1 Introduction and Preliminaries

Blum and Hewitt first proposed two-dimensional automata as a computational model of two-dimensional pattern processing, and investigated their pattern recognition abilities [1]. Since then, many researchers in this field have investigated the properties of automata on a two- or three-dimensional tape. On the other hand, the question of whether processing four-dimensional digital patterns is more difficult than processing two- or three-dimensional ones is of great interest from both theoretical and practical standpoints. Thus, the study of four-dimensional automata as the computational model of four-dimensional pattern processing has been meaningful. From this point of view, we are interested in four-dimensional automata. In the multi-dimensional pattern processing, designers often use a strategy whereby features are extracted by projecting high-dimensional space on low-dimensional space. From this viewpoint, we introduce a new computational model, \( k \)-neighborhood template \( \mathcal{A} \)-type three-dimensional bounded cellular acceptor (abbreviated as \( \mathcal{A}-3BCA(k) \)) on four-dimensional tapes, and discuss some basic properties. (An \( \mathcal{A}-3BCA(k) \) consists of a pair of a converter and a configuration-reader. An \( \mathcal{A}-3DBC\mathcal{A}(k) \) (\( \mathcal{A}-3NBC\mathcal{A}(k) \)) is called a \( k \)-neighborhood template \( \mathcal{A} \)-type three-dimensional deterministic bounded cellular acceptor (\( k \)-neighborhood template \( \mathcal{A} \)-type three-dimensional nondeterministic bounded cellular acceptor). A \( 2-DA \) (\( 2-NA \), \( 2-DO \), \( 2-NO \), \( 2-DTM \), \( 2-NTM \)) is called a two-dimensional deterministic finite automaton (two-dimensional nondeterministic finite automaton, two-dimensional deterministic on-line tessellation acceptor, two-dimensional nondeterministic on-line tessellation acceptor, two-dimensional deterministic Turing machine, two-dimensional nondeterministic Turing machine) Let \( T(M) \) be the set of four-dimensional tapes accepted by a machine \( M \), and let \( \mathcal{L}[\mathcal{A}-3DBC\mathcal{A}(k)] = \{T|T=T(M) \} \) for some \( \mathcal{A}-3DBC\mathcal{A}(k) \) \( M \). \( \mathcal{L}[\mathcal{A}-3NBC\mathcal{A}(k)] \), etc. one defined in the same way as \( \mathcal{L}[\mathcal{A}-3DBC\mathcal{A}(k)] \).)

2 Main Results

We deal with only four-dimensional input tapes which each side-length is equivalent. Then, we get the following theorems.

**Theorem 1.** For each \( A \in \{2-DA, 2-NA, 2-DO, 2-NO, 2-DTM, 2-NTM\} \) and for each \( X \in \{D,N\}, \mathcal{L}[\mathcal{A}-3XBCA(1)] \subseteq \mathcal{L}[\mathcal{A}-3XBCA(5)] \).  

**Theorem 2.** For each \( A \in \{2-DA, 2-NA, 2-DO, 2-NO\}, \mathcal{L}[\mathcal{A}-3DBC\mathcal{A}(5)] \subseteq \mathcal{L}[\mathcal{A}-3DBC\mathcal{A}(9)] \).  

**Theorem 3.** For each \( A \in \{2-DA, 2-NA, 2-DO, 2-NO, 2-DTM, 2-NTM\}, \mathcal{L}[\mathcal{A}-3NBC\mathcal{A}(5)] = \mathcal{L}[\mathcal{A}-3NBC\mathcal{A}(9)] \).  

**References**