Examination of Coil Resistance Measurement during Wireless Power Transfer and Validity of Calculation by Finite Element Analysis

Daigo Yonetsu^{1*}, Yuji Nakano¹, Naoya Hamano¹

¹ Department of Electrical and Electronics Engineering, Kansai University

*yonetsu@kansai-u.ac.jp

Received: October 28, 2021; Accepted: December 22, 2021; Published: January 31, 2022

Abstract. This paper describes a method for measuring the ac resistance of sending and receiving coils during wireless power transfer. This method focuses on the ratio of the input voltage to the coil current. The coil resistances measured by this method were compared with the calculated values obtained by finite element analysis. Further, the validity of the measurement and calculation for the coil constants were examined by solving simultaneous equations for sending and receiving coil current for an equivalent circuit for a wireless power transfer system and obtaining the input and output power and power transfer efficiency.

Keywords: Wireless power transfer, Coil resistance, Equivalent circuit, Finite element analysis

1. Introduction

Wireless power chargers based on electromagnetic induction are increasingly being used in household appliances and electric vehicles. These chargers produce a high-frequency magnetic flux between a pair of coils. The goal of this technique is to realize high power transfer efficiency. Most of the losses that decrease the transfer efficiency are the copper losses at the sending and receiving coils [1]. Thus, it is very important to know the ac resistance of the sending and receiving coils during wireless power transfer.

We calculated the coil constants, including the ac resistance of the sending and receiving coils, by finite element analysis (FEA) [2]-[8]. The calculated values of the self and mutual inductances were found to be reasonable in comparison with values measured by an LCR meter. However, the validity of the calculated ac resistances is not clear because the LCR meter cannot be connected during the wireless power transfer. Although a proximity effect between the sending and receiving coils appears during wireless power transfer, it does not appear during measurement by an LCR meter.

Previous papers have addressed the calculation of the ac resistance in wireless power transfer systems [9]-[12]. In reference [9], the ac resistance of a coil was calculated as the volume integral of the heat density divided by the square of the current. This method is thought to be suitable for calculating the ac resistance of coils made from a single solid wire. When this method was applied to the calculation for coils made from litz wires, a supercomputer, such as the Japanese Earth Simulator, was needed [10]. In references [11] and [12], reasonable methods for the calculation of the ac resistance of coils made from litz wire were proposed. Although the validity of these methods was examined by comparing the results with the measured values, the results could be obtained only for the sending side or receiving side coil alone. Therefore, additional experiments to examine the validity of the calculated ac coil resistance during wireless power transfer are desired.

In this study, the ac resistance of the coils during wireless power transfer was measured by focusing on the ratio of the input voltage to the coil current. After obtaining the coil resistance, the obtained values were compared with the calculation results by a finite element analysis. Also, the calculated self and mutual inductances of the coils were compared with the measured values to examine the validity of the FEA. In addition, the validity of the measurements and the calculation method for the self and mutual inductance and the ac resistance of the coils were examined by solving the simultaneous equations or the sending and receiving coil current of the equivalent circuit for the wireless power transfer system and obtaining the input and output power and power transfer efficiency.

2. Measurement method

2.1 Equivalent circuit for measuring coil resistances

The equivalent circuit for a wireless power transfer system connected with a sinusoidal voltage source and a load resistance is shown in Fig. 1. The symbols in Fig. 1 are shown in phasor mode because the input voltage and current for the coils have a sinusoidal waveform.

2.2 Equations for coil resistance from measured value

The equations for the coil current are derived from the equivalent circuit as follows:

$$\begin{pmatrix} (r_{L1} + r_{C1}) + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) & j\omega M \\ j\omega M & (r_{L2} + r_{C2} + R_L) + j\left(\omega(L_1 + L_{RL}) - \frac{1}{\omega C_2}\right) \end{pmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}, \quad (1)$$

where the capacitances C_1 and C_2 are adjusted so that the imaginary parts of the equation are zero. In the other words, the following equations hold:

$$C_1 = \frac{1}{\omega^2 L_1}, \quad C_2 = \frac{1}{\omega^2 (L_2 + L_{RL})}. \quad (2)$$

In this situation, theoretically, the phase of current I_1 is the same as that of the input voltage V_1 , and that of current I_2 lags that of the input voltage V_1 by 90 deg.

V_1 : input voltage [V]	ω : angular frequency [rad/s]
I_1 : current of sending coil [A]	L_{RL} : inductance including in load resistance [H]
I_2 : current of receiving coil [A]	R_L : load resistance [Ω]
L_1 : self-inductance of sending coil [H]	r_{L1} : resistance of sending coil [Ω]
L_2 : self-inductance of receiving coil [H]	r_{L2} : resistance of receiving coil [Ω]
M : mutual inductance between coils [H]	r_{C1} : resistance including in capacitor C_1 [Ω]
C_1 : capacitance in sending side [F]	r_{C2} : resistance including in capacitor C_2 [Ω]
C_2 : capacitance in receiving side [F]

Figure 1: Equivalent circuit for measuring coil resistance

Figure 2: Simulation model for FEA

By solving Equation (1) while satisfying Equation (2), the ratio of the input voltage V_1 to the current I_1 and I_2 is obtained as follows:

$$\left| \frac{I_1}{V_1} \right| = \frac{r_{L2} + r_{C2} + R_L}{(r_{L1} + r_{C1})(r_{L2} + r_{C2} + R_L) + \omega^2 M^2}, \quad (3)$$

$$\left| \frac{I_2}{V_1} \right| = \frac{\omega M}{(r_{L1} + r_{C1})(r_{L2} + r_{C2} + R_L) + \omega^2 M^2}, \quad (4)$$

$$\left| \frac{I_1}{I_2} \right| = \frac{r_{L2} + r_{C2} + R_L}{\omega M}. \quad (5)$$

Here, the effective value of the input voltage V_1 and the currents I_1 and I_2 can be measured by a power meter. The circuit constants excluding the coil resistances r_{L1} and r_{L2} can be measured by an LCR meter. By substituting the measured values into Equations (3) and (4), these equations change into the simultaneous equations for the coil resistances r_{L1} and r_{L2} . By dividing Equation (3) by Equation (4), Equation (5) was derived. From Equation (5), the ac resistance r_{L2} can be obtained easily. Moreover, the ac resistance r_{L1} can be obtained from either simultaneous equation.

3. Calculation by FEA method

3.1 Simulation model

The validity of the measurement method for the coil resistance during wireless power transfer needed to be confirmed. Thus, these values were compared with the calculation results obtained by FEA.

Figure 2 shows the simulation model for calculating the coil resistances by FEA. In this study, the sending and receiving coils for the wireless power transfer were made from enamel-insulated wire with a diameter of 1 mm. The number of turns was 10 and the inner diameter was 100 mm for both coils. The interval between the sending and receiving coils was 50 mm. The simulation model was chosen based on the above situation. The copper part had a conductivity of 5.76×10^7 S/m. The mesh length for the copper part was about 8.43% of the skin depth. The number of nodes was 4,778,631.

3.2 Calculation method for coil resistance

After the finite element analysis, the coil resistances r_{L1} and r_{L2} were calculated as the volume integral of the heat density divided by the square of the current in the copper part of the sending and the receiving coils [9]. The calculation formulas were as follows:

$$r_1 = \sum_{e=1}^{n_1} \frac{1}{\sigma} |J_0^e + J_e^e|^2 V^e / I_1^2, \quad (6)$$

$$r_2 = \sum_{e=1}^{n_2} \frac{1}{\sigma} |J_e^e|^2 V^e / I_2^2, \quad (7)$$

where σ is the conductivity of copper; J_0^e and J_e^e are the input and eddy current densities, respectively, of element e ; and I_1 and I_2 are the coil current flowing in the sending and receiving coils, respectively. The coil currents were calculated as the area integral of the current density that was the sum of the input and eddy current density in each element, including the sending and receiving coils.

3.3 Calculation method for self and mutual inductances of coils

To calculate the self and mutual inductances of the sending and receiving coils, the FEA did not consider the conductivity of copper. After the FEA without the conductivity of the copper, the self and mutual inductances of the sending and receiving coils were calculated from the magnetic vector potential distribution as follows:

$$L_1 = T_{c1} \sum_{e=1}^{n_1} V^e A^e, \quad (8)$$

$$M = T_{c2} \sum_{e=1}^{n_2} V^e A^e, \quad (9)$$

where T_{c1} and T_{c2} are respectively the number of turns per unit area for the sending and receiving coils, n_1 and n_2 are respectively the number of copper elements in the sending and receiving coils, V^e is the volume of element e , and A^e is the magnetic vector potential in ele-

ment e . In this study, $T_{c1}=T_{c2}$ and $n_1=n_2$. The self-inductance of the receiving coil was set to L_1 .

4. Validity comparison for experimental and calculation results

4.1 Measured waveforms and values

The sending coil was connected to the variable capacitor C_1 and the input voltage source, the receiving coil was connected to the variable capacitor C_2 , and the load resistance was 2.03Ω . The frequency of the input voltage was set to 50 kHz.

When the capacitances C_1 and C_2 were set to 282 nF and 278 nF, respectively, the waveforms of the input voltage and the current were obtained as shown in Fig. 3. From Fig. 3, the phase of the sending current appears to be almost the same as that of the input voltage, and that of the receiving current lags the input voltage phase by about 90 deg. The values measured at this time are shown in Table 1.

Figure 3: Waveforms of input voltage and coil current

Table 1: Measured voltage, current, and circuit constants

$ V_1 $	$ I_1 $	$ I_2 $	L_1	L_2	M
0.843 V	0.419 A	0.331 A	$35.0 \mu\text{H}$	$35.3 \mu\text{H}$	$6.93 \mu\text{H}$
L_{RL}	R	r_{C1}	r_{C2}	C_1	C_2
$0.311 \mu\text{H}$	2.03Ω	0.107Ω	0.530Ω	282 nF	278 nF

Table 2: Measured and calculated ac coil resistances

	measured value	calculated value	error
r_{L1}	0.186Ω	0.169Ω	9.14 %
r_{L2}	0.196Ω	0.172Ω	12.2 %

Table 3: Measured and calculated inductances

	measured value	calculated value	error
L_1	35.0 μH	35.2 μH	0.57 %
L_2	35.3 μH	35.2 μH	0.28 %
M	6.93 μH	6.86 μH	1.01 %

4.2 Measured coil resistances and comparison with FEA

After solving Equations (3) and (4) by inserting the measured values shown in Table 1, the ac resistances of the sending and receiving coils were obtained as shown in Table 2. In addition, the values calculated by the FEA are shown. From Table 2, the ac resistances of the sending and receiving coils measured by the proposed method were almost equal to the calculated FEA values, although the measured values were larger than the calculated values. It was thought that the calculation region for FEA was enough, and that the differences between the measured and calculated values occurred due to the difference with and without wiring between the power supply and the sending coil and between the receiving coil and the charging device.

4.3 Measured and calculated inductance of coils

To show the validity of calculating the coil constants by FEA, the calculated self and mutual inductances were compared with the values measured by the LCR meter. If we assume that the inductance measured with the ends of the sending and receiving coils connected was L_a , and the inductance measured with one of the ends changed to the other was L_b , the measured value of the mutual inductance was as follows:

$$M = \left| \frac{L_a - L_b}{4} \right| \quad (10)$$

Here, for example, the value of L_a became $L_1 + L_2 + 2M$, the value of L_b $L_1 + L_2 - 2M$.

Table 3 shows the measured and calculated inductances of the coils. From Table 3, the values calculated by the FEA were in good agreement with the values measured by the LCR meter. From these results, the fidelity of the FEA was thought to be sufficient.

4.4 Verification of validity of measured coil resistance

The measured and calculated coil constants were inserted into simultaneous equations (1) and (2), respectively. After solving the simultaneous equations, the input power P_{in} , output power P_{out} , and power transfer efficiency η were obtained from the following equations:

$$P_{\text{in}} = \text{Re}[\mathbf{V}_1 \mathbf{I}_1], \quad (11)$$

$$P_{\text{out}} = \text{Re}[(R_L + j\omega L_{\text{RL}}) \mathbf{I}_2 \mathbf{I}_2^*], \quad (12)$$

$$\eta = P_{\text{out}} / P_{\text{in}} \times 100. \quad (13)$$

Table 4: Measured and calculated current, input and output power, and power transfer efficiency

	measured value	calculated value (using measured coil constants)	calculated value (using calculated coil constants)
$ I_1 $	0.419 A	0.419 A	0.427 A
$ I_2 $	0.331 A	0.331 A	0.337 A
P_{in}	0.353 W	0.353 W	0.360 W
P_{out}	0.217 W	0.217 W	0.230 W
η	61.7 %	61.7 %	63.9 %

Table 5: Frequency dependence of measured ac resistances of coils

frequency	r_{L1}	r_{L2}
50 kHz	0.186 Ω	0.196 Ω
75 kHz	0.274 Ω	0.322 Ω
100 kHz	0.379 Ω	0.388 Ω

Table 4 shows the measured and calculated current, input and output power, and power transfer efficiency. The calculated values using measured coil constants were in perfect agreement with the measured values. In contrast, although the values calculated using the calculated coil constants by the FEA were almost the same as the measured values, all of the calculated values were larger than the measured values. The reason was considered to be that the calculated coil resistances were smaller than the measured values, so that the current and power became larger, and the copper losses for the coils became smaller than the measured values, which increased the power transfer efficiency.

4.5 Frequency dependence of ac coil resistances

Finally, the frequency dependence of the measured ac coil resistance is shown in Table 5. The coil resistance was measured by the method described in section 2. Conditions such as the input voltage, the resistance and inductance of the load, and the self and mutual inductance of the sending and receiving coils determined by measurement, were almost the same.

From Table 5, both coil resistances increased monotonically as the frequency increased. Also, the resistance of the receiving coil was slightly larger than that of the sending coil at any frequency. This was considered to be due to skin and proximity effects.

5. Conclusion

This study developed a method to measure the coil resistance during wireless power transfer. The results confirmed the validity of this measurement method. In addition, the calculated values of the coil resistance by a FEA were appropriate. Finally, the calculated current, power,

and power transfer efficiency obtained by the measured coil constants were in perfect agreement with the values measured by the power meter. The frequency dependence of the ac coil resistance was determined correctly using the proposed method.

References

- [1] D. Yonetsu, Y. Yamamoto : Behavior prediction for low-power non-contact charger, *Transactions on IEE Japan*, vol. 134-D, no. 11 (2014), pp. 948-955.
- [2] D. Yonetsu, Y. Yamamoto : An Estimation Method for Power Transmission Efficiency of Inductive Power Transfer System using Coils Made of Non-insulated stranded Wire, *The 15th Biennial IEEE Conference on Electromagnetic Field Computation* (2012), p.5.
- [3] D. Yonetsu, T. Hara, T. Fujiwara, K. Ishizu and T. Hanada : Determination of Optimum Operating Frequency for Non-contact Charger, *Journal of the Japan Society of Applied Electromagnetics and mechanics*, Vol.20, No.2 (2012) , pp.377-384.
- [4] D. Yonetsu, Y. Yamamoto : Simulation for Power Transfer Efficiency of Inductive Charging System, *The 33rd JSST Annual Conference: International Conference on Simulation Technology* (2014), pp.36-37.
- [5] D. Yonetsu, Y. Yamamoto : Examination about Current Dependence of Power Factor and Power Transfer Efficiency for Non-Contact Charger, *The 34th JSST Annual Conference: International Conference on Simulation Technology* (2015), pp.202-205.
- [6] D. Yonetsu, Y. Yamamoto : Investigation of Current Dependence of Power Factor and Power Transfer Efficiency for Non-Contact Charger (2) , *The 35th JSST Annual Conference: International Conference on Simulation Technology* (2016), pp.31-34.
- [7] D. Yonetsu: Investigation of relationship between coupling coefficient and power factor for non-contact chargers, *2016 IEEE PELS Workshop on Emerging Technologies: Wireless Power Transfer (WoW)* (2016), pp.114-117.
- [8] D. Yonetsu, S. Okamoto, K. Noguchi : Investigation about Power Transfer Efficiency of Wireless Charger for Each Wire Rod, *2018 IEEE PELS Workshop on Emerging Technologies: Wireless Power Transfer (WoW)* (2018), pp.1-5.
- [9] Y. Kawase, T. Mori and T. Ota: Magnetic Field Analysis of Coupling Transformers for Electric Vehicle Using 3-D Finite Element Method, *IEEE Trans. on Magnetics*, vol. 34, no. 5 (1998), pp.3186-3189.
- [10] Y. Kawase, T. Yamaguchi, D. Kato, S. Tsukada, K. Ito, K. Ida, Y. Fukui, N. Nishikawa: Numerical Analysis of Stranded Wire Coil with Magnetic Substrate Using 3D Finite Element Method, *18th International Conference on Electrical Machines and System (ICEMS)* (2015), pp.985-988.
- [11] J. Acero, P. J. Hernández, J. M. Burdío, R. Alonso, L. A. Barragán : Simple Resistance Calculation in Litz-Wire Planar Windings for Induction Cooking Appliances, *IEEE Trans. on Magnetics*, vol. 41, no. 4 (2005), pp.1280-1288.
- [12] S. Hiruma, Y. Otomo, H. Igarashi: Eddy Current Analysis of Litz Wire Using Homogenization-Based FEM in Conjunction With Integral Equation, *IEEE Trans. on Magnetics*, vol. 54, no. 3 (2018), pp.1-4.