Three-Dimensional Shape Optimization of Claw-Pole-Motors

Yuki Hidaka^{1,2*}, Hajime Igarashi²

¹Advanced Technology R&D Center, Mitsubishi Electric Corporation, 661-8661, Japan

²Graduate School of Information Science and Technology, Hokkaido University, 060-0814, Japan

*Hidaka.Yuki@ea.MitsubishiElectric.co.jp

Received: March 16, 2017; Accepted: December 20, 2017; Published: March 31, 2018

Abstract. This paper presents three-dimensional shape optimization of claw-pole motors. In the present method, the iron core of a claw-pole motor is subdivided into uniform voxel finite elements. The material attribute of each element, {1(core), 0(air)}, is determined from the shape parameters which vary in the optimization process. Thanks to this approach, time-consuming for mesh generation in the optimization process can be avoided. Moreover appropriately optimization algorithm is selected for reducing the computational costs. The core shape of a claw-pole motor is optimized so that the induced voltages at low and high speed conditions are maximized.

Keywords: Finite element method, claw-pole motor, voxel finite element, shape optimization

1. Introduction

Because of considerable increase in electric loads of automotives, improvements in the performance of automotive alternators have been strongly required. Claw-pole motors, which have three-dimensional forged iron cores with complicated geometry, are widely used for the alternator. To improve the performance of claw-pole motors, which has been designed mainly based on the knowledge and experience of engineers, design optimization of the iron core structure based on three-dimensional electromagnetic field analysis would be effective [1-3].

Stochastic population methods such as the genetic algorithm (GA) has widely been used for optimization of core structures in rotating machines. In the parameter optimization of motors using GA, finite element meshes are repeatedly generated to modify the core shapes [4-5]. Although this approach is effective for two-dimensional optimization, it would be too time consuming to design the three-dimensional structure of claw-pole motors.

In this paper, we propose a shape optimization method for claw-pole motors. In this method, the rotor shape of a claw-pole motor is expressed by shape parameters. The claw-pole motor is subdivided into uniform voxel finite elements (FEs) [6-7], and the material attribute of each element, $\{1 \text{ (core)}, 0 \text{ (air)}\}$, is determined from the shape parameters. Because this method does not need the time-consuming for mesh generation in the optimization process, it is suitable especially for three-dimensional optimizations. There are no previous works in which three dimensional complex shapes as rotor of claw-pole motors are divided into voxel FEs. By using voxel FEs for parameter optimization of claw-pole motors, the design concept of claw-pole motor can be obtained with low calculation costs. From this point, the optimization model of this paper is defined by selected parameters which mainly affect to the performance of claw-pole motors. Moreover, we employ the micro GA (μ GA) [8] for optimization. Gradient based optimization methods are effective for reducing the calculation costs of optimization process. However, the performance of the methods are lower than that of stochastic based optimization methods. Since μ GA needs small number of individuals in search of the optimal solution, computational costs for the fitness evaluation can be reduced. In addition to this, searching process is based on stochastic theory. For this reason, the solutions which have higher evaluation than that of the gradient based methods can be obtained by using μ GA.

The present method is applied to design of the three-dimensional core structure of a claw-pole motor. It will be shown that we can improve performance of the claw-pole motor at low and high speed conditions.

2. Analysis of Claw-pole motors

2.1. Formulation

Figs. 1 and 2 show the equivalent circuit of an automotive alternator and the rotor of the claw-pole motor used for the alternator. The magnetic fields, generated in the rotor core by the supplied DC current, interlinkage with the stator windings. The rotation of the rotor induces voltages in the stator windings. To improve the power generation characteristic, the induced voltage of the stator windings should be increased. We start with a simple optimization problem; the line voltage of the open circuit is maximized, which allows us to avoid time-consuming solution of the circuit equations coupled with FE equations. The magnetostatic field generated by the claw-pole motor is determined from

$$\nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J} \quad (1)$$

where ν , $\mathbf{A}$ and $\mathbf{J}$ denote magnetic reluctivity, vector potential and current density, respectively. We consider the line voltage between U and V phase coils which is computed from

$$V_{uv} = -\frac{\partial}{\partial t}(\phi_u - \phi_v) \quad (2)$$

where ϕ_λ is the flux for the λ -phase coil. The amplitude of the other line voltages is the same as V_{uv} , which is maximized in the optimization.

Fig. 1. Equivalent circuit of automotive alternator

(a) Assy

(b) CAD model

Fig. 2. Rotor of claw-pole motor.

2.2. Domain and Boundary condition

The supplied current and flux path of the claw-pole motor are shown in Fig. 3(a). In the FE analysis of (1), the axis inverted boundary conditions

$$\begin{aligned} B_r(r, \theta, z) &= -B_r(r, p + \theta, -z) \\ B_\theta(r, \theta, z) &= -B_\theta(r, p + \theta, -z) \\ B_z(r, \theta, z) &= B_z(r, p + \theta, -z) \end{aligned} \quad (3)$$

are imposed on the periodic boundary [9], where B and p denote magnetic flux density and pole numbers. The magnetic fields in the whole motor domain can be known from analysis of the one-pole model, shown in Fig. 3(b), to whose boundary (3) is imposed.

Fig. 3. Magnetic field analysis of claw-pole motors

3. Methodology

3.1. Problem and type of optimization

Shape optimization can be mainly divided into two types which are topology and parameter optimizations. In the topology optimization [5], device shapes are expressed as binary pixel images or arbitrarily shape functions. Optimization process is conducted by changing the binary status or variable of the shape functions. It seems effective that this type of optimization is used for obtaining innovative device shapes. Since there are no certain parameters which are defined as designing parameters, it is not appropriate for production designing process. In the parameter optimization [7], device shapes are expressed in terms of geometric parameters and optimization is conducted by changing them. For this reason, it seems effective that commonly designing work is conducted by using this optimization. In this paper, parameter optimization method of claw-pole motors is developed for reducing the human costs of production design process. The rotor of the claw-pole motor is parameterized by 8 design variables as shown in Fig. 4.

There are no previous work in which shape optimization technique is applied for magnetic designing of claw pole motors. This is because multi-case calculation of claw-pole motors takes enormous CPU time because of the three-dimensional complex structure. For this reason, efficient algorithm and mesh generation method must be chosen.

Fig. 4. Optimization model and shape parameters

3.2. Mesh generation

In the present optimization, the rotor is subdivided into uniform voxel FEs as shown in Fig. 5, which has been used for analysis of an IPM motor and three-dimensional core of a transformer [6-7]. The material attribute {1 (core), 0 (air)} assigned to each voxel is determined from the shape parameters. If the gravitation center of a voxel is judged inside the core (air) region, then the attribute of the voxel is set to 1 (0). Because the FE mesh is fixed while the material attributes are changed in this process, time-consuming for mesh generation is not required.

The process in FE analysis of claw-pole motor is constructed as (A) Mesh generation, (B) Making boundary condition matrix, (C) Equation solving and (D) Post process. An example of the calculation time for each process is shown in Table 1. These values are obtained using tetra mesh generator [10]. By using voxel FEM, the process (A) and (B) can be omitted. Thanks to this, the total calculation costs for optimization can be reduced by 11.3%.

Fig. 5. Voxel FEM

Table 1 calculation conditions

Computer spec	CPU	2.90GHz 15MB Cache 6cores
	Amount of memory	128GB
Mesh in base model for the present method	Number of elements in Rotor	151,040
	Nmber of elements in stator	158,925
	Number of nodes in Rotor	214,272
	Number of nodes in Stator	222,820
Breakdown of the calculation cost (second)	Mesh generation	23
	Making boundary condition matrix	89
	Equation solving	851
	Post-process	21

3.3. Optimization algorithm

In this section, deterministic and stochastic optimizations are compared with mathematical test problems. The test problems are defined by below equations [11].

$$f(x) = \sum_{i=1}^n x_i^2 \quad (4-a)$$

$$f(x) = \sum_{i=1}^{n-1} \left\{ \left(x_i^2 + x_{i+1}^2 \right)^{0.25} \times \left(\sin^2 \left(50 * \left(x_i^2 + x_{i+1}^2 \right)^{0.1} \right) \right) + 1.0 \right\} \quad (4-b)$$

In Eq. (4), $\mathbf{x} = \{x_1, x_2, \dots, x_8\}$ and $n=8$ means design variable and number of parameters. Eq. (4-a), called Sphere function, is basic function for evaluating performance of the optimization algorithms. Eq. (4-b), called Schaffer function with strong multi-peaks, is more practical function. To select the valid optimization method, convergence and evaluation for acquiring solutions are compared. For deterministic algorithm, gradient based method is selected. In the gradient based method, design parameters are updated by below equation.

$$\mathbf{x}^{k+1} = \mathbf{x}^k - \alpha \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} \quad (5)$$

In Eq. (5), k and $\alpha=0.1$ are iteration steps and gradient factor of parameter updating. For stochastic algorithm, ordinal GA and μ GA are selected. Since μ GA has much smaller populations compared with conventional GA, we can suppress computational time to evaluate individuals. To keep diversity in individuals, re-initialization is conducted if the population is trapped to a local minimum [8].

Optimization results for test problems are summarized in Fig. 6. In Fig. (6), the horai-

zontal and vertical axis mean number of evaluation for obtaining the solutions and evaluation of best solution. Optimization is repeated 20 times with changing the initial solution and seed of random numbers. From repeated results, best one the solutions of which has best performance than that of the other results is selected.

In both problems, gradient based method has best convergence characteristic than that of the others. On the other hand, optimized solutions obtained by the gradient based method with problem (4-b) has lower performance than that of the others. Although gradient based method has best convergence characteristic, it is not appropriate for practical problem like shape optimization of claw-pole motors. Final solution obtained by the GA is superior than that of μ GA in both problems. On the other hand, the solution obtained by the GA has lower performance than that of the μ GA until the evaluation count reach to 500times. From table 1, it takes about 136 hours for 500 times FEM analysis of claw-pole motors. In contrasts, it takes about 27 hours for obtaining converged solution in the μ GA. From these results, μ GA is selected for optimization algorithm in the present method.

Fig. 6. Optimization results for test problems.

4. Numerical results

4.1. Optimization problem

The present method is applied to the optimization model shown in Fig. 4. We consider Problem 1 defined by

$$\begin{aligned} F &= V_{uv}^{1000} \rightarrow \max. \\ G &= S_{RC} - S_{ref} > 0 \end{aligned} \quad (6)$$

where S_{RC} denotes the cross-sectional area of the conductor. Moreover S_{ref} is the reference area

whose value is determined so that the current density is lower than 12 [AT/mm²]. In (6), V_{uv}^{1000} denotes the open voltage between U and V phase coil at 1000r/min rotation speed. Table 2 summarizes the analysis condition and specification of the claw-pole motor.

Table 2 Analysis conditions and motor specifications

Stator outer / inner radius (mm)	130.0 / 104.4
Rotor outer / inner radius (mm)	103.8 / 17.4
Core length of stator (mm)	33.4
Core length of rotor (mm)	56.0
Turn number of stator coil (Turn)	3.0
Turn number of rotor coil (Turn)	202.5
Construct value of rotor coil area : S_{ref} (mm ²)	169.6
Assumed space factor of rotor coil	0.60
Field coil current (A)	6.0

4.2. Optimization results

The optimization results obtained by the present method are summarized in Table 3. The larger changes in x_1 , x_4 and x_7 are made from the initial values through the optimization in comparison with other parameters; x_1 and x_7 increase to reduce the magnetic resistance of the center core. On the other hand, x_4 decreases to increase the magnetic resistance of the top and bottom cores. The optimization history and induced voltage waves of the elite individuals at different generations are plotted in Fig. 7. It can be seen from Fig. 7 and also Table 3 that the induced voltage is increased by the optimization.

The magnitude of the magnetic induction in the core region of the elite individual at every five generation is shown in Fig. 8. It is found that the magnetically saturated region localizes near the coil in the rotor of the initial generation, while the saturated region spreads over the whole core in the final rotor. This is due to the fact that the magnetic saturation in the claw part leads to increase in the magnetic flux which interlinkages with the stator windings.

Table 3 Resultant parameters for problem 1

	Initial	5th	10th	Optimized
x_1 (mm)	23.0	24.9	28.5	28.1
x_2 (mm)	15.4	12.7	11.3	15.2
x_3 (mm)	47.0	47.8	52.2	48.7
x_4 (mm)	21.3	19.3	20.7	15.7
x_5 (mm)	38.9	29.6	35.9	36.3
x_6 (mm)	44.0	46.3	47.1	47.4
x_7 (mm)	7.2	7.1	10.1	10.9
x_8 (mm)	3.9	4.4	3.5	3.3
V_{uv}^{1000} (Vrms)	6.4	6.9	7.7	8.19

(a) Optimization history

(b) Induced open voltage

Fig. 7. Optimization results for problem 1.

Fig. 8. Magnetic induction in rotor core.

4.3. Comparison of generated DC current

Because we maximize the open voltage in Problem 1, the optimized motor might have weak properties in the load conditions. To consider this possibility, the performances of the motors corresponding to the elite individuals are evaluated under the load condition by solving FE and circuit equations where the core is subdivided into tetrahedral FEs.

In Fig. 9, the generated DC currents are plotted against the rotation speed. It can be seen that the optimized rotor has the strongest current at low speed regime. This means that the optimized rotor has a property suitable for city driving. Moreover analysis accuracy of the voxel FEs is sufficiently for relative comparison in designing process. On the other hand, the elite rotor at the 10th generation has the strongest current at high speed conditions. This suggests that the changes in the magnetic fields at high speed conditions should be considered for our optimization.

The magnetic distributions in the rotors at 10th and final generations at low and high speed conditions are shown in Fig. 10. The differences in magnetic distribution are due to the load current of the stator coil. The AC currents at 6000r/min generated by the optimized rotor are plotted in Fig. 11(a). The magnetic reluctivity of the rotor core increases because of the magnetic flux generated by the stator currents. As a result, the induced voltage by the optimized rotor reduces as shown in Fig. 11(b). Because the rotor at the 10th generation has a larger volume in comparison with the optimized rotor, the former would have smaller affections from the stator currents.

Fig. 9. Dependence of generated DC currents on rotation speed.

Fig. 10. Magnetic induction at 10th and final generation.

Fig. 11. Analysis results in 6000r/min condition.

4.4. Consideration of different speed conditions

In order to improve the power generation efficiency also at high rotation speed condition, Problem 1 is now modified to Problem 2:

$$F = \frac{V_{uv}^{1000}}{V_{ref}^{1000}} + W \frac{V_{uv}^{6000}}{V_{ref}^{6000}} \rightarrow \max. \quad (7)$$

$$G = S_{RC} - S_{ref} > 0$$

where V_{uv}^{1000} and V_{uv}^{6000} are the induced voltages at 1000 and 6000 r/min. In the optimization, the weighting constant W is set to 1.0. The optimal solution to Problem 1 is included as an individual of the first generation. For computation at 6000r/min condition, the three-phase AC current shown in Fig. 11(a) is assumed to the stator windings to avoid the circuit analysis during the optimization. The optimized shape parameters and open voltages are summarized in Table 4. The rotor optimized

for Problem 2 has higher induced voltage than that for Problem 1 at both low and high rotation speed conditions. The parameters x_1 and x_2 increase to reduce the magnetic resistance in the rotor core. Thanks to this structure, the affection from the load current in the stator core is reduced. The generated DC currents obtained by solving Problem 1 and 2 at low and high speed conditions are shown in Fig. 12. The motor optimized for Problem 2 generates higher currents at both low and high rotation speed conditions.

Fig. 12. Generated DC currents of the motor optimized for Problem 1 and 2

Table 4 Resultant parameters for problem 2

	Initial	Differences in parameters optimized for problem1 and 2
x_1 (mm)	28.9	0.76
x_2 (mm)	15.8	0.60
x_3 (mm)	48.4	-0.28
x_4 (mm)	15.8	0.07
x_5 (mm)	36.4	0.15
x_6 (mm)	47.7	0.25
x_7 (mm)	11.0	-0.01
x_8 (mm)	3.7	0.32
V_{inv}^{1000} (Vrms)	8.38	0.19
V_{inv}^{6000} (Vrms)	33.78	1.02

5. Conclusion

This paper presents the three-dimensional shape optimization of a claw-pole motor. In the present method, the rotor is subdivided into the voxel FEs. The material attribute of the voxel elements are determined from the shape parameters, which are optimized to maximize the induced voltage in the stator windings. The present method does not need time-consuming mesh generation. Moreover the μ GA is selected for the optimization algorithm. Thanks to this, the computational costs for obtaining converged solution are reduced.

It has been shown that the changes in the magnetic field due to the load current at high speed conditions should be considered in the optimization. The induced voltages at low and high speed conditions are successfully maximized by the present method.

In the future work, estimation methods by which the affection of induction current at high rotational speed condition can be considered are developed.

References

- [1] S. Kiippers and G. Henneberger, "Numerical procedures for the calculation and design of automotive alternators," *IEEE Trans. Magn.*, vol. 33, no. 2, pp. 2022-2025, 1997.
- [2] A. Tan-Kim, V. Lanfranchi, S. Vivier, J. Legranger and F. Palleschi, "Vibro-acoustic simulation and optimization of a claw pole alternator," *IEEE Transaction on industry applications*, vol. 52, no. 5, pp. 3878-3885, 2016.
- [3] D. Bilyi, V. Bilyi and D. Gerling, "FEM based model development and co-simulation of automotive multi-phase claw-pole alternator and rectifier," *Transportation Electrification Conference and Expo 2016 (ITEC)*, 2016.
- [4] K. Yamasaki, H. Ishigami, A. Abe, "An Adaptive Finite Element Method for Minor Shape Modification in Optimization Algorithms," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1202-1205, 2008.
- [5] N. Takahashi, T. Yamada, S. Shimose and D. Miyagi, "Optimization of rotor of actual IPM motor using on/off method," *IEEE Trans. Magn.*, vol. 47, no. 5, pp. 1262-1265, 2011.
- [6] S. Odawara, Y. Gao, K. Muramatsu, T. Okitsu and D. Matsuhashi, "Magnetic Field Analysis Using Voxel Modeling With Nonconforming Technique," *IEEE Trans. Magn.*, vol. 47, no. 5, pp. 1442-1445, 2011.
- [7] T. Sato, K. Watanabe, H. Igarashi, T. Matsuo, T. Mifune, K. Kawano, M. Suzuki, Y. Uehara and A. Furuya, "3-D Optimization of Ferrite Inductor Considering Hysteresis Loss," *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 2129-2132, 2013.
- [8] C. A. Coello and G. T. Pulido, "A micro-genetic algorithm for multi objective optimization," in *Proc. EMO*, pp. 126-140, 2001.
- [9] S. Hayashi, Y. Fujita, H. Kometani and S. Sakabe, "Three dimensional Magnetic Field Analysis of a Lundell Alternator," *The Institute of Electrical Engineers of Japan*, vol. 114, no. 12, pp. 1284-1293, 1994.
- [10] Y. Kawase, T. Yamaguchi, M. Yoshida and K. Hirata, "3-D finite element analysis of

rotary oscillatory actuator using a new auto-mesh coupling method," *IEEE Trans. Magn.*, vol. 37, no. 5, pp. 3711-3714, 2001.

- [11] S. Kobayashi, "The frontiers of real-coded genetic algorithms," *Transactions of the Japanese Society for Artificial Intelligence*, vol. 24, no. 1, pp. 147-162, 2009.