A NEW CORRELATION METHOD FOR THE EFFECT OF VIBRATION ON FORCED-CONVECTION HEAT TRANSFER

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Key Words: Heat Transfer, Mass Transfer, Vibration, Momentum Transfer, Energy Dissipation

The forced convection heat transfer from a vibrating sphere, a cylinder and a square-section tube to water was experimentally investigated. The obtained heat transfer data with the vibration effect is well correlated in terms of the energy dissipation calculated from the fluid drag acting on the vibrating bodies. Through the use of the energy dissipation, the heat transfer from vibrating bodies to a fluid flow can be discussed analogously with the mass transfer.

Introduction

It is well known that the heat transfer rate is greater in a turbulent flow than in a laminar flow. Vibration of the heat transfer surfaces gives rise to turbulence in the case of laminar boundary layers and increases the heat transfer rate.

Studies on the effect of vibration on heat transfer can be classified into the following four cases:

- (a) *natural* convection heat transfer with *mechanical* vibration^{3-6,15,16}

- (b) *natural* convection heat transfer with *acoustical* vibration^{7-10,13,20}
- (c) *forced* convection heat transfer with *mechanical* vibration^{1,14,22,23}
- (d) *forced* convection heat transfer with *acoustical* vibration¹¹

Both acoustical and mechanical vibration are basically the same phenomenon; each creates an oscillating relative velocity vector between a heated surface and a fluid.

The studies in this field can be used either to increase the heat transfer rates in industrial units or to avoid the over-design of heat transfer elements subjected to induced vibration.^{2,17} Most previous studies, how-

* Received November 11, 1988. Correspondence concerning this article should be addressed to K. Takahashi. K. Takahashi is at Dept. of Chem. and Chem. Eng., Kanazawa Univ., Kanazawa 920.

ever, have dealt with only cases (a) and (b). A few treat cases (c) and (d), but neither theoretical nor empirical methods of predicting the magnitude of the effect of vibration on forced-convection heat transfer have yet been devised.

In this paper we propose a new correlation method that utilizes energy dissipation, and then discuss how vibration affects the forced convection heat transfer from a sphere, a cylinder and a square-section tube to water.

1. Experimental Apparatus and Procedure

The experimental system, the flow circuit and the vibration apparatus were the same as were used in previous experiments.²⁴⁾ The test section was a cylindrical duct with an inside diameter of 19.0 cm. Vibrations were imparted to the test body through a Scotch-Yoke driven by an electric motor. The maximum amplitude of these vibrations was 4.0 cm. Tap-water was used as a test fluid and the velocity of fluid flow was controlled by an inverter connected to a pump motor. The vibration vector was parallel to the fluid flow direction.

The shapes of the test bodies used were a sphere, a cylinder and a square-section tube. The heat transfer surfaces of these bodies were made of copper. Each test body was heated electrically by an electric resistance heater embedded in it. Cutaway views of the test bodies are shown in Fig. 1. The cylinder and square-section tube were provided with Bakelite resin at both ends to reduce the axial heat conduction losses and the end-effect (that is, the three-dimensional effect). For each of the test bodies, surface temperature was measured at three points by copper-constantan thermocouples soldered onto the test body surface. The surface temperatures observed at those three points agreed well with one another, showing that the uniform temperature condition was satisfied.

The heat input to the test body is given by

$$Q = IV \quad (1)$$

where I is the electric current and V is the applied voltage. The heat transfer coefficient can be calculated from the following equation.

$$h = \frac{Q}{(T_s - T_w)A} \quad (2)$$

where T_s is the surface temperature, T_w the water temperature and A the heated surface area.

The experiments were performed under the following three conditions.

- (1) Heat transfer from stationary bodies to a fluid flow
- (2) Heat transfer from vibrating bodies to a fluid at rest
- (3) Heat transfer from vibrating bodies to a fluid flow

Fig. 1. Cutaway views of test bodies: 1, copper; 2, electric resistance heater; 3, copper lead wire; 4, epoxy resin; 5, Bakelite; 6, ceramic; 7, thermocouple junction; 8, stainless steel support

When experiments under condition (3) were performed, a very slow fluid flow compared with the vibrational velocity was imposed on the test section. This was done because heat would accumulate in the fluid if the experimental time became long enough. Such heat accumulation might cause radical local change in the fluid temperature.

All the experiments were performed under such conditions that the natural convection due to temperature differences within the fluid would not affect the forced-convection heat transfer coefficient.

The experimental conditions are shown in Table 1. Physical properties of the water, thermal conductance, density and viscosity were evaluated at the film temperature T_f , which is the arithmetic mean of T_s and T_w .

2. Results and Discussion

2.1 Heat transfer from stationary bodies in a flowing fluid

The results for forced convection heat transfer without vibration are shown in Fig. 2.

A great deal of work has been done on heat transfer from cylinders^{12,18)} and spheres^{21,26)} and some correlated equations have been proposed, as listed in Table 2. To compare the present results with previous ones, $Nu_f Pr^{-1/3}$ is plotted against the Reynolds number based on the fluid flow velocity $U_0 d/\nu$ in Fig. 2. Nusselt numbers are conventionally assumed to be proportional to the cube root of the Prandtl numbers. The solid lines in Fig. 2 show the previous correlation equations given in Table 2. It can be seen that the present data for the cylinder closely match those of the previous work,¹²⁾ with at most 3 percent deviation.

Table 1. Experimental conditions

	Sphere	Cylinder	Square-section tube
Diameter of test body d [cm]	6.9	1.7, 2.3	2.8
Velocity of bulk flow U_0 [cm/s]	5.0–10.0	5.0–15.0	5.0–13.0
Amplitude of vibration a [cm]	2.0–4.0	1.0–4.0	1.0–4.0
Frequency of vibration f [Hz]	0.5–1.2	0.5–1.2	0.5–1.2
Reynolds number $U_0 d/\nu$ [—]	3000–8000	1000–2600	1500–4500
Vibrational Reynolds number $a\omega d/\nu$ [—]	3000–18000	800–6200	1000–7500
Surface temperature T_s [K]	291.15–300.15	302.15–306.15	291.15–314.15
Temperature difference ΔT [K]	3–7	5–13	5–10

As for square-section tube, its width across corners is adopted as the diameter.

Fig. 2. Correlation between forced-convection heat transfer rate and Reynolds number

Table 1. Summary of correlation for forced-convection heat transfer from literature

Geometry	Investigator	Correlation
① cylinder	McAdams ¹⁸⁾	$Nu = 0.27 Re^{0.6} Pr^{1/3}$ $1000 < Re < 60000$
② cylinder	Hilpert ¹²⁾	$Nu = 0.69 Re^{0.47} Pr^{1/3}$ $40 < Re < 4000$
③ sphere	Ranz-Marshall ²¹⁾	$Nu - 2 = 0.60 Re^{1/2} Pr^{1/3}$ $1 < Re < 10^5$
④ sphere	Tsubouchi ²⁶⁾	$Nu - 2 = 0.57 Re^{1/2} Pr^{1/3}$ $0.6 < Re < 2400$

For the sphere, the present data show values slightly higher than the previous ones.²¹⁾ The correlations of Nu_f in terms of the Reynolds number, $U_0 d/\nu$, obtained through our experiments are made by the following equations:

For spheres

$$Nu_f - 2 = 0.66(U_0 d/\nu)^{1/2} Pr^{1/3} \quad (3)$$

$$3000 < U_0 d/\nu < 8000$$

For square-section tubes

$$Nu_f = 0.65(U_0 d/\nu)^{1/2} Pr^{1/3} \quad (4)$$

$$1500 < U_0 d/\nu < 4500$$

Fig. 3. Effect of vibration on forced-convection heat transfer behavior. The solid lines show average values of heat transfer rate from vibrating bodies to a fluid at rest $Nu_v Pr^{-1/3}$.

For cylinders

$$Nu_f = 0.55(U_0 d/\nu)^{1/2} Pr^{1/3} \quad (5)$$

$$1000 < U_0 d/\nu < 2600$$

2.2 Heat transfer from vibrating bodies

The effects of vibration on forced-convection heat transfer are examined in **Fig. 3**. The heat transfer rate from the vibrating body to the fluid flow was arranged in the form of $Nu_f Pr^{-1/3} (U_0 d/\nu)^{-1/2}$, and plotted against the vibrational Reynolds number, $a\omega d/\nu$. With decreasing vibrational Reynolds number, $Nu_f Pr^{-1/3} (U_0 d/\nu)^{-1/2}$ approaches a constant value regardless of the change in fluid flow velocity. This means that the vibration effect on heat transfer rate is negligible in comparison with the forced-convection

Fig. 4. Correlation for effect of vibration on forced-convection heat transfer based on the energy dissipations. The solid lines show average results of previous experiments on mass transfer. The dotted lines show Eq. (8).

effect. Consequently, the heat transfer rate depends only on the Reynolds number $U_0 d/\nu$ under such vibration conditions. In the high-vibrational Reynolds number region, $Nu_{fv} Pr^{-1/3} (U_0 d/\nu)^{-1/2}$ increases with increasing vibrational Reynolds number $a\omega d/\nu$. In addition, $Nu_{fv} Pr^{-1/3} (U_0 d/\nu)^{-1/2}$ depends on the fluid flow velocity as well.

Also shown in Fig. 3 are the results for mass transfer obtained in our previous experiments.²⁵⁾ The Schmidt number in the previous experiments is 1535, and the Prandtl numbers in the present work are about 6 to 8. Although it is of interest to know how the vibration affects the dependency of the Nusselt number on the Prandtl number, this problem is not discussed here. To do so, it is necessary to obtain much more information over a wider range of Prandtl or Schmidt numbers through many experiments for various systems.

In previous papers we proposed a relationship between the three types of time-averaged energy dissipation E_{fv} , E_f and E_v ²⁴⁾ and reported that the mass transfer from a vibrating body in a fluid flow was well correlated with the energy dissipation ratio.²⁵⁾ The energy dissipation was found to represent inclusively the effects of vibration, fluid flow and configuration of the body on mass transfer behavior.

The ratio of heat transfer rates, Nu_{fv}/Nu_f , against the energy dissipation ratio E_f/E_v are plotted in Fig. 4. Values of Nu_f were calculated from Eqs. (3), (4) and (5). E_f and E_v are the energy dissipations for the

Table 3. Values of drag coefficient C_{dv} in Eq. (7)

Amplitude a [cm]	Sphere $d=6.9$ cm	Square-section tube $d=2.8$ cm	Cylinder	
			$d=1.7$ cm	$d=2.3$ cm
1.0	—	5.5	1.5	1.5
2.0	0.8	4.4	1.8	1.7
3.0	0.7	3.5	2.2	2.1
4.0	0.7	3.1	2.2	2.0

stationary body in a fluid flow and for the vibrating body in a liquid at rest, respectively. The energy dissipations E_f and E_v can be expressed by the following equations²⁴⁾:

$$E_f = (1/2)\rho S_0 C_{df} U_0^3 \quad (6)$$

$$E_v = (2/3\pi)\rho S_0 C_{dv} (a\omega)^3 \quad (7)$$

where C_{df} is the drag coefficient for a stationary body in a fluid flow and is equal to 0.48 for spheres, 1.55 for square-section tubes and 1.17 for cylinders.¹⁹⁾ The values of C_{dv} in Eq. (7), the drag coefficient for a vibrating body in a liquid at rest, is shown in Table 3. Details of the evaluation procedure for the drag coefficients is given in the previous paper.²⁴⁾ The values of C_{dv} for square-section tubes were measured in the same way as in the previous work.²⁴⁾

The value of Nu_{fv}/Nu_f is a measure of the effect of vibration on forced-convection heat transfer. The solid lines in Fig. 4 show average results obtained in our previous experiments in mass transfer.²⁵⁾ The heat

transfer behavior is just the same as that of mass transfer reported previously.

In the range of E_f/E_v greater than 20 in Fig. 4, the value of Nu_{fv}/Nu_f is nearly equal to unity, showing that the influence of vibration on forced-convection heat transfer is negligible. Vibration becomes effective in the forced-convection heat transfer rate only when E_f/E_v is less than 20.

Further examination of Fig. 4 reveals that Nu_{fv}/Nu_f is roughly proportional to $(E_f/E_v)^{-1/6}$ for E_f/E_v values smaller than 20 and that the proportionality constant is 1.5 for any tested body. By using these values, the correlation between Nu_{fv}/Nu_f and E_f/E_v for the sphere, the cylinder and the square-section tube can be expressed by the following single equation within $\pm 15\%$ deviation:

$$\left(\frac{Nu_{fv}}{Nu_f}\right) = 1.5 \left(\frac{E_f}{E_v}\right)^{-1/6} \quad (8)$$

$$0.01 < E_f/E_v < 20$$

Sreenivasan and Ramachandran²³⁾ reported that no remarkable change in the heat transfer rate from a cylinder was observed even at a root-mean-square vibrational velocity as high as 20 percent of the cross-flow velocity. We calculated some values of E_f/E_v from their experimental data in a previous paper.²⁵⁾ Even at the maximum intensity of vibration, E_f/E_v obtained is 104.8. Consequently the conclusion of Sreenivasan and Ramachandran seems quite natural from the above discussion of Fig. 4.

There has been little information concerning the critical level of vibrational intensity, above which the vibration becomes effective in the forced-convection heat transfer rate. Our results given in Fig. 4 are capable of showing this lower bound of effective vibrational intensity through the use of the energy dissipation ratio.

On the whole, the energy dissipation is found to be useful in correlating the heat transfer rates from bodies subjected to both a steady fluid flow and vibration, and will be helpful in analyzing the behavior of heat and mass transfer from vibrating bodies.

Conclusion

The effect of vibration on forced-convection heat transfer from a sphere, a square-section tube and a cylinder was investigated experimentally under uniform surface temperature conditions. The heat transfer coefficient was well correlated with Eq. (8) by use of the energy dissipations.

It was found that the vibration has an effect on forced-convection heat transfer only when the energy dissipation ratio E_f/E_v is less than 20.

Acknowledgment

The authors are greatly indebted to Dr. S. Mori (Kanazawa Univ.) and Dr. M. Sakakibara (Fukui Univ.) for their valuable advice and criticism during the preparation of this paper.

Nomenclature

a	= amplitude of vibration	[m]
C_d	= drag coefficient	[—]
C_p	= specific heat at constant pressure	[J/(kg · K)]
$\mathcal{D}$	= molecular diffusivity	[m ² /s]
d	= diameter of test body	[m]
E	= energy dissipation	[W]
f	= frequency of vibration	[Hz]
h	= heat transfer coefficient	[W/(m ² · K)]
I	= electric current	[A]
k	= mass transfer coefficient	[m/s]
Nu	= Nusselt number, hd/κ	[—]
Pr	= Prandtl number, $C_p\mu/\kappa$	[—]
Q	= heat input	[W]
Re	= Reynolds number, U_0d/ν	[—]
Sh	= Sherwood number, $kd/\mathcal{D}$	[—]
S_0	= projected area of body	[m ²]
T_f	= film temperature, $(T_s + T_w)/2$	[K]
T_s	= temperature of body surface	[K]
T_w	= temperature of water	[K]
U_0	= velocity of fluid	[m/s]
V	= voltage	[V]
κ	= thermal conductivity	[W/(m · K)]
μ	= viscosity	[Pa · s]
ν	= kinematic viscosity	[m ² /s]
ρ	= density	[kg/m ³]
ω	= angular frequency of vibrations, $2\pi f$	[rad/s]

<Subscripts>

f	= fluid flow
v	= vibration
fv	= fluid flow and vibration

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