpy of rock depends only on temperature, and specific heat capacity of rock is uniform and constant, we get

$$\frac{\partial h}{\partial t} = \frac{C_r}{C} \frac{\partial h}{\partial t} \quad (5).$$

We assume that the density of rock is constant. Then we can write accumulation term in the energy balance equation as

$$\begin{aligned} \frac{\partial}{\partial t} [\phi \rho h + (1 - \phi) \rho_r h] \\ = \left[\phi h \left(\frac{\partial \rho}{\partial h} \right)_p + \phi \rho \right. \\ \left. + (1 - \phi) \rho_r \frac{C_r}{C} \right] \frac{\partial h}{\partial t} \quad (6). \end{aligned}$$

If we do not consider heat source within an element except for that specified as

$$q_h = h q_m \quad (7),$$

we can simplify the energy equation (2) as

$$\begin{aligned} \left[(1 - \phi) \rho_r \frac{C_r}{C} + \phi \rho \right] \frac{\partial h}{\partial t} \\ - \nabla h \cdot \left[\frac{k \rho}{\mu} \cdot (\nabla p - \rho g \nabla D) \right] \\ - \frac{\lambda}{C} \nabla^2 h = 0 \quad (8). \end{aligned}$$

The permeability major axes are assumed to be orthogonal. We use following substitutions

Fig. 1 Relation between the coordinates and an element.

for simplification.

$$\frac{k \rho}{\mu} = \frac{\rho}{\mu} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \quad (9).$$

For application of triangular elements, we consider the local coordinates (η, ξ) as in Fig. 1. After substituting (4) for the accumulation term in (1), and multiplying it by an interpolation function N_i , we take the integral for an element.

$$\begin{aligned} \phi \iint_{\text{element}} N_i \left[\left(\frac{\partial \rho}{\partial h} \right)_p \frac{\partial h}{\partial t} \right] d\eta d\xi \\ - \iint_{\text{element}} N_i \left[A_1 \frac{\partial}{\partial \eta} \left(\frac{\partial p}{\partial \eta} + \rho g \sin \beta \right) \right. \\ \left. + A_2 \frac{\partial}{\partial \xi} \left(\frac{\partial p}{\partial \xi} + \rho g \cos \beta \right) \right. \\ \left. + \frac{\partial A_1}{\partial \eta} \frac{\partial p}{\partial \eta} + \frac{\partial A_2}{\partial \xi} \frac{\partial p}{\partial \xi} \right. \\ \left. + \rho g \left(\frac{\partial A_1}{\partial \eta} \sin \beta + \frac{\partial A_2}{\partial \xi} \cos \beta \right) \right. \\ \left. + q_m \right] d\eta d\xi = 0 \quad (10). \end{aligned}$$

Integration by parts for (10) as in ZIENKIEWICZ (1971) results in

$$\begin{aligned} \phi \iint_{\text{element}} N_i \left[\left(\frac{\partial \rho}{\partial h} \right)_p \frac{\partial h}{\partial t} \right] d\eta d\xi \\ + \iint_{\text{element}} \left[A_1 \frac{\partial N_i}{\partial \eta} \frac{\partial p}{\partial \eta} + A_2 \frac{\partial N_i}{\partial \xi} \frac{\partial p}{\partial \xi} \right. \\ \left. + A_1 \frac{\partial N_i}{\partial \eta} \rho g \sin \beta + A_2 \frac{\partial N_i}{\partial \xi} \rho g \cos \beta \right. \\ \left. - N_i q_m \right] d\eta d\xi - \int_{\text{boundary}} N_i \left[A_1 \left(\frac{\partial p}{\partial \eta} \right. \right. \\ \left. \left. + \rho g \sin \beta \right) L_1 + A_2 \left(\frac{\partial p}{\partial \xi} \right. \right. \\ \left. \left. + \rho g \cos \beta \right) L_2 \right] dS = 0 \quad (11), \end{aligned}$$

where L_1 and L_2 are the direction cosines of outward normal of the element boundary (Fig. 1).

Pressure and enthalpy within an element are assumed to be the linear combinations of values at three nodes of the element as

$$p = N_1 p_1 + N_2 p_2 + N_3 p_3 = N_i p_i \quad (12)$$

$$h = N_1 h_1 + N_2 h_2 + N_3 h_3 = N_i h_i \quad (13).$$

Pressure of a node is denoted as

$$p_i = \theta p_i^{t+\Delta t} + (1 - \theta) p_i^t \quad (14).$$

If θ equals to 1, it implies that fully implicit time difference scheme is adopted.

Time derivative of enthalpy is written as

$$\frac{\partial h}{\partial t} = \frac{h^{t+\Delta t} - h^t}{\Delta t} = \frac{N_i h_i^{t+\Delta t} - N_i h_i^t}{\Delta t} \quad (15).$$

Taking substitutions

$$E_{ij} = \iint_v \left(A_1 \frac{\partial N_i}{\partial \eta} \frac{\partial N_j}{\partial \eta} + A_2 \frac{\partial N_i}{\partial \xi} \frac{\partial N_j}{\partial \xi} \right) d\eta d\xi \quad (16),$$

$$F_i = \iint_v \left(A_1 \frac{\partial N_i}{\partial \eta} \rho g \sin \beta + A_2 \frac{\partial N_i}{\partial \xi} \rho g \cos \beta - N_i q_m \right) d\eta d\xi \quad (17),$$

$$H_{ij} = \theta E_{ij} \quad (18),$$

$$I_i = \frac{\phi}{\Delta t} (h_i^{t+\Delta t} - h_i^t) \iint_v N_i N_j \left(\frac{\partial \rho}{\partial h} \right)_p d\eta d\xi \\ - p_i^t \frac{\phi}{\Delta t} \iint_v N_i N_j \left(\frac{\partial \rho}{\partial p} \right)_h d\eta d\xi \\ + (1 - \theta) E_{ij} p_i^t \quad (19),$$

and

$$S_i = \int_v N_i \left[A_1 \left(\frac{\partial p}{\partial \eta} + \rho g \sin \beta \right) L_1 + A_2 \left(\frac{\partial p}{\partial \xi} + \rho g \cos \beta \right) L_2 \right] d\eta d\xi \quad (20),$$

then the discretized mass balance equation (11) becomes

$$H_{ij} p_i^{t+\Delta t} + I_i + F_i - S_i = 0 \quad (21),$$

where $p_i^{t+\Delta t}$ is the unknown variable.

The same procedure can be applied to the energy balance equation (8).

Assuming

$$h_i = \theta h_i^{t+\Delta t} + (1 - \theta) h_i^t \quad (22),$$

and substituting

$$G_{ij} = \iint_v \left[A_1 N_i \frac{\partial N_j}{\partial \eta} \left(\frac{\partial p}{\partial \eta} + \rho g \sin \beta \right) + A_2 N_i \frac{\partial N_j}{\partial \xi} \left(\frac{\partial p}{\partial \xi} + \rho g \cos \beta \right) - \frac{\lambda}{C} \left(\frac{\partial N_i}{\partial \eta} \frac{\partial N_j}{\partial \eta} + \frac{\partial N_i}{\partial \xi} \frac{\partial N_j}{\partial \xi} \right) \right] d\eta d\xi \quad (23),$$

$$S'_i = \int_v \frac{\lambda}{C} N_i \left(\frac{\partial h}{\partial \eta} L_1 + \frac{\partial h}{\partial \xi} L_2 \right) dS \quad (24),$$

$$J_{ij} = \frac{1}{\Delta t} \iint_v N_i N_j \left[(1 - \phi) \rho \frac{C_r}{C} + \phi \rho \right] d\eta d\xi - \theta G_{ij} \quad (25),$$

$$O_i = - \frac{h_i^t}{\Delta t} \iint_v N_i N_j \left[(1 - \phi) \rho \frac{C_r}{C} + \phi \rho \right] d\eta d\xi - (1 - \theta) G_{ij} h_i^t \quad (26),$$

the discretized energy balance equation (8) becomes

$$J_{ij} h_i^{t+\Delta t} + O_i - S'_i = 0 \quad (27),$$

where $h_i^{t+\Delta t}$ is the unknown variable.

Linear interpolation function N_i can be written as

$$N_i = \frac{1}{2M} (b_{i1} + b_{i2} \eta + b_{i3} \xi) \quad (28),$$

where M is the area of the element.

Considering differentials and integrals of N_i , such as

$$\iint_v N_i N_j d\eta d\xi = \begin{cases} \frac{M}{12} & (i \neq j) \\ \frac{M}{6} & (i = j) \end{cases} \quad (29),$$

we can rewrite the coefficients of the equations (21) and (27) as follows;

$$E_{ij} = \frac{1}{4M} (A_1 b_{2i} b_{2j} + A_2 b_{3i} b_{3j}) \quad (30)$$

$$F_i = \frac{\rho g}{2} (A_1 b_{2i} \sin \beta + A_2 b_{3i} \cos \beta) - \frac{M}{3} q_m \quad (31)$$

$$G_{ij} = \frac{1}{6} \left[A_1 b_{2j} \left(\frac{\partial p}{\partial \eta} + \rho g \sin \beta \right) + A_2 b_{3j} \left(\frac{\partial p}{\partial \xi} + \rho g \cos \beta \right) \right] - \frac{\lambda}{C} \frac{1}{4M} (b_{2i} b_{2j} + b_{3i} b_{3j}) \quad (32)$$

$$I_i = \frac{\phi}{\Delta t} a_{ij} \left[(h_i^{t+\Delta t} - h_i^t) \left(\frac{\partial \rho}{\partial h} \right)_p \right] + (1 - \theta) E_{ij} p_i^t \quad (33)$$

$$J_{ij} = \frac{1}{\Delta t} a_{ij} \left[(1 - \phi) \rho \frac{C_r}{C} + \phi \rho \right] - \theta G_{ij} \quad (34)$$

$$O_i = -\frac{h_i'}{\Delta t} a_{ij} \left[(1 - \phi) \rho \frac{C_r}{C} + \phi \rho \right] - (1 - \theta) G_{ij} h_i' \quad (35)$$

$$a_{ij} = \begin{cases} \frac{M}{12} & (i \neq j) \\ \frac{M}{6} & (i = j) \end{cases} \quad (36)$$

Starting from initial values, pressure and enthalpy can be calculated by (21) and (27). All the other variables can be calculated from these two variables by approximation functions for state equations. The calculation of flow can be performed easily by using interpolation function.

3. Considerations for programming

Fig. 2 is the simplified flow chart of the program for the simulation by the procedure described above.

We start with data input (①). Parameters for calculation are, time step interval, maximum time, maximum iteration number, θ , maximum deviation for iteration. As the reservoir parameters, we need the coordinates of nodes, the relation between element numbers and nodal numbers, and rock properties such as permeability, thermal conductivity or specific heat capacity, etc.. Boundary conditions may be either Dirichlet type or Neumann type, pressure and enthalpy conditions. Sinks or sources q_m can be specified. Following the assumption (7), q_m is automatically calculated. Initial pressure and enthalpy values must be specified for each node.

In order to clearly understand the whole conditions we set, we had better have a graphic view of the element system. A printer output of the values we input helps to check them (②).

We must calculate the coefficients b_i of the interpolation function for each element from

Fig. 2 Flow chart of the FORTRAN program for hydrothermal simulation.

the coordinates of nodes and the direction of the local coordinate (η, ξ) specified by the major axes of the permeability tensor (③).

We must set the initial values of pressure and enthalpy into dimensioned variables of the program p 's and h 's (4).

At each iteration ⑥–⑧, $p_i^{t+\Delta t}$ s and $h_i^{t+\Delta t}$ s, are calculated. If the difference between calculated values and the values of the former iteration becomes negligible, we stop the iteration and go on to the next time step.

In the procedure in this paper, mass and energy balance equations are linearized by GALERKIN'S method for pressure and enthalpy respectively, and adapted for the iteration. On the other hand, Newton Raphson iteration may be generally used to solve nonlinear equations as in hydrothermal simulation. In Newton Raphson method, the original nonlinear equation

$$f(x_i) = 0 \quad (37)$$

is used to create a linear equation for one variable x_i^{n+1} by

$$f(x_i^n) + \left. \frac{\partial f}{\partial x_i} \right|_{x_i^n} (x_i^{n+1} - x_i^n) = 0 \quad (38),$$

and x_i is sought by iteration, where x_i specifies one of the unknown variables, and n is the iteration index. This is purely a numerical iteration procedure, and may be used when the equations are difficult to be linearized through the procedure as in this paper. Anyway, it is favorable to consider mass and energy equations independently for the evaluation of the convergence of them. If the change of one of either pressure or enthalpy is very small, the number of iterations can be reduced by skipping those with the negligible change.

To solve the assembled equation, a mathematical subprogram package for linear equations with band matrix is useful. In order to keep the band-width minimum for the efficiency of calculation, we must be careful in assigning the nodal numbers for elements.

At each iteration of every time step, state variables such as temperature, density, and viscosity are calculated from pressure and enthalpy through state equations. We had better use a suitable approximation function es-

pecially for viscosity, because it changes greatly with enthalpy, and affects the movement of water. All the variables must be treated in one unit system such as SI unit in the program, avoiding confusion in calculation.

Some elaborations for calculations, such as automatic time step reduction or restart from filed results are very useful and may reduce computation time required.

Results of calculation are filed and are used for display. In order to get a plain picture of the convection, a display of vectors of water flux and contours of temperature overlaid is most useful. Interactive use of graphic displays with high resolution screen is preferable.

4. Implementation and test

The procedure described in this paper was implemented into a FORTRAN program. The computer system used for programming and execution is the SIGMA system (YANO and HANAOKA, 1987) of Geological Survey. A character display and a graphic display were used as I/O devices. The program consists of about 40 subroutines including those for displaying, which have total 3,500 lines including comment lines.

In order to assure that the simulator would work correctly, program execution tests should be made. However, there are only a few published results that would be suitable for evaluating the ability and validity of the simulator presented here. Although there is a code comparison study for geothermal reservoir simulators (DOE, 1980), it focuses upon the other aspects of geothermal reservoir, rather than the study of natural convection pattern.

Here, the results of program execution tests with parameters similar to YUSA (1983) are described. It may be an integrated function test of the program for a flow pattern on a vertical section with potential flow and thermal convective flow.

Fig. 3a and 3b are calculated results for reproducing the 1,000 and 5,000 years models in the Fig. 2 of YUSA (1983). Parameters used

Fig. 3a Upper: Calculated flow and temperature for the program verification test. Input parameters are the same as the published result of Fig. 2 in YUSA (1983) (left-right inverted). They are shown in Table 1. Kinematic viscosity is constant. Elapsed time is 1,000 years. Length of arrows indicates mass flux density in logarithmic scale in this paper. Temperature contours are for 66°C, 112°C, 158°C and 204°C respectively from above. The right 2 km part is not shown.

Lower: Flow pattern and temperature taken from the result at 1,000 years in Fig. 2 in YUSA (1983). Broken lines are contours of nondimensional stream function. Temperatures for the isothermal lines (solid lines) are the same as above.

Fig. 3b Upper: Calculated flow (distribution of mass flux density) and temperature with the same parameters as in Fig. 3a. Elapsed time is 5,000 years. Contour values are the same as those of Fig. 3a.

Lower: Flow pattern (contours of nondimensional stream function) and temperature taken from the result at 5,000 years in Fig. 2 in YUSA (1983).

Table 1 Input parameters for the models of Figures 3a, 3b, 4, 6a, and 6b.**Reservoir parameters**

thickness=1km

length=5km

 $k=1.0 \times 10^{-10} \text{ cm}^2$ $C_r=0.152 \text{ cal/g} \cdot ^\circ\text{C}$ $\lambda=5.0 \times 10^{-3} \text{ cal/s} \cdot \text{cm} \cdot ^\circ\text{C}$ $\phi=0.1$ $\rho_r=2.7 \text{ g/cm}^3$ $g=0.98 \text{ m/s}^2$ $\nu=2.0 \times 10^{-3} \text{ cm}^2/\text{s}$ for Figures 3a, 3b, and 4.Approximation for μ in Fig. 5 is for Figures 6a and 6b.Approximation for ρ in Fig. 5 is for all models.**Boundary conditions**

sides: insulated for both heat and mass

surface: temperature= 20°C ,

pressure=1 atm for Fig. 3a, 3b, 6a and 6b

pressure increases leftward for Fig. 4 (1 atm at the right end and the gradient is 1.1661 atm/km)bottom: temperature= 250°C for 1km from left end**Initial conditions**temperature= 20°C , pressure=hydrostatic pressure **Fig. 4** Upper: Calculated flow (distribution of mass flux density) and temperature with the same parameters as the published result of Fig. 3 in YUSA (1983) (left-right inverted). The parameters are shown in Table 1. Elapsed time is 5,000 years. Contour values are the same as those of Fig. 3a.

Lower: Flow pattern (contours of nondimensional stream function) and temperature taken from the result at 5,000 years in Fig. 3 in YUSA (1983).

are shown in Table 1. The degree of reproduction is quite reasonable. The main hot water plume was formed at the right hand edge of the heat source on the bottom at 1,000 years, and it moved to the edge of the aquifer at later time. This model treats only the thermal convective flow. The change of pattern is almost the same as the result of YUSA (1983).

Fig. 4 is a calculated result for the reproduction of the 5,000 years model in Fig. 3 in YUSA (1983). It involves both thermal convective flow and potential flow. Two major hot water plumes were produced on the heat source. This pattern also follows the result of YUSA (1983) quite well.

As was shown above, it seems that the program works quite reasonably, though there are slight differences between the published result and the calculation by the program. The minor differences are considered to be caused

by the differences of mesh configuration, accuracy of approximation and calculation, or some slight differences of parameters used.

In this calculation, kinematic viscosity of

Fig. 5 Change of dynamic viscosity, kinematic viscosity and density of water with temperature.

Fig. 6a Calculated flow and temperature with viscosity change shown in Fig. 5. The input parameters, elapsed time and contour values are the same as those in Fig. 3a.

Fig. 6b Calculated flow and temperature with viscosity change shown in Fig. 5. The input parameters, elapsed time and contour values are the same as those in Fig. 3b.

water is set constant as for water at 125°C, but it is not appropriate as a realistic property of geothermal water. The kinematic viscosity of water changes depending on temperature as in Fig. 5 and it affects the flow pattern significantly. Fig. 6a and 6b are computed results using the approximation function for viscosity, for the same models of Fig. 3a and 3b. It is observed that the change of viscosity affects the pattern and also the rate of the change of the pattern quite significantly.

Fig. 7a Calculated flow and temperature on a basement slope. Parameters used are shown in Table 2. Elapsed time is 5,000 years. Temperature contour interval is 20°C (the same for following figures).

Fig. 7b Calculated flow and temperature at 30,000 years under the same conditions as Fig. 7a.

5. On a procedure for postulated steady state

Steady state convection models are often used as natural state models, in case the geologic setting may be considered to be constant for a long time range while the hydrothermal system evolves and reaches to a steady state.

A simulation for quite a long time span showed a process of reaching to a steady state. Fig. 7a-7d are calculated flow pattern and temperature for different time starting with an initial condition, by the simulator presented in

Fig. 7c Calculated flow and temperature at 100,000 years under the same conditions as Fig. 7a.

Fig. 7d Calculated flow and temperature at 500,000 years under the same conditions as Fig. 7a.

Table 2 Input parameters for the models of Figures 7a, 7b, 7c, 7d, 8, and 9.**Reservoir parameters**

surface elevation (m) from left

860, 855, 848, 840, 835, 827, 820, 813, 806, 800, 792, 785, 780, 775, 775.
775, 775, 775, 775, 775, 774, 773, 772, 771, 770, 770, 770.

bottom elevation (m) from left

-500, -530, -600, -700, -800, -900, -1000, -1180, -1390, -1570,
-1750, -1930, -2110, -2290, -2470, -2650, -2840, -3030, -3220,
-3455, -3690, -3795, -3900, -4010, -4120, -4120, -4120.

length 6.5km

 $k=2.0$ md (5.0 md for Fig. 9) $C_r=0.23$ cal/g°C (not used for Fig. 8) $\lambda=5.0 \times 10^{-3}$ cal/s · cm · °C $\phi=0.2$ (not used for Fig. 8) $\rho_r=2.7$ g/cm³ (not used for Fig. 8) $g=0.980665$ Approximations for μ and ρ in Fig. 5 are used.**Boundary conditions**

sides: insulated for both heat and mass

surface: temperature=10°C

pressure=1 atm

bottom: temperature=200°C

Initial conditions (and iteration starting values for Fig 8)

temperature=10°C, pressure=hydrostatic pressure.

this paper (YANO, *et al.*, 1987a). The parameters used are shown in Table 2. The upflow zone is formed at about 30,000 years after the initial condition. It continues to develop and is pushed downward to the intermediate location on the basement slope. The steady state is reached after about 500,000 years.

If the accumulation terms of the conservation equations were omitted, that is, a steady state is postulated in the simulator itself, we get two discretized equations from (21) and (27) as

$$E_{ij}p_i + F_i - S_i = 0 \quad (39)$$

$$G_{ij}h_i + S'_i = 0 \quad (40).$$

They are identical to the equations shown by HANAOKA (1980). A steady state of a convection system is calculated by applying boundary conditions to these equations.

Fig. 8 shows a steady state model (YANO *et al.*, 1987b) calculated with equations (39) and (40). The parameters used are the same for the models of Fig. 7, except that the storage

Fig. 8 Steady state flow and temperature on a basement slope calculated by the equations (39) and (40). Parameters used are shown in Table 2.

type parameters such as specific heat, porosity and density of rock are not necessary. In this model, the heat supplied from the slope of the caldera basement and the pressure gradient forced by the upheavement of the

watertable at the caldera rim are constantly combined to form an upflowing hot zone on the basement slope.

As was shown above, there are two procedures to obtain steady state of a natural geothermal system. One is to use general conservation equations ((21) and (27)) having accumulation terms, and the other is to postulate steady state in the equations to be used ((39) and (40)). The former requires long calculation time starting from the initial condition to reach to the state considered to be constant. The latter seeks that state straightly from appropriate set of initial values and needs rather shorter time. But there are many cases in which the initial values can not be appropriate when the flow pattern is too much complicated or there is no steady state under the boundary condition as in the case of oscillatory convection (YANO, 1987). In those cases, the calculation by the latter procedure runs into idle non-convergent iterations. For example, if the permeability for the model of Fig. 8 is changed to higher value as 5md, we can not get to converged values by the postulated steady state procedure. But if we apply the procedure developed here, we can get a convection pattern at an arbitrary time. Fig. 9 is one of the transient flow patterns. In this

Fig. 9 Calculated transient flow and temperature on a basement slope with higher permeability value than that for Fig. 7 and 8. Parameters used are shown in Table 2. Elapsed time is 500,000 years.

case, the condition seems to have no steady state solution. An oscillatory convection is observed if we trace the change. But as far as the elements can deal the approximation, the procedure can get a result satisfying the physical equations. It seems that tracking natural physical pass to a state is the reasonable way for a numerical calculation.

6. Conclusion

A numerical discretization procedure for the simulation of two-dimensional hydrothermal convection was developed using FEM. It was implemented into a FORTRAN program and was verified by comparing its calculated flow pattern with published results. It was found to be applicable both to the simulation of the evolution of a geothermal system and to the calculation of its final steady state. It is based on general conservation equations having accumulation terms, and thus can avoid troubles on convergence which might occur in case the steady state is postulated in the program.

The procedure can easily be implemented into computer programs. The main routines would be those for forming coefficient matrices, assembling and solving the linearized equations. Miscellaneous subroutines for facilitating data input and output display may help practical use of the simulator.

There are many aspects remained in the research of liquid dominated geothermal systems, such as the effect of the shape and structure of the reservoir, the relation between heat source and the evolution of hydrothermal convection, *etc.* If there are enough exploration data that can specify parameters for a geothermal reservoir, the simulation study may be useful for further understanding of it. The procedure has a capability for treating them through approximations without excessive simplifications.

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Nomenclature

h	: fluid enthalpy
p	: fluid pressure
ρ	: fluid density
μ	: fluid dynamic viscosity
ν	: fluid kinematic viscosity
C	: fluid specific heat capacity
q_m	: mass source
q_h	: heat source
T	: temperature
λ	: thermal conductivity of reservoir
k	: permeability tensor
ϕ	: porosity
C_r	: rock specific heat capacity
ρ_r	: rock density
h_r	: rock enthalpy
g	: gravitational acceleration
D	: depth
t	: time
M	: element area
N_i	: interpolation function used in the Galerkin approximation
b_{ij}	: coefficients of interpolation function
β	: horizontal angle of the major axis of permeability tensor
θ	: coefficient associated with finite difference approximation of time derivative

Unit conversion

λ	: $10^{-3} \text{ cal}/(\text{cm} \cdot \text{s} \cdot ^\circ\text{C}) = 0.4184 \text{ W}/(\text{m} \cdot ^\circ\text{C})$
C	: $1 \text{ cal}/(\text{g} \cdot ^\circ\text{C}) = 4.184 \times 10^3 \text{ J}/(\text{kg} \cdot ^\circ\text{C})$
k	: $1 \text{ md} = 9.86923 \times 10^{-16} \text{ m}^2$
p	: $1 \text{ atm} = 1.01325 \times 10^5 \text{ Pa}$

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要 約

有限要素法による垂直 2 次元熱水シミュレー
ションのための実用的手続き

矢 野 雄 策

序 論

地熱資源評価において熱水流動の数値シミュレーションは重要な意義を持つ。熱水の物理的挙動を扱うには入力・実行が容易かつ効率的な 2 次元断面解析のシミュレータが有効であり、それには有限要素法が適している。

基礎方程式と離散化

熱水系に対する質量とエネルギーの保存を表す一般的な微分方程式が FAUST and MERCER (1979) によって示されている ((1)式および(2)式)。花岡 (1980) は、定常状態の液相单相熱水系をシミュレートするために、これらの方程式の時間依存項と蒸気に関する項を除き、3 角形要素を用いたガラキン有限要素法によって離散化する手順を示した。ここでは、系の変化状態も扱うために、MERCER and FAUST (1975) による時間微分の近似法を用い、通常の保存式の形で基礎方程式を離散化した。液相单相熱水系に適当である近似 ((3)式~(7)式) によってエネルギー保存式は簡略化される ((8)式)。両保存式に内挿関数を掛けエレメント領域で積分したものを解の対象となる方程式とし、部分積分を施す (質量

保存式について(10)式および(11)式)。未知数を圧力およびエンタルピとし、エレメント内は各ノードの値で近似する ((12)式および(13)式)。各時間の値で未知数およびその微分を近似すれば ((14)式、(15)式および(22)式)、求めるべき離散化方程式 (21) 式および (27) 式が得られる。これらの方程式の係数行列は内挿関数の微分と積分を用いて (30) 式~ (36) 式から求められる。

プログラミング上の考慮点

Fig. 2 は上記のように離散化した方程式を解くためのプログラムの流れ図である。大きく分けた手順として、データの入力、入力データの表示とチェック、内挿関数の計算、初期値のセット、方程式の演算、次の時間ステップへの移行があげられる。プログラムを実用化するためには、データ作成と計算に対して、様々な工夫が必要である。

実行とテスト

作成したプログラムに対し、由佐 (1983) と同様な入力データを用い、その結果との比較を行った (Fig. 3a, Fig. 3b, Fig. 4)。これによって、プログラムの正常動作を確認した。また、動粘度一定を仮定したこの結果と、粘度に近似関数 (Fig. 5) を用いた結果の相違も観察した (Fig. 6a, Fig. 6b)。

定常を仮定する手続きとの比較

ここで求めた最終的な離散化方程式 ((21) 式および (27) 式) から時間依存項を除けば、(39) 式および (40) 式が得られ、これは花岡 (1980) と一致する。このように、定常状態を方程式自体に仮定すると、本手法のように初期値から時間を追って求める (Fig. 7a-d) 最終的な定常状態を一挙に求められる (Fig. 8) が、反面、適当な計算開始値が予測できない場合や最終的な定常状態の無い場合等では計算の収束が得られない。通常の保存則を追ってシミュレートすれば、適当な空間近似、時間ステップが存在する限り、任意の時点の結果の収束値が得られる (Fig. 9)。