A MODEL FOR STEAM BUBBLE FORMATION AT A SUBMERGED ORIFICE IN A FLOWING LIQUID

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A model for describing steam bubble formation at a submerged orifice in flowing liquid is developed. It is assumed that the bubble grows continuously in the radial direction, translates continuously in the vertical direction, and is surrounded by a thin liquid layer unaffected by the bulk liquid motion. The model allows fluctuation of pressure, temperature, and density of the vapor in a bubble in equilibrium with the liquid at the bubble wall. The temperature profile in the boundary layer is not limited to a quadratic function as assumed in Denekamp's model but varies in accordance with the propagation of heat. The detachment criterion assumed is that the bubble neck is equal to zero for the first bubble of the pairing and equal to half of the bubble radius for the bubble containing the second bubble of the pairing.

The prediction values from the present model using an explicit finite-difference technique are compared with experimental data in the literature. The results are in good agreement and show a significant improvement over Denekamp's model.

Introduction

In chemical engineering, gas-liquid contacting devices have been widely used for the transfer of matter or heat across an interface. In processes such as distillation and direct-contact heat transfer, which is of interest in regard to atomic reactors, the vapor phase is often dispersed by bubbling through orifices submerged in a liquid of relatively small depth. In those cases the coalescence and breakdown of bubbles are not serious, and not only the size of the bubble at detachment but also the instantaneous bubble size (i.e., the instantaneous surface area of the bubble) becomes important in heat and mass transfer up to the bubble detachment. Because the amount of heat and mass transferred during the period of bubble formation is considerably large, it is of interest to investigate the dynamics coupled with heat and mass transfer.

Though many workers have investigated both gas bubble formation at a submerged orifice without heat or mass transfer across the bubble wall and vapor bubble growth (or collapse) in boiling process there are few publications about the combined process. Some investigators have studied the collapse of a vapor bubble formed and detached at a submerged orifice with high-speed cinematography, but they did not report the data up to the time of bubble detachment. The only reported theoretical study of steam bubble formation at a submerged orifice was made by Denekamp et al. They assumed a nonequilibrium model at the bubble wall and a quadratic temperature profile in the boundary layer. They did not consider the effect of the mass flux on the radial velocity of the liquid. It is assumed that the bubble is spherical throughout the time of bubble formation and detachment when the lower base of the bubble touches the orifice. The only viscous drag is considered in drag force. The pressure difference between the bubble and the liquid at the orifice plate is neglected.

The objective of the present work is to study theoretically the vapor bubble formation at a single submerged circular nozzle connected to a large vapor chamber. The present model improves Denekamp's model by lessening the restrictions. The vapor in the bubble is assumed to be in equilibrium with the liquid at the bubble wall. The temperature profile in the boundary layer is not limited to a quadratic function. The effect of mass flux on the radial velocity of the liquid and on the pressure at the bubble wall, and the effect of the pressure difference between the bubble and the liquid at the orifice, are considered. An experimentally suggested drag force is used. The bubble is allowed to grow from a hemisphere and to detach as both a single bubble and paired bubble (a bubble containing the leading and the following bubble in pairing). To examine the validity of the proposed model, computed results based on the model were compared with the previous work. The present work is limited to the formation of a vapor bubble without mutual influence of bubbles formed at a

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neighboring nozzle. But it may provide statistical information about a bubbling plate.

1. Model Development

The system investigated is shown schematically in Fig. 1. From a nozzle, steam is bubbled into a subcooled liquid which flows continuously.

Though the nonspherical model\textsuperscript{8,17} for bubble formation at a submerged orifice is somewhat realistic, a spherical model is taken for simplicity because a complete solution to the problem requires that the continuity, momentum, and energy equations for liquid and vapor must be solved simultaneously and coupled to mass and energy transfer across the interface. As discussed earlier by other workers\textsuperscript{7,13} a spherical model similar to that of McCann and Prince\textsuperscript{15} is taken instead of the two-stage model.\textsuperscript{11}

Various values of length of the bubble neck at the moment of bubble detachment have been proposed to fit the calculated results to the experimental\textsuperscript{4,5,11,12} Motion pictures have shown that\textsuperscript{2,6,9} the shape of the second bubble in pairing is not a sphere but rather a small cylinder. It is assumed that the first (leading) bubble leaves the orifice at the time \( y = R \) and that the second (following) bubble is connected to the first bubble and the orifice. The combined bubble is assumed to be detached when \( y = 1.5 \, R \) is reached, as discussed by Gaddis and Vogelpohl.\textsuperscript{7} Figure 2(a) shows the model of bubble growth schematically.

Concerning heat transfer, Denekamp \textit{et al.}\textsuperscript{6} studied a nonequilibrium model with a quadratic temperature profile in the thermal boundary layer. But the nonequilibrium effect is negligible\textsuperscript{6,25,29} except in the case of extremely low pressure. For this reason and the rapid circulation of the vapor in the bubble, it is assumed that the vapor in the bubble is saturated with the liquid at the bubble wall and is uniform in temperature and pressure. The distribution of temperature around the bubble wall is assumed as shown in Fig. 2(b). The temperature profile in the boundary layer is not an exact quadratic because the vapor phase may fluctuate in pressure and temperature. A model to describe steam bubble formation at a submerged orifice was developed with the following additional assumptions:

(1) The liquid is a Newtonian fluid and incompressible.
(2) The vapor is incompressible.\textsuperscript{19}
(3) The thermophysical properties of the liquid are constant during the bubbling period.\textsuperscript{3}
(4) Viscous heating and kinetic energy of the liquid are negligible.\textsuperscript{22}
(5) The distribution of temperature and pressure in liquid phase is one-dimensional. It is only a function of radial distance and time.
(6) Heat conduction across the orifice is ignored.

(7) Energy exchange across the vapor-liquid interface is dominated by mass transfer.\textsuperscript{5}

2. Basic Equations

Total mass balance for the vapor in the bubble is

\[
d\left(\rho_v V_b\right) \over dt = \rho_v q_w + j A_b
\]

where

\[
q_w = K\left(P_c - P_b\right)^{1/2} \left(\frac{P_c - P_b}{P_c - P_b}\right)
\]

where \( K \) is the orifice constant and is obtained from experiment.\textsuperscript{12}

The equation of motion for the surrounding liquid is obtained in integrated form\textsuperscript{16} as

\[
P_{IR} - P_{IX} = \left(\frac{P_b - 2\alpha}{R}\right) - P_{IX}
\]

\[
= \rho_l \left(R \frac{dv_{IR}}{dt} + \frac{3}{2} \frac{v_{IR}^2}{R}\right) + 2j v_{IR} + \frac{4\mu_t}{R} v_{IR}
\]

The liquid velocity at the bubble wall can be replaced by the velocity of the bubble wall as shown in Fig. 3.
The bubble wall is affected by sudden velocity change of the evaporating (or condensing) molecules. From Newton’s third law of motion, the repulsive pressure on the bubble wall by the steam vapor evaporated from (or condensed on) the bubble wall is \( j/i \times j/\rho_b \) and that by the water moved toward (or away from) the bubble wall is \( j/i \times j/\rho_i \). The terms \( j/\rho_b \) and \( j/\rho_i \) are the velocity of the steam and of the water relative to the bubble wall respectively. The pressure just inside the bubble wall is \( P_b = (j/i) \rho_b - (j/i) \rho_i \). The pressure just inside the bubble wall is \( P_{IR} \), as given by (Eq. 4) (Eq. 3) follows by substitution of Eq. (4) into Eq. (3) gives
\[
\frac{\partial T_i}{\partial t} = \frac{\partial^2 T_i}{\partial \eta^2} + \left[ \frac{\rho_i}{\eta + R} \frac{dR}{dt} \left( \frac{dR}{dt} - j \frac{dR}{dt} - \frac{j}{\rho_i} \right) \left( \frac{R^2}{\eta + R} \right) \right] \frac{\partial T_i}{\partial \eta}
\]
(8)

The boundary conditions are \( T_i(t, \eta = 0) = T_{IR} \) and \( T_i(t, \eta \rightarrow \infty) = T_{1\infty} \).

The heat balance at the bubble wall is
\[
k \frac{\partial T_i}{\partial r} \bigg|_{r=R} = jL
\]
(9)

From the liquid-vapor equilibrium, the following equations are obtained.
\[
T_{IR} = T_b = T^*(P_b) \quad \text{or} \quad P_b = P^*(T_{IR})
\]
(10)

where the superscript * indicates equilibrium with the condition in parentheses.
The upward or downward forces on are bubble are balanced as
\[
\frac{d}{dt} \left( V_b \rho_b + V_i \rho_i \right) \left( \frac{dy}{dt} - v_f \right)
= -\sqrt[3]{V_b \rho_b g + V_i \rho_i g + \pi \epsilon^2 (P_b - P_i) + \pi \epsilon^2 \rho_b v_b | v_b |}
- 2\pi \rho_i \sigma \left( 1 + \frac{24}{Re} \right) (0.5 \pi R^2 \rho_i) \left( \frac{dy}{dt} - v_f \right)
\]
(11)

where \( (V_b \rho_b g + V_i \rho_i g/2) \) is virtual mass. The terms on the right-hand side of Eq. (11) are gravitational force, buoyant force, excess pressure force, force due to vapor momentum, surface tension force and drag force in that order. The drag coefficient was taken from Gaddis and Vogelpohl’s suggestion.

3. Initial Conditions

Initial conditions are somewhat complex and generally unknown because the actual conditions are affected by the temperature distribution and the flow field associated with the preceding bubble, the orifice, the wall of the liquid column, etc. However, to avoid complexity it is assumed that at the beginning of each bubble formation the velocities are small\(^1\) and that the initial shape of the bubble is a hemisphere, the radius of which is equal to the radius of the orifice. It is assumed that the liquid temperature at the bubble wall immediately becomes the same as the vapor temperature.

Equations (1), (2), (6), (8), (9), (10) and (11) can be solved with the initial conditions, \( R = r_0 \), \( dR/dt = 0 \), \( y = 0 \), \( dy/dt = 0 \), \( P_b = P_i + \rho_i gH + 2\sigma/\rho_v \), \( T_i(t = 0, \eta) = T_{1\infty} \), and \( j = (4\pi/Re)^{1/2} \{ P^*(T_{1\infty})/T_{1\infty}^{1/2} - P_b/T_{1\infty}^{1/2} \} \) at \( t = 0 \). Because initially the rate of mass condensation (or evaporation), \( j(t = 0) \), is unbounded, it is assumed to follow the Hertz-Knudsen equation.\(^{27} \)
4. Numerical solution

An explicit finite-difference method which is an extension of the methods proposed by previous investigators\(^{3,8,17}\) is used. The procedure for numerical computation at time \(t_1\) to time \(t_2\) with a time interval \(\Delta t\) is carried out as shown in Appendix.

The largest \(\Delta t\) and \(\Delta \eta\) were tested with a stability criterion and it was found that \(10^{-6}\) m and \(10^{-6}\) sec are enough (within 0.1% error) respectively. Initially, \(\Delta t\) was taken as small as \(10^{-3}\) sec and \(\Delta \eta\) as \(10^{-7}\) m because of the rapid change in temperature of the liquid phase. \(\Delta \eta\) was made larger from the bubble wall to the liquid bulk.

5. Comparison with Experimental Results and Discussion

The model was tested by comparison with the experiments of Denekamp et al.\(^{6}\) The inner diameter of the nozzle was \(1.52 \times 10^{-3}\) m and the outer diameter was about \(2.36 \times 10^{-3}\) m, as taken from their paper.

Figure 4 shows the instantaneous growth of a steam bubble. The orifice constant is an experimental parameter that depends on the orifice geometry, gas properties and gas flow rate. From Fig. 4, it is evident that the present model gives a better representation of the instantaneous bubble size. Denekamp et al.\(^{6}\) suggested that the bubble detachment time from their model is for the leading bubble of the pairing while for estimating the flow rate of the steam it is taken as the detachment time of a paired bubble. But our model gives the detachment time of the leading bubble as the time when \(y = R\) and the detachment time of a paired bubble as the time at \(y = 1.5 \times R\). The present model gives more reasonable results. Figure 5 is similar to Fig. 4. Both Denekamp's model and the present model show poor agreement with bubble size at the latter time in Fig. 5. But the present model was more useful in predicting the detachment time of the leading bubble and the paired bubble.

Table 1 shows the bubble radius at detachment, the bubble forming time, and the steam flow rate. Only the leading bubble was observed and reported in the experiments.\(^{6}\) The experimental results are compared with the theoretical values from both Denekamp's model and the present model. The present model shows a significant improvement over Denekamp’s. The difference between experimental data and theoretical results by the present model is about 60% of the difference by Denekamp’s model for detachment radius and about 80% for detachment time. The error range of the steam flow rate in the present model is about 12% while that in Denekamp's is about 35% except for run No. 4. In the case of run No. 4 the bulk flow velocity is much higher \((v_f = 0.314\) m/s) than the others \((v_f \leq 0.116\) m/s). At high bulk velocity a vortex may be formed at the nozzle tip, suppressing bubble detachment.

Concerning the plate efficiency and the plate capacity of a bubbling plate it is important to estimate not only the condensation rate of the steam but also the ratio of the steam condensed to the steam entering through the orifice. Figure 6 shows the amount of steam condensed during bubble formation and the percentage of condensation. Because \(T_s - T_{in}\) of experimental run No. 1 is greater than that of run No. 2 with other conditions the same, the steam condensation rate of experimental run No. 1 \((3.9 \times 10^{-6}\) kg/s) is greater than that of run No. 2 \((2.65 \times 10^{-6}\) kg/s). But the percentage of condensation of run No. 1 (30.8%) is smaller than that of run No.
### Table 1. Comparison of calculated results with experimental data of Denekamp et al.⁹

<table>
<thead>
<tr>
<th>Exp. run. No.</th>
<th>Exp. data</th>
<th>Present work</th>
<th>Denekamp et al.</th>
</tr>
</thead>
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<tr>
<td>Detachment radius of 1st bubble ( \times 10^3 ) (m)</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<tr>
<td>Detachment time of 1st bubble ( \times 10^3 ) (sec)</td>
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<td></td>
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<tr>
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<td>32.4</td>
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<td>Average steam flow rate ( \times 10^3 ) (kg/sec)</td>
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### Conclusion

An improved model for vapor bubble formation at a submerged orifice has been developed. The present model is more realistic and has fewer restrictions than Denekamp’s model. It is assumed that initially the bubble is a hemisphere with radius equal to that of the orifice, and that it grows continuously in the radial direction and moves continuously in translatory motion. The vapor in the bubble is assumed to be in equilibrium with the liquid at the bubble wall. A boundary layer unaffected by the bulk liquid is assumed, and the bubble detachment is taken to be when the bubble neck becomes half the bubble radius. The results predicted from the present model for moderate liquid velocity (less than 0.116 m/s) is in good agreement with the experimental results. The predicted results of instantaneous bubble size, detachment bubble size, detachment time, and average steam flow rate show that the present model a significant improvement over Denekamp’s model.

The present model should provide a useful method for predicting the characteristics of a phase-change bubble in the single-bubbling region or in the pair-bubbling region.

This model should provide the basis for design of a bubbling plate.

### Appendix

The computational procedure of the basic equations is as follows:

Step 1. From Eq. (6), \( d^2 R/dr^2 \) is calculated; then

\[
R_2 = R_1 + \left( \frac{dR}{dr} \right)_1 (\Delta t) + \frac{1}{2} \left( \frac{d^2 R}{dr^2} \right)_1 (\Delta t)^2
\]

\[
\left( \frac{dR}{dr} \right)_2 = \left( \frac{dR}{dr} \right)_1 + \frac{1}{2} \left( \frac{d^2 R}{dr^2} \right)_1 (\Delta t)
\]

Step 2. From Eq. (11), \( d^3 y/dr^3 \) is calculated; then

\[
y_2 = y_1 + \left( \frac{dy}{dr} \right)_1 (\Delta t) + \frac{1}{2} \left( \frac{d^2 y}{dr^2} \right)_1 (\Delta t)^2
\]

\[
\left( \frac{dy}{dr} \right)_2 = \left( \frac{dy}{dr} \right)_1 + \frac{1}{2} \left( \frac{d^2 y}{dr^2} \right)_1 (\Delta t)
\]

Step 3. From new coordinates, the bubble volume, bubble surface area and rate of bubble expansion are calculated.

Step 4. From Eq. (8) the liquid temperature \( T_{12} \) at all locations is obtained by explicit method except the temperature at the bubble wall.

Step 5. \( \rho_{e2} \) is assumed. Then \( T_{12} \) is obtained from Eq. (10) and \( q_{e2} \) from Eq. (2). \( L_2 \) and \( \rho_{e2} \) are obtained from the data package.

Step 6. With Eq. (9), \( j_2 \) can be calculated by the three-point method:

\[
j_2 = k \frac{1}{L_2} \int \frac{dT_i}{\partial r} \left[ r = R \right] \left[ \frac{\partial T_i}{\partial r} \right]_{r=0} = \frac{k}{L_2} \left( \frac{\partial T_i}{\partial r} \right)_{r=0} - \frac{3}{2} \left[ T_{12} + 2T_{12}2T_{12}2T_{12} \right]
\]

Step 7. Because the time increment is small enough, the total mass balance Eq. (1) can be checked as

\[
\frac{\rho_{e2} \Delta T_{12} \rho_{e2} \Delta T_{12}}{\Delta t} = \frac{(\rho_{e2} \Delta T_{12} + j_2 \Delta T_{12}) + (\rho_{e2} \Delta T_{12} + j_2 \Delta T_{12})}{2}
\]

If this equation is unsatisfied, a new pressure is assumed and

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Fig. 6: Amount of steam condensed during bubble formation and percentage of condensation ———, run No. 1; ———, run No. 2; ———, run No. 3.
iterated from step 5.

Step 8. If the total mass balance is satisfied, \( \frac{d}{dt}(\rho_v \cdot V^2) \) and \( \frac{d}{dt}(\rho_v \cdot V^2) \) are calculated.

Step 9. With an approximation \( \frac{d}{dt}(\rho_v \cdot V^2) \) and \( \frac{d}{dt}(\rho_v \cdot V^2) \), step 1 is repeated for the next time step until \( y = 1.5R \) is reached.

Nomenclature

- \( A_b \) = surface area of bubble [m²]
- \( g \) = acceleration due to gravity [m/s²]
- \( h \) = average distance from liquid surface to bubble [m]
- \( H \) = orifice submergence [m]
- \( j \) = mass flux across interface [kg/m²·s]
- \( k \) = thermal conductivity of liquid [W/m·K]
- \( K \) = orifice constant
- \( L \) = latent heat [J/kg]
- \( l_c \) = bubble neck length [m]
- \( M \) = molecular weight [kg/mole]
- \( p^* \) = pressure at saturation condition [Pa]
- \( P _p \) = pressure in bubble [Pa]
- \( P_v \) = pressure in vapor chamber [Pa]
- \( P_{pa} \) = pressure affected by phase change [Pa]
- \( P_{liq} \) = liquid pressure [Pa]
- \( P_{s,liq} \) = liquid pressure at bubble surface [Pa]
- \( P_s \) = pressure at orifice [Pa]
- \( q_s \) = system pressure [Pa]
- \( q_v \) = volumetric flow rate of vapor through orifice [m³/s]
- \( r \) = radial distance from center of bubble [m]
- \( r_n \) = inner radius of nozzle [m]
- \( r_o \) = outer radius of nozzle [m]
- \( R \) = bubble radius [m]
- \( R_s \) = gas constant [J/mol.K]
- \( t_l \) = bubble detachment time of leading bubble [s]
- \( t_f \) = bubble detachment time of following bubble [s]
- \( t \) = time [s]
- \( \Delta t \) = time increment [s]
- \( T^* \) = temperature at saturation condition [K]
- \( T_v \) = vapor temperature in bubble [K]
- \( T_{pa} \) = vapor temperature in vapor chamber [K]
- \( T_{liq} \) = liquid temperature [K]
- \( T_{liq} \) = liquid temperature at bubble wall [K]
- \( T_{liq} \) = bulk liquid temperature [K]
- \( V_s \) = bubble volume [m³]
- \( v_f \) = velocity of bulk liquid [m/s]
- \( v_s \) = vapor velocity through orifice [m/s]
- \( v_r \) = radial velocity of liquid [m/s]
- \( v_{l,liq} \) = radial velocity of liquid at bubble wall [m/s]
- \( y \) = displacement of bubble center from orifice [m]
- \( \alpha \) = thermal diffusivity of liquid [m²/s]
- \( \eta \) = radial distance from bubble wall [m]
- \( \Delta \eta \) = increment of radial distance [m]
- \( \rho_l \) = liquid density [kg/m³]
- \( \rho_{v} \) = density of vapor in bubble [kg/m³]
- \( \rho_{v,liq} \) = density of vapor in vapor chamber [kg/m³]
- \( \sigma \) = surface tension [N/m]
- \( \mu_l \) = liquid viscosity [Pa·s]

\[\langle \text{Subscriber} \rangle, \quad 1, 2 \quad = \text{values before and after a time increment}\]

Literature Cited

21) Rayleigh, Lord: *Phil. Mag.*, 34, 94 (1917).