On the Recognizability of Arbitrary Three-Dimensional Patterns

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1 Introduction and Preliminaries

Due to the advances in computer vision, robotics, and so forth, it has become increasingly apparent that the study of three-dimensional pattern processing should be very important. Thus, the study of three-dimensional automata as the computational model of three-dimensional information processing has been significant. During the past about thirty years, automata on a three-dimensional tape have been obtained [3]. On the other hand, it is well-known that whether or not the pattern on a two- or three-dimensional rectangular tape is connected can be decided by a deterministic one-marker finite automata [3]. As far as we know, however, it is unknown whether a similar result holds for recognition of the connectedness of patterns on three-dimensional arbitrarily shaped tape. In this paper, we consider whether or not the pattern on a three-dimensional arbitrarily shaped tape is connected can be decided by a deterministic multi-marker finite automaton.

Let \( \Sigma^{(3)} \) be a set of points in the three-dimensional Euclidean space with integer coordinates. Each point in \( \Sigma^{(3)} \) is called a vertex. Each unit-length segment connecting two vertices is called an edge. Each region of unit area enclosed by twelve edges is called a voxel. Each voxel can have an input symbol ‘0’ or ‘1’, or a boundary symbol ‘#’. A voxel is called 0-voxel(1-voxel, or #-voxel) if it has symbol 0 (1, or #). Two-voxels are 6-adjacent (or 27-adjacent) if they share a common edge (or a common vertex) [1]. A 6-adjacent (or 27-adjacent) path is a sequence of voxels \( c(1), c(2), \ldots, c(i) \) such that for each \( 1 \leq j \leq i - 1 \), \( c(j) \) and \( c(j + 1) \) are 6-adjacent (or 27-adjacent). A three-dimensional arbitrarily shaped pathwise-connected tape \( (p-tape) \) \( T \) is a set of 0,1-voxels surrounded by #-voxels, where any two 0,1-voxels in \( T \) are connected by a 6-adjacent path with only 0,1-voxels in \( T \). (Note that \( T \) can contain some ‘holes’ in its inside.) the pattern \( P \) on a p-tape \( T \) is the set of all 1-voxels that appear there. For the pattern \( P \) on a p-tape, a 1-component \( C \) is any maximal set of 1-voxels such that any 1-voxels in \( C \) are connected by a 6-adjacent path with only 1-voxels in \( C \). A pattern \( P \) is connected if and only if any two 1-voxels are connected by a 6-adjacent path with only 1-voxels in \( P \). That is, \( P \) is connected if and only if there exists only one 1-component. A k-marker finite automaton \( M(k) \) consists of a finite control with a read-only input head and \( k \) (labelled) markers operating on a p-tape \( T \). \( M(k) \) is started on a 0,1-voxel in its start state with carrying its markers. The markers can be placed on or collected back to the finite control from only the voxel the input head is currently scanning. In each step, \( M(k) \) can change its internal state, place a marker ‘carried’ by the finite control (or collect back a marker (if it exists) to the finite control) on (or from) the voxel the input head is currently scanning, and move the input head to a 6-adjacent cell, according to the current state, the symbol and the presence of marker on the voxel currently scanned by the input head. \( M(k) \) is called deterministic if its next-move function is deterministic, otherwise it is called nondeterministic. We assume that \( M(k) \) can visit any #-voxel which is 6-adjacent to some 0,1-voxel in \( T \), but can never fall off the tape \( T \) beyond these #-cells.

2 Main Results

By using the same technique as in the proof of Theorem 3.1 in [1], we get the result.

**Theorem 1.** whether or not the pattern on a p-tape is connected can be decided by a deterministic three-marker finite automaton.

It is shown in [2] that there is no deterministic one-marker finite automaton which is able to search all mazes (i.e., p-tapes). Moreover, it is shown in [3] that the set of all three-dimensional connected tapes is not recognizable by any three-dimensional nondeterministic multi-inkdot finite automaton (an inkdot machine is a conventional machine capable of dropping an inkdot on a given input tape for a landmark, but unable to further pick it up[3]). This result means

**Theorem 2.** Whether or not the pattern on a p-tape is connected cannot be decided by any deterministic one-marker finite automaton.

References

