Parameter optimization of a 3D coastal model using Green's functions for modelling river plume dynamics

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Abstract. River plumes flowing into channels with strong tidal currents cause pulsed decreases in the salinity around the river mouth. This study constructs a numerical model for reproducing the river plumes of Asahi and Yoshii Rivers flowing into Bisan Seto, a tidal channel in the Seto Inland Sea. Recently inverse estimation of the model parameters using Green's functions has been applied to numerical modelling of sea shelves as a simple and effective method of parameter optimization. This paper examines the method that aims to improve the precision of the model results, but uses a model domain (ca. 50 km) that is much smaller than in previous studies. The reproducibility of the tidal elevations and currents was improved significantly by the parameter optimization procedures. The reproducibility of the temporal changes in salinity was also improved, although they showed strong nonlinearity. These results indicate that the inverse estimation of model parameters using Green's functions can greatly improve the reproducibility of the modelling of river plumes flowing into tidal channels.

Keywords: River plume, Seto Inland Sea, Parameter optimization, Green's function

1. Introduction

In coastal seas with convoluted shorelines, tidal elevations and currents vary greatly. The

Seto Inland Sea, Japan, resembles an archipelago in that it has a number of narrow straits where strong tidal currents are generated. Any river plume flowing into such a strait is strongly influenced by tidal currents, and as a result, its flow paths change significantly and it quickly disperses. Bricker *et al.* [2] call such a river plume a “pulsed river plume”.

The river plumes flowing into the Seto Inland Sea play an important role in supplying nutrients essential for seaweed culture, which is a major coastal industry. Takagi *et al.* [12] continuously monitored salinity at a number of stations in an area of seaweed culture in Okayama prefecture, and fully explored the pulsed river plume there. High-precision modelling of the river plume is the next step required if we are to forecast the supply of nutrients and so help to manage river discharge.

The most important element in the realistic reproduction of the tidal currents in the small-scale model domain is the accuracy of the tidal elevations given at the horizontal boundaries. Parameters such as bottom friction must be carefully selected to obtain precise tidal currents. Here, we use Green’s functions to determine an optimal set of parameters for modelling river plumes, following the studies of Menemenlis *et al.* [8] and Hirose [5].

This method estimates the optimized values using linear least squares. Moon *et al.* [9] used this method and succeeded to reproduce tidal levels in East Asian marginal seas. The model domain in this study (ca. 50 km) is much smaller than was used in these previous studies, and our aim here is to reproduce the pulsed change in salinity, which shows strong nonlinearity.

We examine here the applicability of using Green’s functions in a small-scale coastal model, and evaluate the reduction of the cost functions. Through this approach we construct a numerical model to reproduce the river plumes of Asahi and Yoshii Rivers flowing into Bisan Seto, a tidal channel in the Seto Inland Sea.

2. Model configuration and data for model optimization

This study applies a numerical model, the Princeton Ocean Model with a generalized coordinate of sigma (POMgcs; [7]), which combines the z- and sigma-coordinate to simulate tidal currents and the salinity distribution. The model domain is shown in Figure 1b. This is the first case to apply POMgcs to simulate tidal currents and river plume around Kojima Bay, while tidal and residual currents in the Seto Inland Sea are simulated using POM [3] in previous studies (e.g. [6, 4]). The Coriolis parameter is $8 \times 10^{-5} \text{ s}^{-1}$ at a latitude of 34° . The horizontal grid size is 1 km^2 , and eight non-uniform sigma levels are used in the vertical direction. The depth data were obtained by smoothing from bathymetry data (ca. 500 m) supplied by the Japan Ocean Data Center. Since the main aim of this study was to simulate tidal currents and river plume in a short time and to examine the applicability of the parameter optimization using Green’s functions, the grid resolution was set to be a lower limit. The model

was forced by the four tidal constituents of surface height along the eastern and western open boundaries (K_1 , O_1 , M_2 , and S_2 , shown by broken lines in Figure 1b). The tide data at eastern (Sanbonmatsu) and western (Marugame) boundaries were derived from harmonic constants data provided by the Japan Coast Guard. As the model aims to reproduce a river plume controlled mainly by tidal elevation and currents, we did not set wind as a driving force here.

Harmonic constants of tidal elevations and currents derived from five tide gauges (Ushimado, Kogushi, Uno, Jozozaki, Obe) and three current time series at 5 m from the surface (locations in Figure 1b) provided by the Japan Coast Guard were used for the optimization of the model. Salinity was measured every 10 min from 7th to 31st December 2007 by instruments moored at 0.5 m from the surface at 16 stations [12] and we used salinity data at stations 4 to 9 and 11 to 13 (locations in Figure 1c) for the optimization.

Figure 1: (a) Location and (b) map of the Seto Inland Sea. The computational domain is surrounded by a black border, and the eastern and western open boundaries are shown by broken lines. The triangles and rectangles indicate the locations of the stations where the harmonic constants of the tidal currents and tidal levels, respectively, were obtained. The white rectangles indicate data used for the boundary conditions, and the black rectangles indicate data used for the parameter optimization. (c) Enlarged view of the study area. The circles numbered 1–16 mark the salinity monitoring stations. Star marks (*) in Figures 1(b) and 1(c) show the stations where the modeled and observed results are shown in Figure 4.

Figure 2 shows an example of the observed salinity variation with tidal elevation (derived from the tide gauges) and daily mean river discharge (observed by the Ministry of Land,

Infrastructure Transport and Tourism). Salinity measured at all 16 stations showed pulsed decreases at every low tide, while several peaks coalesced into a single peak only when the river discharge increased (e.g., over the period 23–25 December 2007; Figure 2a). The magnitude of the decrease in salinity during neap tides was less than that associated with the spring tides, even though river discharge showed little variation over the neap–spring cycle.

The calculation period was set to one month from 1st December 2007, and the calculated results after 10th December were used for the optimization of the model and the discussion.

Figure 2: Temporal variations of (a) discharge ($\text{m}^3 \text{s}^{-1}$) from the Asahi and Yoshii rivers (these locations are shown in Figure 1b). (b) Salinity observed at station 6 (shown in Figure 1c) and tidal elevations (m) over the period 7–31 December 2007.

3. Parameter optimization using Green's functions

We first conducted a baseline integration using the default physical parameters: horizontal eddy viscosity and diffusion, vertical eddy viscosity, bottom drag coefficient, and harmonic constituents obtained by observation described in Section 2 at the eastern and western open boundaries. We then conducted the sensitivity experiment with perturbations in both the bottom drag coefficient and the harmonic constituents along the open boundaries (Table 1). In

addition to these parameters, the stable increment of water level at the western boundary was also perturbed as a controllable factor, following the distribution of tidal energy flux in the Seto Inland Sea [6].

Table 1: List of sensitivity experiments

Experiment	Parameter	Baseline	Perturbation
1	M ₂ amplitude at western boundary (%)	100	90
2	M ₂ phase at western boundary (°)	0	-10
3	S ₂ amplitude at western boundary (%)	100	90
4	S ₂ phase at western boundary (°)	0	-10
5	K ₁ amplitude at western boundary (%)	100	90
6	K ₁ phase at western boundary (°)	0	-10
7	O ₁ amplitude at western boundary (%)	100	90
8	O ₁ phase at western boundary (°)	0	-10
9	M ₂ amplitude at eastern boundary (%)	100	90
10	M ₂ phase at eastern boundary (°)	0	-10
11	S ₂ amplitude at eastern boundary (%)	100	90
12	S ₂ phase at eastern boundary (°)	0	-10
13	K ₁ amplitude at eastern n boundary (%)	100	90
14	K ₁ phase at eastern boundary (°)	0	-10
15	O ₁ amplitude at eastern boundary (%)	100	90
16	O ₁ phase at eastern boundary (°)	0	-10
17	addition of water level at west. bound. (m)	0.00	0.02
18	bottom drag coefficient ($\times 10^{-3}$)	2.6	3.6

The differences between the baseline and sensitivity experiments essentially provide the column vectors of the Green's functions matrix [5]. Here, we created the Green's functions matrix for six cases using the combinations of selected datasets for tidal elevation, current, and salinity (optimizations A–F; Table 2).

The linear inversion method of Menemenlis *et al.* [8] was applied and the solutions for the 18 parameters for each optimization A–F are given in Table 3. The optimized bottom drag coefficients are all larger than the default value of 2.6×10^{-3} . The optimized increment of water level at the western boundary remained stable at 0.4–0.5 cm in all cases that used tidal currents and salinity to generate the Green's functions matrix (i.e., optimizations B–F). The

optimized amplitude and phases at the eastern and western open boundaries were determined with little variation among the cases. The solutions were independent of the values for perturbation in the sensitivity experiments (shown in Table 1), for example, the optimized values did not change when the values for perturbation were set to be 110 % for the amplitude or +10 degree for the phases.

Table 2: List of optimizations

Case	Data used for making Green's functions matrix
Baseline	—
Optimization A	tidal elevations
Optimization B	tidal currents
Optimization C	tidal elevations + tidal currents
Optimization D	salinity
Optimization E	salinity + tidal currents
Optimization F	tidal elevations + tidal currents + salinity

Table 3: Optimized values of factors and increments used to obtain optimized parameters for optimizations A–F (shown in Table 2).

Parameter	Optimized values					
	Op.A	Op.B	Op.C	Op.D	Op.E	Op.F
M ₂ amplitude at west. bound. (%)	8.7	95	88	99	98	97
M ₂ phase at west. bound. (°)	-5.1	2.0	0.2	0.1	0.0	-0.5
S ₂ amplitude at west. bound. (%)	86	97	92	83	84	83
S ₂ phase at west. bound. (°)	-5.6	6.1	-4.4	10.4	8.8	7.6
K ₁ amplitude at west. bound. (%)	100	91	104	97	95	100
K ₁ phase at west. bound. (°)	-4.8	1.2	2.2	0.7	0.0	1.5
O ₁ amplitude at west. bound. (%)	110	104	101	114	113	110
O ₁ phase at west. bound. (°)	-1.8	-0.5	1.9	1.6	2.3	0.9
M ₂ amplitude at east. bound. (%)	93	117	96	108	108	104
M ₂ phase at east. bound. (°)	-11.5	-11.5	-15.3	-16.0	-13.7	-14.0
S ₂ amplitude at east. bound. (%)	101	106	100	156	149	148
S ₂ phase at east. bound. (°)	-26.5	19.5	-25.6	-7.7	-5.0	-9.1
K ₁ amplitude at east. bound. (%)	111	93	108	82	83	88

K ₁ phase at east. bound. (°)	-1.0	-4.3	-2.5	-10.6	-10.9	-8.5
O ₁ amplitude at east. bound. (%)	97	106	102	97	98	95
O ₁ phase at east. bound. (°)	-2.8	-7.2	-4.7	1.0	0.9	-0.4
addition of water level at west.bound. (m)	-0.012	0.004	0.005	0.004	0.004	0.005
bottom drag coefficient (× 10 ⁻³)	3.1	3.9	3.9	4.8	4.3	4.3

Calculations using the optimized parameters were then run, and the cost functions for tidal elevation, current, and salinity were calculated using the observed and modelled results for each optimization case.

The cost functions for tidal elevation, current, and salinity (J_E , J_C , and J_S , respectively) are defined as

$$J_E = \sum_{n1,t} \frac{(E_{n1,t}^O - E_{n1,t}^M)^2}{R_{n1,t}^E},$$

$$J_C = \sum_{n2,t} \frac{(U_{n2,t}^O - U_{n2,t}^M)^2}{R_{n2,t}^U} + \sum_{n2,t} \frac{(V_{n2,t}^O - V_{n2,t}^M)^2}{R_{n2,t}^V},$$

$$J_S = \sum_{n3,t} \frac{(S_{n3,t}^O - S_{n3,t}^M)^2}{R_{n3,t}^S},$$

where E , U , V , and S are the elevation, eastward and northward velocities, and salinity, respectively; the superscript O represents observational data measured for their hourly mean zonal and meridional components; the superscript M represents the model's equivalents; and the subscripts $n1$, $n2$, $n3$, and t indicate measurement identities ($n1 = 5$, $n2 = 3$, $n3 = 9$) and temporal indices, respectively. Data at 0.5 m from the surface for salinity and 5 m for tidal currents were extracted from the model results. The number of the stations where salinity was measured was 16 as shown in Figure 1c, while we used salinity data at stations 4 to 9 and 11 to 13 which are located in the region where the reproduction of salinity are needed. We used every 1 hour data for 20 days (10-30 December 2007). The error variances ($R_{n1,t}^E$, $R_{n2,t}^U$, $R_{n2,t}^V$, $R_{n3,t}^S$) are assumed here to be the same as the data variance. The assumption is based on the adjustment of R , according to the fundamental rule of data assimilation that cost function J should be the same order as degrees of freedom [1]. Menemenlis [8] showed that three cases which error variance is equal to the data variance, equal to the variance of the model-data difference and proportional to it, give similar estimates.

The cost functions of the tidal currents significantly decreases in all of the optimization cases (Table 4), with a notable and substantial decrease shown in the case that considered only salinity in the formulation of the Green's functions matrix (optimization D). The cost func-

tions for salinity decrease in all cases, but the amount of decrease is relatively small in the case that used only tidal elevation to generate the Green's functions matrix. Also, note the clear decreases in the cases that used tidal current to generate the Green's functions matrix (optimizations B–C). Although the cost functions of tidal elevation decrease in some cases and increase in others, the values in all cases were much smaller than those of tidal current and salinity, because the inconsistencies between the observed and modelled tidal elevations were not significant originally.

Table 4: Cost function and its reduction relative to the baseline experiment

Case	Cost functions		
	tidal elevation	Tidal currents	salinity
Baseline	227	1961	4200
Optimization A	191 (16 % ↓)	1415 (16 % ↓)	4097 (2 % ↓)
Optimization B	304 (34 % ↑)	981 (50 % ↓)	3856 (8 % ↓)
Optimization C	185 (19 % ↓)	1022 (48 % ↓)	3901 (7 % ↓)
Optimization D	307 (35 % ↑)	1087 (45 % ↓)	3760 (10 % ↓)
Optimization E	303 (33 % ↑)	1100 (44 % ↓)	3792 (10 % ↓)
Optimization F	266 (17 % ↑)	1112 (43 % ↓)	3810 (9 % ↓)

4. Parameter optimization using Green's functions

The river plume calculated using the optimized parameter (optimization C) is shown in Figure 3. The low-salinity area indicated by dark shading moves southward during the ebb tide, and moves toward the east until low tide. It then moves west after low tide, because the tidal currents are directed to the west during the flood tide. The dark shading area disappears from the model domain until the next ebb tide comes. The changing low-salinity area modelled here is qualitatively consistent with the results of field observations in this area [12], and the next paragraph gives the evaluation of the consistency.

Figure 4 shows the improvement in the reproducibility of the river plume by the optimization. The phase of the tidal elevations and the amplitude of the tidal currents after optimization are closer to the observed results than the values in the baseline experiment. The mean root-mean-square (RMS) errors between the optimized model results and observations for the tidal elevations at five stations and currents of two components at three stations every 1 hour between 10th and 30th December 2007, are 0.12 m and 0.11 ms⁻¹, respectively. The phase of the minimum of salinity at low tide and the range of the decrease in salinity, are also clearly improved after optimization. These improvements at some stations are probably the main

reason for the reductions in the cost functions for salinity. The mean root-mean-square (RMS) errors between optimized model results and observations for salinity every 1 hour between 10th and 30th December 2007, are 0.84 psu at the stations as same as these used to calculate cost function. It is important to remember, of course, the peaks of salinity in the model results did not perfectly agree with the observed results, but the reproducibility of the model was not bad as compared to the previous study which modeled river plumes (e.g. [10, 11]).

Figure 3: Horizontal distributions of salinity and vector plot of tidal currents (cm s^{-1}) from 02:00 to 13:00 on 26th December 2007, modelled using optimized parameter (optimization C). The numbers on each panel indicate the time in hours from low water at Uno (shown in Figure 1b).

Figure 4: Temporal variations of (a) tidal elevations (m), (b) tidal currents, and (c) salinity from 00:00 on 25th to 12:00 on 27th December 2007 at the stations shown in Fig. 1(b) and 1(c) by star marks. Positive and negative values of tidal currents represent eastward and westward movements, respectively. The black, white, and bold black lines indicate the observed results and those calculated for the baseline case and optimization C, respectively.

5. Summary and discussion

This study examined the applicability of the Green's functions approach to a coastal model with smaller spatial scale than used in previous studies. The cost functions of tidal elevation and current reduced after optimization of the model parameters, suggesting that the Green's functions improved model performance. The cost function of salinity at the river mouth also reduced following the optimization of tidal elevation and current, and the pulsed changes of salinity and the tidal cycle of the river plume were well reproduced.

We also compared the optimized results in cases using tidal elevation, current, and salinity separately to generate the Green's functions matrix. The reproducibility of the pulsed decrease in salinity improved when the optimization considered tidal elevation and current data. The accuracy of the tidal current was also improved in the case that used only salinity to generate the Green's functions matrix. These findings suggest that salinity variations were caused mainly by tidal currents—which show linearity—and that the Green's functions approach can accurately reproduce these variations in salinity, despite their strong nonlinearity.

Here, we only applied tidal elevation at the lateral boundaries of the model to simulate the river plume under the assumption that it is mainly controlled by tidal elevation and currents, and the model successfully reproduced the river plume. We recognize, however, that the minor inconsistencies between the modelled and observed results imply that the driving force other than tidal currents, such as wind and density currents, may influence on temporal changes in salinity. The future simulation of these currents, which are typically 1–2 orders of magnitude smaller than the tidal currents and also irregular, will require the optimization of their related model parameters in addition to those related to tidal currents.

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