STRESS RELAXATION OF EXPRESSED CAKE

TOSHIRO MURASE, MASASHI IWATA, TAKEHITO ADACHI
AND MASAHIRO WAKITA
Department of Chemical Engineering, Nagoya University, Nagoya 464-01

Key Words: Solid Liquid Separation, Relaxation, Consolidation, Expansion, Cake Stress, Korean Kaolin, Solka Floc, Numerical Analysis

When expression is stopped before a material reaches its equilibrium compression state, the cake stress will decrease as the material relaxes. In this paper, the change of cake stress with time was measured after an interruption of a constant-pressure or constant-rate expression operation of a semisolid material. This relaxation process is analyzed by considering the elastic deformation of the material and its irreversibility and non-linearity. It is found by numerical calculation that local expansion appears near the drainage surface while local consolidation appears near the center of the cake. The calculated stress in a transient stage falls faster than the empirical stress. The theoretical values of the equilibrium cake stress, however, agree well with the empirical ones. It is found that the equilibrium cake stress does not depend on the mode of expression (constant-pressure or constant-rate expression) but is uniquely determined by the average consolidation ratio when the expression is interrupted.

Introduction

When expression is stopped before attaining an equilibrium compression state of a material and its thickness is held constant, the cake stress decreases as the material relaxes. This stress relaxation is important to understanding the mechanical properties of a particulate bed. Schwartzberg investigated the stress relaxation phenomena of apple chunks, chopped alfalfa and drip-grind coffee grounds, while Burton studied those of Victorian brown coal experimentally. Banks approached the relaxation problem theoretically by use of a linear model in which compressibility, permeability and consolidation coefficient were treated as constants. He explained that stress relaxation can be recognized as the process of liquid redistribution until liquid pressure gradients at the end of expression fall to zero. However, there was a discrepancy between the experimental results and his theory, and he also suggested that complicated effects (the non-linear compression of the material, the irreversibility of its deformation, and the side wall friction) require consideration.

In the previous paper, expansion of a consolidated material was investigated, and it was shown that the expansion process can be analyzed by the same mathematical treatment as in the consolidation process. Here we develop a conceptually similar model for the relaxation process considering the irreversibility and non-linearity of deformation that successfully correlates our data.

1. Experimental Procedure

Experiments were conducted by use of a compression cell consisting of a cell cylinder and a piston of 6 cm diameter; the piston was fixed at the cross-head of a materials testing machine. Liquid can flow out of a material through both top and bottom drainage surfaces. A slurry of Korean kaolin–Solka floc (BW-200) mixture was used as an experimental material. The mixture was pre-consolidated in the cell under a pressure $p_e(e_1)$ of 98.1 kPa, resulting in a semisolid material with a uniform void ratio.

Constant-pressure expressions were conducted under a pressure $p_{\text{max}}$ of 0.98–6.86 MPa for 10–60 s. The movement of the piston was then stopped and the decrease of cake stress was measured by a load cell fitted on the cross-head. The relaxation process after an interruption of constant-rate expression operation was also investigated. The semisolid material was compressed at a constant expression rate $i_q$ of $2.27 \times 10^{-4}$–$1.17 \times 10^{-2}$ cm/s under increasing pressure until the pressure reached a fixed value $p_{\text{max}}$ of 0.98–6.86 MPa; the movement of the piston was then stopped and the change of cake stress was traced.

2. Theory

Figure 1 shows a result of stress relaxation after an interruption of constant-pressure expression, where $L$ denotes the thickness of compressed cake; $L_1$, its initial value; $L_0(P_{\text{max}})$, the hypothetical equilibrium thickness corresponding to the expression pressure $P_{\text{max}}$; $P$, the cake stress; and $\theta_0$, the time required after the expression is started. The movement of the piston was stopped when $\theta_0 = 60$ s ($L = 0.994$ cm). After the interruption of expression, the cake stress $P$ decreased

* Received February 1, 1989. Correspondence concerning this article should be addressed to T. Murase.
and gradually approached the equilibrium stress $p_\infty$, which was measured after one day's relaxation.

Since a liquid pressure gradient exists in the compressed cake at the instant of piston stoppage, liquid continues to flow toward the drainage surface. However, liquid cannot pass through the surface because the volume of the cake is not changing. Therefore, there is a redistribution of liquid over the length of the cake until liquid pressure gradients diminish to zero.\(\text{(1)}\)

The apparent velocity of liquid $u$ based on the material coordinate $\omega$ can be represented by $\text{(1)}$

$$u = \frac{1}{\mu \rho_s} \frac{\partial p_L}{\partial \omega}$$

where $\mu$ is the viscosity of the liquid; $\alpha$, the specific resistance of the compressed cake; $\rho_s$, the true density of the solid; and $p_L$, the liquid pressure. Based on the force balance in an infinitesimal layer of the cake, the cake stress $p$ is equal to the sum of the liquid pressure $p_L$ and the solid compressive pressure $p_s$ on an arbitrary plane in the cake.

$$p = p_L + p_s$$

Since $p$ is independent of $\omega$, Eq. (1) can be rewritten as

$$u = \frac{1}{\mu \rho_s} \frac{\partial p_L}{\partial \omega}$$

The mass balance of liquid in an infinitesimal layer of the cake leads to a continuity equation of the form

$$\frac{\partial e}{\partial \theta_R} = \frac{\partial u}{\partial \omega}$$

where $e$ is the local value of the void ratio and $\theta_R$ is the time required after the relaxation is started.

Considering only the elastic deformation of the material and combining Eq. (3) with Eq. (4) yield the basic differential equation for the relaxation process.

$$\frac{\partial p_s}{\partial \theta_R} = C_{er} \left\{ \frac{\partial^2 p_s}{\partial \omega^2} - \frac{1}{\alpha} \frac{dp_s}{d\omega} \left( \frac{dp_s}{\partial \omega} \right)^2 \right\}$$

where $C_{er}$ is the relaxation coefficient represented by

$$C_{er} = \frac{1}{\mu \rho_s (-d\omega/d\rho_s)}$$

Equation (5) is of the same form as the basic consolidation equation based on the modified Terzaghi model, in which only the elastic deformation of a particular layer is considered.

In solving Eq. (5), the initial and boundary conditions are

I.C.: $p_s = p_{s,1}$ at $\theta_R = 0$

B.C.: $\partial p_s / \partial \omega = 0$ at $\omega = 0$ and $\omega_0 / i$

Here $p_{s,1}$ in Eq. (7) is the solid compressive pressure distribution at the instant of piston stoppage, which can be obtained from the numerical calculation of a consolidation equation of the same form as Eq. (5). Equation (8) means that there is no flow through the drainage surface ($\omega = 0$) and the center of the compressed cake ($\omega = \omega_0 / i$); $i$ denotes the number of drainage surfaces ($i = 2$ in this paper).

To solve Eq. (5), the right-hand side is represented by finite difference forms in place of the derivatives. Numerical calculations based upon the Runge-Kutta-Gill method are then made.\(\text{(6)}\) For considering the irreversibility of the material deformation we utilize the equilibrium compression and expansion data shown in Fig. 2. If the material is compressed in the cell at a constant $p_s$-value, the corresponding $e$-value on the compression line in the figure is attained throughout the material after a sufficiently long time. The expansion data were obtained by reducing the compressive pressure in steps from that on the compression line. The relation between local void ratio $e$ vs. local solid compressive pressure $p_s$ in deforming materials coincides with that in Fig. 2 if
only elastic deformation occurs in the relaxation process. In the analysis based on the linear model,\(^1\) it has been shown that both local consolidation and local expansion occur simultaneously as the material relaxes. If a local value of \(p_r\) increases with time, we will accordingly use the slope of the compression data as the value of \((-de/dp_r)\) in Eq. (6); if a local \(p_r\) decreases with time, we will use the slope of the expansion data as \((-de/dp_r)\).

\(z\) in Eqs. (5) and (6) is a function of \(e\) and is represented by

\[
z = 1.48 \times 10^7 \exp\{(3.32 - e)/0.166\}
\]

for the Korean kaolin-Solka floc mixture.

3. Discussion

Figure 3 shows changes of \(p_r\) and \(e\)-distributions with time after an interruption of constant-pressure expression, which are the results of numerical calculations based on Eq. (5). Here \(io/\omega_0\) represents an arbitrary position in the material \((io/\omega_0 = 0\) at the drainage surface, and \(io/\omega_0 = 1\) at the center of the material). At the beginning of relaxation \((\theta_R = 0)\), the material is compact and dry near the drainage surface, while it is wet near the center of the material. It can be seen from the figure that \(p_r\)-values near the drainage surface decrease and that values near the center increase. As expected from the linear theory,\(^1\) both local expansion and local consolidation occur simultaneously. In the relaxation process, the following equation holds.

\[
\int_0^{\text{exp}} \{e - e(\theta_R = 0)\} \, d\omega = 0
\]

In the linear theory, a compressibility coefficient \((-de/dp_r)\) is treated as a constant. In this case, Eq. (10) is equivalent to

\[
\int_0^{\text{exp}} \{p_r - p_r(\theta_R = 0)\} \, d\omega = 0
\]

On the basis of the linear theory, consequently, the \(p_r\)-distribution in the equilibrium state becomes uniform with its mean value at \(\theta_R = 0\) (the broken line in Fig. 3). As shown in Fig. 3, however, the equilibrium \(p_r\)-value \((\theta_R = \infty)\) of the present study is lower than the mean value, because the change in the compressibility coefficient with time and the position, i.e. the non-linearity of deformation, is considered in the numerical calculation. It is also interesting that the layer near the drainage surface is more compact than that near the center of the material even in the equilibrium state. This is the result of the calculation considering the irreversibility of deformation, i.e. utilizing the compression and expansion data shown in Fig. 2. Since the \(p_r\)-value at the drainage surface is zero, the overall cake stress \(p\) coincides with the \(p_r\)-values at \(io/\omega_0 = 0\) in Fig. 3.

Figures 4 and 5 compare the theoretical cake stress \(p\) based on Eq. (5) with the empirical data. Figure 4 is the result of the interruption of constant-pressure expression, while Fig. 5 represents the constant-rate expression which was interrupted when the expression pressure \(p\) reached 2.94 MPa. The theoretical lines tend to fall faster than the observed ones. This is probably because the creep effect of the material and the friction between the material and the cell were not considered in the analysis. \(U_c\) in the figures is the average consolidation ratio defined by

\[
U_c = \frac{L_1 - L}{L_1 - L_0(\Delta p)}
\]

Solid lines in the expression stage in the figures are the estimated value of \(U_c\) and \(p\) from the numerical calculation of the basic consolidation equation.\(^6\)

The extent of relaxation is closely related to the internal condition of the cake at the beginning of
Fig. 4. Comparison of theoretical and experimental stress change after an interruption of constant-pressure expression.

Fig. 5. Comparison of theoretical and experimental stress change after an interruption of constant-rate expression.

Fig. 6. Relation between extent of relaxation and average consolidation ratio at the end of constant-pressure expression.

Fig. 7. Relation between extent of relaxation and average consolidation ratio at the end of constant-rate expression.

Fig. 8. Equilibrium cake stress $p_e$ vs. average consolidation ratio $U_{co}$ at the end of expression.

relaxation. Figures 6 and 7 represent changes of cake stress $p$ with time during expression and relaxation stages. Here the constant-pressure expression was interrupted at $\theta_e = 10$ s, 30 s, and 60 s, while the constant-rate expressions were stopped when $p$ had attained 2.94 MPa. $U_{co}$ in the figures indicates the average consolidation ratio at the instant of interruption, and is a measure of the extent of consolidation. The smaller the value of $U_{co}$, the smaller the value of $p_\infty$, i.e., the greater the extent of relaxation. These results are summarized in Fig. 8. The broken line in the figure represents the theoretical value of $p_\infty$, which varies from a pre-consolidation pressure $p_0(e_1)$ at $U_{co} = 0$ to $p_{\text{max}}$ at $U_{co} = 1$. Theoretical $p_\infty$ for relaxation after constant-pressure expression and after constant-rate expression fall on the same line. The theoretical values of the equilibrium cake stress agree well with the experimental values. This supports the analytical method in this paper, which considers the irreversibility and non-linearity of the cake deformation. It can be concluded from the figure that $p_\infty$ does not depend on the mode of expression (constant-pressure or constant-rate expression) but is uniquely determined by the value of $U_{co}$. Since $p_\infty$ depends only upon the internal condition of the cake at the
beginning of relaxation, Fig. 8 suggests that the following assumption used in the analysis of constant-rate expression is valid: 7)

"The internal condition of expressed material is uniquely determined by the instantaneous expression pressure ($p_{max}$ in Fig. 8) and the amount of removed liquid ($U_{c0}$) at the moment, and is not related to the pressure history."

**Conclusions**

The relaxation process after an interruption of expression operation was investigated. This can be recognized as a process where the liquid and the solid compressive pressure gradients at the end of expression diminish to zero.

1. In the relaxation process, there are a local expansion near the drainage surface and a local consolidation near the center of the cake. If the irreversibility of the cake deformation is considered, $e$-distribution is not uniform even at the end of relaxation.

2. The equilibrium cake stress is uniquely determined by the average consolidation ratio $U_{c0}$ when expression is interrupted.

Further research is needed to quantify the effect of creep deformation and friction between the material and the cell cylinder on the relaxation process.

**Nomenclature**

- $C_{ER}$ = relaxation coefficient defined by Eq. (6) [m$^2$/s]
- $e$ = local void ratio [-]
- $e_1$ = initial void ratio of a semisolid material [-]
- $e_0$ = local void ratio at the beginning of relaxation [-]
- $i$ = number of drainage surfaces ($i=2$ in this paper) [-]
- $L$ = thickness of material [m]
- $L_1$ = initial thickness of material [m]
- $L_e(p_{max})$ = equilibrium thickness of material under pressure $p_{max}$ [m]
- $p$ = expression pressure or overall cake stress [Pa]
- $p_L$ = local hydraulic pressure [Pa]
- $p_{max}$ = expression pressure under constant-pressure operation, or maximum expression pressure under constant-rate operation [Pa]
- $p_L$ = local solid compressive pressure [Pa]
- $p_{L1}$ = local solid compressive pressure at the beginning of relaxation [Pa]
- $p_{L1}(e_1)$ = pre-consolidation pressure under which a homogeneous semisolid material with a void ratio $e_1$ is obtained [Pa]
- $p_e$ = equilibrium cake stress [Pa]
- $q$ = rate of deliquoring per unit medium area [m/s]
- $u$ = local value of apparent relative velocity of liquid to solid [m/s]
- $U_e$ = average consolidation ratio [-]
- $U_{e0}$ = $U_e$-value at the beginning of relaxation [-]
- $\alpha$ = local specific resistance of filter cake [m/kg]
- $\theta_T$ = time required after expression is started [s]
- $\theta_{S1}$ = $\theta_T$ at the end of constant-pressure expression [s]
- $\theta_R$ = time required after relaxation is started [s]
- $\mu$ = viscosity of liquid [Pa·s]
- $\rho_s$ = true density of solid [kg/m$^3$]
- $\omega_s$ = net solid volume per unit medium area lying from medium up to an arbitrary position in the material [m]
- $\omega_0$ = total solid volume in the material per unit sectional area [m]

**Literature Cited**


(Submitted at the 21st Autumn Meeting of The Society of Chemical Engineers, Japan, at Fukuoka, October 1988).