Coupled Stick-Slip Simulation and Multifractal Analysis for Earthquakes

Atsushi Tanaka1,*

1Graduate School of Science and Engineering, Yamagata University, Japan

*tanaka@yamagata-u.ac.jp

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Abstract. The Great East Japan Earthquake on Mar. 11, 2011 in Japan was extremely huge and brought us enormous damage. In that earthquake, a lot of faults north and south were destroyed successively. Though many numerical models of earthquakes have been proposed, they are all inappropriate for many faults breakdown. Thus in this paper, a simple but interactive coupled stick-slip model based on 2-dimensional Burridge-Knopoff model is proposed. The emergence of catastrophic earthquakes can be found in our simulation. In addition multifractal analysis of earthquake data gives us new interpretation of earthquakes and the possibility of the prediction of catastrophic earthquake.

Keywords: Earthquake, Coupled stick-slip model, Multifractal analysis, Generalized dimension

1. Introduction

We have been troubled by a lot of earthquakes from the earliest times. In 1923, the Great Kanto Earthquake, its damage extended to all over Kanto area and more than 100,000 people were killed. Within recent years in the Indian Ocean Earthquake 2004 and the Great East Japan Earthquake 2011, not only huge shock of earthquakes but also huge tsunami caused enormous damage over a wide range. Though many researches of earthquakes have been made and it is clarified that the main origin is the crustal movement inside the earth, we have not reached to detailed clarification of its mechanism yet.

Research of earthquake is divided roughly into two categories, data analysis of past earthquakes and clarification of the mechanism by theory or simulation. Concerning the former, the abstraction of short-term features by the analysis of electromagnetic waves generated in earthquakes, the long-term prediction by the analysis of magnitude distributions, characterization of regions by the analysis of spacial distributions and so on have been made so far. The method of fractal analysis has been mainly used in those studies. Concerning the latter, many geological and physical researches have been made. To express a fault frac-
ture Burridge and Knopoff proposed simple and realistic model and they obtained important results and derivative models were subsequently proposed [1].

However since most of previous models concentrate only on one fault, they cannot express the breakdown of several faults like that of in the Great East Japan Earthquake. In addition, multifractal analysis is able to capture only uniform parts of phenomena, and is not able to analyze the characteristics of various distributions. Hence in this paper, we propose a model in which several faults are coupled to some extent, and investigate the mechanism of huge earthquakes from simulation results. Moreover using multifractal analysis which is the extension of fractal analysis, we aim to obtain new findings from the detailed analysis of earthquakes.

2. Several Models of Earthquakes

2.1. Mechanism of Earthquakes and Modeling

Since there is a mantle convection inside the earth, ocean plates are daily moving and subducted under continental plates. The strain energy accumulates gradually and reaches the limit, the it causes a fracture event, i.e. an earthquake. Simply thinking a moving ocean plate is thought to be a stick-slip that repeats long range stick and short time slip. However, this simple stick-slip is impossible to express the diversity of magnitude of earthquakes. In fact, speaking of the distribution of magnitude of earthquakes, the number of the occurrence of earthquakes $N$ whose magnitude $m$ are more than $M$ is known empirically to be estimated as follows,

$$\log N(m \geq M) = a - bM$$

where $a$ and $b$ are some constants and $M$ is the magnitude. This means smaller earthquakes tend to occur more exponentially and it is called Gutenberg-Richter rule [2]. In the area where earthquakes occur frequently, they say earthquakes occur periodically though, the periods varies considerably, so that makes the prediction difficult and it is impossible to be explained by a simple stick-slip. In 1985, Burridge and Knopoff proposed the stick-slip model using a lot of springs and blocks [1]. This model enabled us to explain Gutenberg-Richter rule and irregular periods, and it became the basis of several models.

2.2. Abstract of Stick-Slip Model

In the stick-slip model, one fault is expressed by one system. A moving floor plate expresses an ocean plate, and the intensity of the fixed springs expresses the strength of the rock of the continental plate. In 2-dimensional model, each block is connected to the ceiling plate and 4-neighboring blocks with springs, and the floor keeps moving unidirectionally. If it continues, the elastic force of the spring connected to some block exceeds the static friction force and begins to slip. Then according to the slip distance, some springs become to transfer the energy, and neighboring blocks begin to slide as a result. We consider this series of successive slides as one earthquake and the magnitude of earthquake is expressed by the number of slipped blocks.
2.3. Application of Stick-Slip Model and Other Previous Works

Since stick-slip model is relatively simple, several application and extension have been performed and proposed. For example, Osawa et al. showed that for the region where the actual magnitude distribution does not fit Gutenberg-Richter rule, the distribution can be expressed by changing its parameters. And they showed concrete aspect of the process of slides.

Because simulations of the stick-slip (Burridge-Knopoff) model require solving Newton’s equations of motion and they are time consuming, several cellular automaton (CA) models have been proposed that simplify the effect of the friction force and so on [4, 5, 6, 7, 8, 9]. In these models \( P(s) \), the distribution of the number of blocks in an event, does not exhibit power law scaling [10]. A generalization of these CA models to include more realistic long-range stress transfer [11, 12, 13] yields considerable differences with the nearest-neighbor CA models and with the original Burridge-Knopoff model. The CA and Burridge-Knopoff models lack several elements that would make them more realistic. Then Xia et al. proposed the generalized Burridge-Knopoff model with long-range stress transfer [14]. This model is more realistic than the original Burridge-Knopoff model and several long-range CA models because of the presence of inertia and dynamic friction force, which exhibits much richer scaling behavior.

3. Coupled Stick-Slip Model

As described in the previous section, they deal with only one fault and observe temporal, spacial and scale distributions in the past models. However in the Great East Japan Earthquake 2011, the succession of dislocation earthquakes brought us such a huge disaster. There are a lot of faults in the world, especially almost everywhere in Japan and surroundings, and there sometimes happen the pull-in effects in huge earthquakes. Thus we have proposed a new model with coupled faults in order to investigate the mechanism of huge earthquakes induced by the pull-in effect and especially their periodicity [3].

In the previous work, we assume that each fault system is 2-dimensional stick-slip model with 50 \( \times \) 50 geometry, and has one moving plate. For simplicity we ignore the effect of friction forces. Neighboring systems overlap vertically to some extent each other and their vertical blocks connect with springs. That means an earthquake event occurred in some system affect other neighboring systems though, only two systems were coupled, and we could not obtain definite result. Hence we increase the number of coupling faults up to 5, and enlarge the geometry up to 100 \( \times \) 100. The schematic diagram of the model is shown in Fig.1.

3.1. Numerical Simulation and Results

In simulation, first we set placements of all blocks 0, and fix the static friction of them \( F_N \) as \( 5.0 \times 10^5 \). Our simulation procedure is as follows.

1. Set static frictions for 100 \( \times \) 100 blocks.
2. Set spring constants of springs between the ceiling plate and blocks and between blocks.

3. For each block, calculate the sum of elastic force of surrounding springs, and generate the slip if it is larger than the static friction.

   (a) For slipping blocks, calculate the displacement of the cell after the slip.

   (b) Check whether all other blocks would slip again.

   (c) Count the number of new slipped blocks, and repeat (a) and (b) until it becomes 0.

4. Record the sum of released energy.

5. Increase the displacement of all blocks by 1.

6. Repeat from 3 to 5.

   We consider the procedures from 3 to 5 as 1 step and carry out simulation until 5 \times 10^5 steps. Here spring constants are given by random number according to normal distribution with the mean \( k \) and the variance \( \sigma_k^2 \), and we vary \( k \) and \( \sigma_k \) and measure the change of the periodicity and \( b \) value in (1).

   Two neighboring systems differ only by the speeds of the moving plates and we vary the ratio \( v_A/v_B \) from 1 to 0.95. We also vary the number of rows of overlapping blocks from 5 to 40, and the spring constants between them are as half as those in the system. In our simulations, the number of faults is fixed as 5, thus the total blocks in the system is 50,000, and we define the term huge earthquake as an earthquake with more than 500 slipped blocks. Here we show the time evolution of the scales of earthquakes, that is the number of slipped blocks, in the coupled system with the fixed ratio \( v_A/v_B = 0.95 \), and the blue dotted lines indicate the threshold of huge earthquakes (Fig. 2). Though we can confirm new larger scales
of earthquakes with a longer quasi-periodic motion (Fig. 2(a)), in the stronger coupled case no huge earthquakes are found except the starting point (Fig. 2(b)). In contrast, if we vary the ratio of the moving plates $v_A/v_B$, no noticeable changes can be found in simulations (Fig. 3).

Figure 2: Time evolution of the scales of earthquakes for different coupling blocks.

Figure 3: Time evolution of the scales of earthquakes for different speed ratio of moving plates.

4. Analysis of Earthquake Data

4.1. Abstract of Fractal

The idea of fractal proposed by B. Mandelbrot in 1975 enabled us to extend smoothly the long existing concept of repeated structure [15]. The character has been found not only in mathematics by also physics, chemistry, biology and other natural phenomena, and that has helped us to clarify the pattern formation and nonlinear phenomena in nature. Moreover, many other fractal objects have been found not only in natural phenomena but also in artificial things or human activities, e.g. the distribution of human population, the frequency of
occurrence of words in sentences, errors in electrical communication and the movement of stock change. The concept of fractal dimension which characterize fractal objects proposed by B. Mandelbrot is a natural extension of special dimension, and it is a very effective quantity to measure the complexity. Fractal dimension, which is extended to non-integer number different from usual spacial dimension, has become to be great contributory to compare several scales of patterns. This method to weigh several things using fractal dimension is called fractal analysis, and an effective methodology in medical and engineering fields.

4.2. Multifractal

The concept of fractal hypothesizes the repetition of the some structure, and expresses the whole with one index. However most natural phenomena are not always homogeneous, so they cannot be measured only by fractal analysis. For example, in Fig.4 both have the same fractal dimension, but (b) is more inhomogeneous clearly. In addition, if some physical quantity distributes on a fractal space, we cannot take into account these weight(Fig.5). So Halsey et al. extended fractal into the concept of multifractal [16].

![Koch curve](image1.png) ![Wakasa bay in Fukui prefecture](image2.png)

Figure 4: The difference between homogeneous and inhomogeneous fractal objects.

Now we extend fractal dimension to define multifractal. Similar to the box-counting method which is used to measure fractal dimension, we cover the pattern with one block, then divide it into small blocks. In fractal measurement, we only count the number of blocks which contain the part of the pattern at each step. With that method, the information how much parts of the pattern are contained is missing. Thus in multifractal measurement, we calculate the probability measure in each step. The probability measure $P_i(\epsilon)$ of $i$-th block, $\epsilon$ on a side is defined as follows,

$$P_i(\epsilon) = \frac{\text{Sum of physical quantities on pattern in } i\text{-th block}}{\text{Whole sum of physical quantities on pattern}}.$$  \hspace{1cm} (2)

Then we calculate the distribution function $Z_q(\epsilon)$ using $P_i(\epsilon)$,

$$Z_q(\epsilon) = \sum_{i=1}^{N(\epsilon)} [P_i(\epsilon)]^q.$$  \hspace{1cm} (3)
where $N(\varepsilon)$ is the number of blocks, $\varepsilon$ on a side. From the definition, the distribution function is the sum of $q$-th power of the probability measure, so the larger $q$ becomes, the more dominant the term with large probability measure becomes, and vice versa. That is, we can characterize the probability distribution by adjusting the value $q$.

If the pattern is fractal, in the limit of $\varepsilon \to 0$, the distribution function is scaled with $q$ dependent exponent $\tau_q$ as follows,

$$Z_q(\varepsilon) \sim \varepsilon^{\tau_q} \quad (\varepsilon \to 0).$$

(4)

Since $Z_1(\varepsilon) = \sum_{i=1}^{N(\varepsilon)} P_i(\varepsilon) = 1$ when $q = 1$, $\tau_q$ becomes 0. Thus if we define $\tau_q$ as,

$$\tau_q = (q - 1)D_q,$$

(5)

then from (4) and (5), we obtain

$$D_q = \frac{1}{q - 1} \lim_{\varepsilon \to 0} \frac{\ln Z_q(\varepsilon)}{\ln \varepsilon}.$$

(6)

$D_q$ is called generalized dimension, and a quantity which characterize the probability distribution of patterns, and one of the most important values in multifractal analysis. If $q = 0$, the distribution function becomes the number of blocks which contain pattern, so $D_0$ is equal to fractal dimension.

### 4.3. Some Previous Works using Multifractal Analysis

As mentioned in the previous section, multifractal analysis can treat more features of patterns than fractal analysis. It has been applied to not only physical phenomena but also biology, economics and etc., and has produced a lot of results. Here we will show some examples.

At first, though it is well-known that brain waves have fractal and chaotic characters, it is so irregular and weak that its chaotic characters cannot be expressed by only one scaling
exponent. So Oga et al. showed each frequency element of brain waves has a multifractal characters and referred its features [17].

In economics, Ohnishi et al. proceeded multifractal analysis of price movement of stock during 31 days, and correlated it with up and down movement of 25 days after, they insisted that they can predict the improvement in stock price up to 80% probability [18].

In pattern formation, Kashiwaya et al. proceeded multifractal analysis of crack patterns of rocks, and found several features which cannot been found using fractal analysis, and concluded they can estimate geometrical difference of patterns definitely [19].

Furthermore, multifractal analysis has been performed in earthquake study for a long time. In 1991, Hirata showed the spacial distribution of earthquakes has a multifractal property and exhibited the effectiveness of multifractal analysis in earthquake study [20]. In recent years, Roy et al. insisted that they could estimate the possibility of future catastrophic earthquakes in Iran and its neighboring region based of temporal and spacial clustering patterns of earthquakes [21]. After that, they calculated the region in Himalaya where a cluster of small earthquakes becomes a core and can cause a large earthquakes [22].

4.4. \(D_q\) Spectrum

Generalized dimension \(D_q\) can be calculated every time \(q\) is fixed, so \(D_q\) is a function of \(q\) and the graph of \(D_q\) is called \(D_q\) Spectrum. From its definition if \(q\) is large, the spots where the probability measure is high are emphasized and if \(q\) is small then vice versa. The larger the change of the probability measure is, the larger that of the spectrum is. Therefore we can measure the variation of complexity by observing the variation width of the spectrum. In uniform fractal, \(D_q\) spectrum does not change by \(q\), hence it comes to fractal dimension \(D_q = D_0\).

As an example of \(D_q\) spectrum, we show in Fig.6 the analytic result of M1.0 ~ 3.0 earthquakes in Tohoku region in April and May 2003. Both spectra vary like S-shaped curves, however the latter(green line) bends more and its variation is large. This change is considered to be the resulting consequence of M7.1 earthquake in April that year.

5. Multifractal Analysis of Earthquakes

5.1. Analytical Data and Method

Earthquake data we use in this paper is the meteorological bureau centralization earthquake center list contributed by National Research Institute for Earth Science and Disaster Prevention. We picked up the occurrence data, latitude, longitude, magnitude from 1,534,152 earthquakes between 2002/1/1 and 2012/6/30. As described in the previous section, since \(D_q\) spectrum bends like S-shaped curve and it is characterized by \(D_{-5}, D_5\) and \(D_0\), we pay attention to them for our analysis. We selected special regions for analysis eastern part of Hokkaido, Kou-Shin-Etsu and Pacific coast in Tohoku where they encountered large earthquakes last decade.
Figure 6: Examples of $D_q$ spectrum in Tohoku region. The red and green lines mean results in April and May 2003 respectively.

5.2. Time Series Behavior of Generalized Dimension

To observe the change of generalized dimension which characterize the distribution with time means to investigate the change of characters of earthquakes, and that might result in the prediction of earthquake consequently. We accumulated earthquake data of these regions every month and investigated the time series behaviors of $D_{-5}, D_0$ and $D_5$ using multifractal analysis. We show the results in Tohoku regions in Fig.7. The red, green and blue lines in the figure mean $D_5, D_0$ and $D_{-5}$ respectively. The vertical lines mean 5-largest earthquakes. We can tell $D_{-5}$ rise and $D_5$ descend after large earthquakes as a whole. However after M9.0 catastrophic earthquake $D_5$ does not descend. Much the same true on M8.0 earthquake in Hokkaido. On the other hand, $D_0$ does not change with time.

5.3. Change of Generalized Dimension on Analytic Ranges

In a real earthquake, the range where the faults were destroyed is called the focal region, and it can be identified almost precisely using GPS. You know of course the shock of an earthquake spread widely from there. Moreover in a large earthquake, successive afterquakes are also large and the damage of them cannot be neglected. Hence to identify the region where afterquake tend to take place, that is called aftershock area, is very important from the point of view of disaster prevention and policy though, the clear definition has not been given yet. Here we investigate the change of generalized dimension by changing analytic ranges, and try to define the aftershock area more precisely.

Figure 8 shows the changes of the number of earthquakes(green) and $D_5$(red). In ordinary days(Fig.8(a)), if we narrow the analytic ranges $D_5$ does not change that much to some
degree though, it decreases suddenly from some range. On the contrary, in a large earthquake (e.g. Sanriku 2003, Fig.8(b)), sudden increase of $D_5$ can be found. Same tendency can be found in other large earthquakes as a whole. Such a clear difference between change of $D_5$ means that the appearance of the distributions of earthquakes from that boundary. Hence it is natural to define the aftershock area as the range where we can see sudden increase of $D_5$. Here we partitioned whole region into squares 11km on a side, and colored them according to the values of $D_5$ (Fig.9). Before large earthquakes, they all have low $D_5$, hence we can simply and clearly define the aftershock area as high $D_5$ value area.

6. Discussion

In this section, we have some discussions on simulation and analysis. At first concerning simulation of coupled stick-slip model, we obtained results of the effect of couplings of one fault model. Of course, the method of coupling is so simple that we should describe the interaction between faults more precisely, however we obtained some degree of positive significance as a first step. The decrease of huge earthquakes in higher coupled system can be explained as follows. If the number of overlapped blocks are large enough, the stress releases happen more often, thus huge earthquakes are hardly found.

Concerning data analysis, generalized dimension has the property of emphasizing large and small magnitude places of earthquakes and it exhibits a characteristic change in all three regions in this paper. Since $D_0$ corresponds to fractal dimension and it is almost constant, it is clarified that fractal analysis is insufficient only by itself for the analysis of earthquakes. The fact that $D_5$ does not decrease in M9.0 and M8.0 large earthquakes can be explained as follows. In earthquakes around M7.0, the regions of large energy release concentrate only
on the neighborhood of earthquake center. Then $D_5$ which is the measure to emphasize the places of large energy release turns out small as a result. On the other hand, in larger than M8.0 earthquakes, earthquake centers widely spreads and never concentrate, thus $D_5$ does not change.

If we narrow the analytic regions of generalized dimension, we can confirm clear difference of the change of $D_5$ between ordinary days and earthquakes. In this paper, we define the aftershock area as the region with large $D_5$ over some threshold, as a result it is possible to specify the aftershock area more precisely comparing with the present definition and quantitative estimates can be proceeded. So far due to the ambiguity of the definition of the aftershock area, there happened the claim for earthquake insurance several times. Of course the aftershock area will change depending on the threshold, so we should consider what threshold is appropriate qualitatively. Moreover precise identification of the aftershock area can be obtained using finer partitioning.

Finally, it is reported that large afterquakes tend to occur at the edge and the neighborhood of the aftershock area. Therefore if we can determine clear and precise definition of the aftershock area, it is expected much of leading to the prediction of earthquakes. Moreover we can develop our simulation model from the analytic results of earthquake data.

7. Conclusion

In summary in order to investigate the mechanism of the occurrence of huge earthquakes induced by the pull-in effect such as the Great East Japan Earthquake and their periodicity, we improve our model, coupled stick-slip model in several ways, the geometry, the number of overlapping blocks and etc. As a result the pull-in effect of earthquakes is observed and the emergence of new quasi-periodicity of huge earthquakes is confirmed in the less overlapping system. The more the number of overlapping blocks increases, the more often
the stress are released, thus few huge earthquakes are found as a result.

In analysis of earthquake data, we proceeded multifractal analysis instead of fractal one, then we succeeded to find the difference between large earthquakes and huge ones using generalized dimension $D_q$. Moreover we showed the effectiveness of new definition of the aftershock area using the same measure.

For future works, we need to explain the occurrence of huge earthquakes in detail, and confirm the validity of our simulation comparing with real earthquake data for a long period. And we will analyze earthquake data using another characteristic index $f(\sigma)$, which is defined as fractal dimension of the spacial distribution of scale characteristics $\sigma$ at each local point. It is an important global quantity characterising the distribution of locally defined characteristic exponents(see [23]).

References


