A System Evaluation for Construction Methods of Multiclass Problems using Binary Classifiers

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Abstract: Construction methods of the multiple classification systems using binary classifiers are discussed and evaluated by the system evaluation model based on rate-distortion functions. Suppose the multiple classification system constructed by \( M(\geq 3) \) categories and \( N(\geq M - 1) \) binary classifiers, then they can be solved by the matrices \( W \), where the matrices \( W \) are given by the table of \( M \) code words with length \( N \). Applying the bench-mark data (News paper articles of the 2000 Yomiuri Shinbun), the relationships between the probability of classification error \( P_r \) and the number of the binary classifiers \( N \) for a given \( M \) are investigated, and we show that the systems have desirable properties such as “Flexible”, “Elastic”, and so on.
<table>
<thead>
<tr>
<th>Functions</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>(f_0(x))</td>
<td>Classification function</td>
</tr>
<tr>
<td>(f_1(x))</td>
<td>Regression function</td>
</tr>
</tbody>
</table>

**Support Vector Machine (SVM)**

\[
\begin{align*}
\text{SVM} = & \sum_{i=1}^{N} \alpha_i y_i \langle x_i, x \rangle + b \\
= & \langle w, x \rangle + b
\end{align*}
\]

**Code Word Table**

<table>
<thead>
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<th>Code Word</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</tbody>
</table>

**Trivial Elastic**

\[d = r(d; L)\]

**Marginal Elastic**

\[E = \frac{\|w\|^2 + C}{\|\alpha\|^2}\]

**Effective Elastic**

\[E = \frac{\|w\|^2}{\|\alpha\|^2}\]
1. Exhaustive Code ($M=5, N_{\text{max}}=15, D=7$)

<table>
<thead>
<tr>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
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</tbody>
</table>

4.2 Exhaustive Code (exhaustive) ($M=5, N_{\text{max}}=15, D=7$)

4.3 Exhaustive Code (exhaustive) ($M=5, N_{\text{max}}=15, D=7$)

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5. Exhaustive Code ($M=5, N_{\text{max}}=15, D=7$)
5.1 Hamming

\[ p_e = \begin{cases} \frac{1}{2^m} & \text{if } m \leq 3 \\ \frac{1}{2^{m+1}} & \text{if } m \geq 3 \end{cases} \]

5.2 Reed-Muller (RM)

\[ p_e = \frac{1}{2^M} \]

5.3

\[ p_e = \begin{cases} \frac{1}{2^M} & \text{if } M = 0 \\ \frac{1}{2^{M+1}} & \text{if } M > 0 \end{cases} \]

\( M \)