Numerical simulation on generation and propagation of vortex synchrotron radiation

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Abstract. It was theoretically shown that a single free electron in circular/spiral motion emits optical vortex. Such process can be found in relativistic electron beam crossing a helical undulator and the emitted radiation is known as undulator radiation. We have carried out experimental studies to characterize the synchrotron radiation at the UVSOR-III electron storage ring. We also have performed numerical simulations of the radiation process using a code Synchrotron Radiation Workshop (SRW), widely used in the synchrotron radiation community. The simulation reproduces the radiation properties very well. The simulation is expected to predict further characteristics of undulator radiation as optical vortex.

Keywords: Optical vortex, Synchrotron radiation, Electron beam

1. Introduction

An optical vortex, a light beam with a phase singularity at the center of helical wavefront carries orbital angular momentum (OAM) [1]. Since the discovery, extensive studies not only on vortex of optical beam but also of electron beam [2] and neutron beam [3] have been carried out. These vortices are created by converting plane waves using spiral phase plates, computer-generated holograms or other devices. On the other hand, Katoh et al. have theoretically shown that a single free electron in circular/spiral motion radiates an optical vortex [4]. Such process can be found in various field of sciences and natures, and among them the most popular one in laboratories is synchrotron radiation from a relativistic electron beam crossing a helical undulator. The emitted radiation is known as synchrotron radiation or undulator radiation. The radiation is expected to be naturally optical vortex without any manipulation.
2. Vortex synchrotron radiation from helical undulator

Synchrotron radiation is electromagnetic radiation emitted from a relativistic electron forced to move along a curved trajectory by applied magnetic field. Electron storage rings are routinely used to provide synchrotron radiation to users. An undulator is a device with periodic magnetic structure used to produce brighter synchrotron radiation. The static magnetic field alternates along the length of the undulator with a wavelength (or a period length) \( \lambda_u \). Electron traversing the periodic magnet structure is forced to undergo oscillation and thus to emit synchrotron radiation. The resonant wavelength of \( n \)-th harmonics of synchrotron radiation from an undulator is given by

\[
\lambda = \frac{\lambda_u}{2\pi\gamma}(1 + \frac{K_x^2 + K_y^2}{\gamma^2} + \theta^2 \gamma^2)
\]

where \( \gamma \) and \( \theta \) are Lorentz factor of the electron beam and the observation angle from the radiation axis, respectively. \( K_x \) and \( K_y \) are deflection parameters in the horizontal and the vertical direction, which is proportional to the magnetic field and the period length [5]. For a planar undulator, \( K_x = 0, K_y \neq 0 \) or \( K_x \neq 0, K_y = 0 \) and for a helical undulator \( K_x = K_y \neq 0 \). According to the theories [4, 6], \( n \)-th harmonics of a helical undulator radiation is expected to possess \( n-1 \) topological charge.

3. SRW simulation

Numerical simulation on electron radiation in helical undulators can be carried out using a code Synchrotron Radiation Workshop (SRW), widely used in the synchrotron radiation community [7]. In the simulation, the motion of a relativistic electron in the magnetic field (ideal sinusoidal field or real magnetic field with field errors) is solved and the electric field of the radiation from the electron is obtained using well known equation:

\[
\vec{E}_\omega = \frac{i e \omega}{c} \int_{-\infty}^{+\infty} \frac{1}{R} \left[ \vec{\beta} - \vec{n} \left( 1 + \frac{ic}{\omega R} \right) \right] \exp[i \omega(\tau + R/c)] d\tau,
\]

where \( \omega, \vec{\beta}, \vec{n} \) and \( R \) are a frequency, instant relative velocity, unit vector directed from instant electron position to an observation point and distance from the electron to the observation point, respectively. In the next step of the simulation, the numerically deduced complex vector field is used for further manipulation or propagation through optical elements. The process can be implemented using the methods of Fourier optics. The software is interfaced by IGOR Pro [8] or Python [9].

The second harmonic radiation from a helical undulator is calculated using SRW. In the simulation, electromagnetic radiation of wavelength 350nm from an electron 500MeV in a helical undulator is calculated. The period length and number of the periods of the undulator is
88mm and 10, respectively and the fundamental wavelength is adjusted to 700nm. Fig. 1 shows the simulation results on intensity distribution (a), phase distribution (b) and corrected phase distribution (c). The simulation condition is shown in Fig. 2 but in this case single undulator is used. The donuts shaped intensity distribution shown in the figure indicates the phase singularity in the center of the radiation. The spiral phase distribution reflects that the undulator radiation is partially spherical wave with the source point located at the center of the undulator (Fig. 1 (b)). In order to correct the spherical phase structure, an ideal virtual lens with focal length \( f \) equal to the length from the undulator center to the observation point is used and the resulting phase structure is shown in Fig. 1 (c). The operation is implemented by subtracting phase shift by the lens: \( \pi(x^2 + y^2)/f\lambda \), from the phase at a position \((x, y)\) in Fig. 1 (b), where \( \lambda \) is the wavelength of the radiation. As is evident from the figure, the second harmonic radiation possesses topological charge 1. Similarly, it is easy to show that \( n-th \) harmonic radiation from a helical undulator possesses topological charge \( n-1 \).

### 4. Comparison with experiment

We have carried out a series of experiments to characterize radiation from a helical undulator as optical vortex at the UVSOR-III electron storage ring. At first, we have observed the interference between a reference beam and a vortex beam which is an established method to show its phase structure. To apply this method to the undulator radiation, two consecutives helical undulators were used: one was tuned to a fundamental resonance and another was tuned to a harmonic resonance. In the experiment, the storage ring was operated with 500MeV and undulator radiation of UV region around 350nm was detected using a CCD camera (Fig. 2).

The experimental results are compared with SRW simulation ones. The interference between
the fundamental and the second harmonics is shown in Fig. 3. As seen in the figure, the simulation reproduces well the experimentally deduced spiral pattern as a result of interference between the vortex beam and the spherical beam. We also made experiments on the interference between the fundamental and the 3rd harmonics and the experimentally deduced interference patterns are well reproduced by the SRW simulations. After the successful experiment on the interference pattern, we have demonstrated Young’s double slit diffraction experiments using radiation from single helical undulator. As is already reported by Sztul and Alfano [10], a singularity in the middle of the strip pattern is expected in double slit diffraction with a vortex beam. We have made SRW simulations in advance and designed a double slit optimal for the diffraction observation. As is already reported in a separated paper [4], singular diffraction pattern was observed and the experimental results are well reproduced by the SRW simulation.

5. **Summary and outlook**

Properties of harmonic radiation from a helical undulator as an optical vortex are experimen-
tally investigated. The experimental results are compared with the simulations using SRW and good agreements are confirmed. Recently we have succeeded in production of a vector beam by superposition of two undulator radiation [11]. In the study, SRW simulation played an important part. The simulation is expected to predict further characteristics of undulator radiation as optical vortex.

References