GAS ENTRAINMENT RATE AND FLOW PATTERN IN A PLUNGING LIQUID JET AERATION SYSTEM USING INCLINED NOZZLES

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For an inclined plunging jet aeration system using liquids of low viscosity, the effects of operating conditions on the gas entrainment rate Q_g when nozzles having various values of length-to-diameter ratio L_n/D_n were employed were evaluated experimentally. The changes in Q_g were related to those in the jet shape before plunging and the liquid velocity distribution at the point where the gas sheath breaks up. An empirical equation was presented to predict the penetration depth Z of bubbles entrained by the diffusing jet. Bubble entrainment behavior was further examined in terms of the liquid velocity distribution in the submerged two-phase region and the size of entrained bubbles.

Introduction

Gas entrainment by a liquid jet is a more attractive means of aeration than are conventional gas-sparging systems, due to its simplicity in construction and operation.¹⁾ Many studies have been carried out on the characteristics of an inclined plunging liquid jet system.^{5-10,16-18)} Reviewing them, however, it is found that studies which investigated the gas entrainment and flow characteristics for systems using a liquid other than water are very scarce.^{2,7)} In addition, it seems that no expressions for predicting the gas entrainment rate Q_g and the maximum bubble penetration depth Z in systems using inclined nozzles of various lengths are yet available. To provide a sounder and more general basis for this type of aeration method, further studies of the effects of liquid physical properties and nozzle length L_n on Q_g or Z are necessary. In this study, as a step forward in evaluating characteristics such as the gas holdup and the gas absorption rate in the inclined liquid jet aeration system, the effects of operating conditions on Q_g when nozzles having various values of L_n were employed were first investigated experimentally for various liquids of low viscosity. The gas entrainment behavior was also studied in terms of the changes in jet shape before plunging and in the liquid velocity distribution near the plunging point. The relationship between Z and the operating conditions was then investigated. Bubble entrainment behavior was fur-

ther examined on the basis of the liquid velocity distribution of the diffusion jet and the size of entrained bubbles.

1. Experimental

The experimental setup is shown in Fig. 1a. Figure 1b shows schematically the outline of an inclined jet system, in which the operating and measuring variables are denoted. The apparatus consisted a liquid pump, a liquid bath ($1.5 \times 1.0 \times 0.5 \text{ m}^3$) made of transparent acrylic resin (TAR) and interchangeable

Fig. 1a. Experimental apparatus. 1, Aeration tank; 2, thermometer; 3, pump; 4, valve; 5, flow meter; 6, nozzle; 7, nozzle cap (=cylinder); 8, replacement vessel

Fig. 1b. Schematic drawing of inclined jet system

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cone-shaped nozzles (made of PVC). Each nozzle was equipped with a nozzle cap, i.e., a variable-length cylinder (0.1 m in diameter, made of TAR) having two pipes (0.015 m in diameter) for introducing entrained gas. The experimental ranges are summarized in **Table 1**. Fourteen kinds of liquids were used. **Table 2** shows the physical properties of these liquids. The gas entrainment rate Q_g was measured by the replacement method.¹³⁾ The liquid velocity near the plunging point and the liquid velocity distribution in the submerged two-phase region were measured by a Pitot tube. The maximum bubble penetration depth Z , defined as the vertical distance from the bath surface, was measured by a scale fitted to the bath wall. Observations of the shape of the discharging jet, the length l_j of the gas sheath at the plunging point and bubble diameter measurement were carried out photographically. All the experiments were carried out at 298 K with air as the gas phase.

2. Results and Discussion

2.1 Effects of operating conditions on the gas entrainment rate Q_g

In addition to the effects of parameters such as F_r , L_j/D_n , L_n/D_n and O_h on the dimensionless gas entrainment rate Q_g/Q_L ,¹³⁾ the effect of the nozzle angle θ was further investigated. **Figure 2** shows the relationship between Q_g/Q_L and F_r with L_j/D_n and θ as parameters. Q_g/Q_L tended to be large with increasing F_r and L_j/D_n and with decreasing θ , but rates of their increase differed in the range of F_r . **Figure 3** shows the relationship between Q_g/Q_L and L_n/D_n . The L_n/D_n values above which Q_g/Q_L is independent of L_n/D_n were about 17, 20 and 25 respectively for the nozzle angles of 75°, 60° and 45°. These transition L_n/D_n values, denoted as $(L_n/D_n)_{tr}$, were correlated by the following equation.

$$(L_n/D_n)_{tr} = 16.5(\sin \theta)^{-1.23} \quad (1)$$

Q_g/Q_L values were found to be almost constant at L_n/D_n values above those calculated from Eq. (1). Regardless of F_r , L_j/D_n , L_n/D_n and θ , Q_g/Q_L tended to decrease gradually with increasing Ohnesorge number O_h , satisfying the relation:

$$Q_g/Q_L \propto O_h^{-0.19} \quad (2)$$

2.2 Bending point of discharging jet

In the low F_r region, the liquid jet flow in the cylinder is bent downward due to the effect of gravity and the angle between the liquid jet flow, and the bath surface at the plunging point becomes larger than the designated nozzle angle θ . This bending point, defined as point B in Fig. 2, tended to appear in the range of high F_r with increasing L_j and with decreasing D_n and θ . The bending point appeared in the range of high F_r with an increase in L_n/D_n for the range of L_n/D_n

Table 1. Experimental conditions

Nozzle diameter D_n	0.007–0.0118 m
Nozzle length-to-diameter ratio L_n/D_n	5–50
Nozzle angle θ	45°–75°
Nozzle exit velocity V_n	2.0–13.5 m/s
Jet length L_j	0.1–0.3 m

Table 2. Physical properties of the liquids

	ρ [kg/m ³]	σ [mN·m ⁻¹]	μ [mPa·s]
Water	997.0	71.96	0.890
0.5–2.0 vol% methanol solns.	994.3–996.4	67.96–70.77	0.908–0.949
0.5–2.0 vol% ethanol solns.	994.1–996.3	65.80–70.26	0.909–0.961
1.0–2.0 wt% glucose solns.	1002.0–1006.0	73.00–73.80	0.908–0.932
0.005–0.015 kg/m ³ Tween 60 solns.	995.9–996.0	61.86–68.00	0.880–0.886
0.002 kmol/m ³ KCL soln.	997.9	73.41	0.889

Fig. 2. Relationship between Q_g/Q_L and F_r at $D_n=0.0118$ m and $L_n/D_n=50$

Fig. 3. Relationship between Q_g/Q_L and L_n/D_n

below $(L_n/D_n)_{tr}$, but was almost independent of L_n/D_n above that range. There was little difference in the bending point due to those of the liquid. The bending point expressed in the form of the Froude number

$(F_r)_B$ was found to be correlated within 10% error to variables such as D_n , L_j , L_n/D_n and $\sin \theta$ by the following equations.

For nozzles of L_n/D_n between 5 and $(L_n/D_n)_{tr}$:

$$(F_r)_B = 0.200 D_n^{-0.88} L_j^{0.40} (L_n/D_n)^{0.11} (\sin \theta)^{-0.65} \quad (3)$$

where the unit of D_n and L_j is m.

For nozzles of L_n/D_n above $(L_n/D_n)_{tr}$:

$$(F_r)_B = 0.200 D_n^{-0.88} L_j^{0.40} (\sin \theta)^{-0.75} \quad (4)$$

Equations (3) and (4) may be usable for predicting the lower operational limit of the inclined liquid jet system.

2.3 Transition points of gas entrainment curve

As for the dependence of F_r on Q_g/Q_L , it is found from Fig. 2 that the curve can be divided into three distinct regions: (I) a low F_r region, (II) a transitional F_r region, and (III) a high F_r region at two transition points P_1 and P_2 . These two transition points tended to appear in the range of high F_r with increasing L_j and with decreasing of D_n and θ . For the effect of L_n/D_n , the transition points shifted slightly to low F_r with increasing L_n/D_n for the range of L_n/D_n below $(L_n/D_n)_{tr}$, but were almost independent of L_n/D_n above that range. There was little difference in the transition points due to those in the liquids tested. Two transition points P_1 and P_2 expressed in the form of the Froude numbers $(F_r)_1$ and $(F_r)_2$ respectively were correlated within 10% error to variables such as D_n , L_j , L_n/D_n and $\sin \theta$ by the following equation.

*For nozzles of L_n/D_n between 5 and $(L_n/D_n)_{tr}$:

$$(F_r)_1 = 0.250 D_n^{-0.92} L_j^{0.13} (L_n/D_n)^{-0.09} (\sin \theta)^{-0.22} \quad (5)$$

$$(F_r)_2 = 0.531 D_n^{-0.93} L_j^{0.21} (L_n/D_n)^{-0.07} (\sin \theta)^{-0.21} \quad (6)$$

For nozzles of L_n/D_n above $(L_n/D_n)_{tr}$:

$$(F_r)_1 = 0.531 D_n^{-0.71} L_j^{0.19} (\sin \theta)^{-0.27} \quad (7)$$

$$(F_r)_2 = 0.591 D_n^{-0.84} L_j^{0.20} (\sin \theta)^{-0.25} \quad (8)$$

The tendency for the gas entrainment curve above $(F_r)_B$ to be divided into three regions has been observed in earlier works^{5,7,16,17)} using long nozzles of $L_n/D_n \geq 50$. Van de Sande and Smith^{16,17)} reported that V_n at the transition point P_1 was between about 4.0 and 5.0 m/s and the transition from II to III began to occur near a value of the Weber number ($= \rho_a V_n^2 D_n / \sigma$) of 10. On the other hand, Kumagai and Endoh⁷⁾ showed that a Weber number of 10 at the transition point P_2 was questionable. And as the correlations for predicting the two transition points P_1 and P_2 , the following equations expressed in the form of the Reynolds numbers $(R_e)_1$ and $(R_e)_2$ were presented.⁷⁾

Fig. 4. Comparison of $(V_n)_1$ and $(V_n)_2$ values calculated from Eqs. (5)–(8) with those calculated from Eqs. (9) and (10)

$$(R_e)_1 = 3.21 \times 10^2 D_n^{0.832} L_j^{0.128} (\sin \theta)^{-0.301} v^{-0.624} \quad (9)$$

$$(R_e)_2 = 1.11 \times 10^1 D_n^{0.700} L_j^{0.053} (\sin \theta)^{-0.052} v^{-0.890} \quad (10)$$

The correlation accuracy of their own data with Eqs. (9) and (10) is about ± 50 and $\pm 25\%$ respectively.⁷⁾ For reference, a comparison of transition nozzle velocities $(V_n)_1$ and $(V_n)_2$ corresponding to two transition points P_1 and P_2 calculated respectively from Eqs. (5)–(8) and Eqs. (9) and (10) is shown in Fig. 4. The values of $(V_n)_2$ estimated from Eq. (10) tended to be larger than those from our correlations. To investigate the effect of v , further experiments using liquid of a higher v such as was employed by Kumagai and Endoh⁷⁾ will be necessary.

2.4 Experimental equations for Q_g/Q_L

It can be generally assumed that the factors which influence Q_g/Q_L are F_r , L_j/D_n , L_n/D_n , O_h and θ . Therefore, the following dependence was assumed.

$$Q_g/Q_L = f(F_r, L_j/D_n, L_n/D_n, O_h, \sin \theta) \quad (11)$$

According to the previous discussions of Q_g/Q_L for regions I, II and III, the dependences of Q_g/Q_L as a function of these parameters were determined. Q_g/Q_L was expressed in the following form within 20% error.

$$Q_g/Q_L = A F_r^{f(\sin \theta)} (L_j/D_n)^B (L_n/D_n)^C O_h^D (\sin \theta)^E \quad (12)$$

$$f(\sin \theta) = F(\sin \theta)^G \quad (13)$$

The values of empirical constants A , B , C , D , E , F and G which change according to the range of L_n/D_n and V_n are summarized in Table 3.

Some reports have dealt with Q_g in inclined in regions I and II for an air-water system. Kumagai and Endoh^{5,7)} measured Q_g jet systems. Van de Sande and Smith¹⁸⁾ measured Q_g in regions I–III for various liquids. For the data in the three regions which include these existing results for water, Fig. 5 shows a comparison between the results calculated from Eqs.

Table 3. Values of A , B , C , D , E , F and G in Eqs. (12) and (13)

L_n/D_n [—]	V_n [m/s]	A [—]	B [—]	C [—]	D [—]	E [—]	F [—]	G [—]
$5 \leq L_n/D_n < (L_n/D_n)_{tr}$	$V_n < (V_n)_1$	1.6×10^{-3}	0.58	0.31	-0.19	-3.80	0.80	2.00
	$(V_n)_1 \leq V_n < (V_n)_2$	6.8×10^{-3}	0.58	0.31	-0.19	-1.30	0.23	1.00
	$V_n \geq (V_n)_2$	1.5×10^{-3}	0.58	0.31	-0.19	-4.30	0.72	2.30
$L_n/D_n \geq (L_n/D_n)_{tr}$	$V_n < (V_n)_1$	5.9×10^{-3}	0.51	0	-0.19	-3.80	0.80	2.00
	$(V_n)_1 \leq V_n < (V_n)_2$	2.3×10^{-2}	0.51	0	-0.19	-1.30	0.23	1.00
	$V_n \geq (V_n)_2$	5.2×10^{-3}	0.51	0	-0.19	-4.30	0.72	2.30

(12) and (13) and those observed. Since the data of Kumagai and Endoh^{5,7)} in region III for water were difficult to read from their reported figures, Q_g values calculated from Eq. (14)^{5,7)} were regarded here as the data to be compared.

$$Q_g = 3.0 \times 10^{-5} (\rho D_n^2 V_n^2) (L_j/D_n)^{0.35} (\sin \theta)^{-0.3} \quad (14)$$

The correlation accuracy of Eq. (14) is about $\pm 40\%$.^{5,7)} As seen in **Figs. 5a** and **5b**, good coincidence between $(Q_g/Q_L)_{cal}$ and $(Q_g/Q_L)_{obs}$ is seen for regions I and II. Also, the correlation accuracy is found to be high compared with that of existing empirical equations.^{5,7)} Regarding the fact that the data of Kumagai and Endoh^{5,7)} in region III tend to be larger than those calculated from our correlation, one of the causes seems likely to be due to a difference in experimental techniques for measuring Q_g . For discussion of cases using a liquid of higher ν , further experimental works are required.

2.5 Relation between change of jet surface and gas entrainment behavior

The effects of F_r and L_j/D_n on the shape of the discharging jet were almost the same as for the vertical water jet.¹³⁾ That is, the surface disturbance or the sinuosity of the jet tended to become larger with increasing F_r and L_j/D_n . For the effect of liquid physical properties expressed in the form of O_h , the surface disturbance or the degree of sinuosity of the jet tended to become slightly smaller with increasing of O_h . The rate of gas entrained is affected by the jet shape before plunging.^{3,4,11,12)} Considering that the larger the change of jet surface the larger the amount of gas entrained, one of the causes for the differences in Q_g/Q_L due to those of parameters such as F_r , L_j/D_n and O_h seems to be related to the change of jet surface before plunging. For the effect of L_n/D_n , in the same manner as in the vertical water jet,¹³⁾ it was observed that the change of jet surface when nozzle of $L_n/D_n \geq 15$ were employed was almost the same.

2.6 Gas entrainment behavior in terms of the liquid velocity distribution near the plunging point

Figure 6 shows velocity profiles of the jet measured. The profiles are found to differ somewhat depending on the liquid. The changes of the center-line velocity

Fig. 5. Comparison of Q_g/Q_L values calculated from Eqs. (12) and (13) with those observed

Fig. 6. Liquid velocity profiles near the plunging point

V_m with x were then investigated by plotting V_m/V_j in **Fig. 7**. The transition x/D_j values from which the V_m/V_j ratio becomes smaller than 1.0 can be shown to differ according to the operating conditions. The relation between the transition distance x_t in **Fig. 7**

Fig. 7. Relationship between V_m/V_j and x/D_j

Fig. 8. Relationship between x_t and F_r at $\theta=45^\circ$ and $L_n/D_n=15$

and F_r with L_j/D_n as a parameter is shown in Fig. 8. The points B, P_1 and P_2 correspond respectively to the bending point and the two transition points in Q_g/Q_L defined previously. As is clear from a comparison between Figs. 2 and 8, the way of changing of x_t is very similar to that of changing of Q_g/Q_L . Good correspondence was also seen between the changes of x_t with O_h and those of Q_g/Q_L . The changes of x_t with L_n/D_n and θ as well were found to correspond to those of Q_g/Q_L (Fig. 9).

2.7 Relation between the length of gas sheath l_f and the transition distance x_t

Figure 10 shows typical photographs of the gas sheath, taken through an L-shaped periscope immersed into the liquid at a point a short distance from the plunging point. The position at which the gas sheath breaks up can be regarded as the distance of l_f designated in each photograph. The effects of F_r , L_j/D_n , L_n/D_n , O_h and θ on l_f were similar to those on x_t . Figure 11 shows the relation between l_f and x_t . These results indicate that the length of gas sheath l_f is approximately equal to the transition distance x_t .

2.8 Effects of operating conditions on the bubble penetration depth Z

The effects of θ and O_h on the dimensionless bubble penetration depth Z/D_n were further investigated, along with the effects of F_r , L_j/D_n and L_n/D_n .¹³⁾ The dimensionless bubble penetration depth Z/D_n at $L_j/D_n \leq 20$ tended to decrease with L_j/D_n , but was almost constant at $L_j/D_n > 20$. Z/D_n decreased with

Fig. 9. Relationship between x_t and L_n/D_n

Fig. 10. Photographs of gas sheath

Fig. 11. Comparison between l_f and x_t

L_n/D_n , if L_n/D_n was smaller than $(L_n/D_n)_{tr}$, but was almost constant above $(L_n/D_n)_{tr}$. Z/D_n was found to increase with increasing F_r and θ . There was little difference in Z/D_n due to those of the liquid physical properties.

2.9 Experimental equations for Z/D_n

From the above results, Z/D_n was expressed in the following form within 15% error.

$$Z/D_n = (L_j/D_n)^a (L_n/D_n)^b 10^{f(F_r)} (\sin \theta)^c \quad (15)$$

$$f(F_r) = d + e(\log F_r) + f(\log F_r)^2 \quad (16)$$

The values of empirical constants a , b , c , d , e and f which change according to the range of L_n/D_n and L_j/D_n are summarized in Table 4. Using nozzles of $L_n/D_n \geq 50$, Kumagai and Endoh¹⁰⁾ measured Z in an air-water system and have correlated Z by the following equation within about 35% error.

Table 4. Values of a , b , c , d , e and f in Eqs. (15) and (16)

L_n/D_n [—]	L_j/D_n [—]	a [—]	b [—]	c [—]	d [—]	e [—]	f [—]
$5 \leq L_n/D_n < (L_n/D_n)_{tr}$	$8.4 \leq L_j/D_n < 20$	-0.27	-0.11	1.64	1.56	0.028	0.30
	$20 \leq L_j/D_n < 43$	0	-0.11	1.00	1.29	0.220	0.39
$L_n/D_n \geq (L_n/D_n)_{tr}$	$8.4 \leq L_j/D_n < 20$	-0.27	0	1.64	1.37	0.040	0.32
	$20 \leq L_j/D_n < 43$	0	0	1.00	1.54	0.720	0.55

$$Z = 50(D_n V_n)^{f(L_j/D_n)} (L_j/D_n)^{-0.55} (\sin \theta)^{1.5} \quad (17)$$

$$f(L_j/D_n) = 1.7(L_j/D_n)^{-1/6} \quad (18)$$

Figure 12 shows a comparison between Z/D_n values calculated and observed, in which the data of Kumagai and Endoh¹⁰⁾ are also plotted. Z/D_n could be correlated by Eqs. (15) and (16) within about 15% error.

2.10 Bubble entrainment behavior in terms of the liquid velocity distribution in submerged two-phase region

Figure 13 shows a velocity profile of liquid in the bubble penetration region. For the region where V_m exists on the jet axis, the changes of V_m/V_j with x/D_j were investigated. **Figure 14** shows the typical relationship between V_m/V_j and x/D_j . There was little difference in the rate of deceleration of V_m/V_j at L_j/D_n above 28.57, as seen in the figure. As to the effect when the liquids were varied, the degree of deceleration tended to be large in the liquids in which small bubbles are easily formed. For the cases using nozzles of various L_n/D_n values at different angles, the changes of V_m/V_j with x/D_j were almost the same in nozzles of L_n/D_n above $(L_n/D_n)_{tr}$. The degree of deceleration tended to be large with increasing θ due to the increased buoyancy effect of entrained bubbles.

2.11 Flow behavior of bubbles near point Z

Bubbles that reached point Z begin to rise to the bath surface at the angle α from its position. This angle α is determined by the magnitude of the freely rising velocity V_r of the bubble and that of the horizontal velocity V_h at point Z. The mean diameter d_B of bubbles near point Z was evaluated on the basis of photographs. From the values of d_B and the liquid physical properties, V_r was estimated.^{14,15)} The values of V_h were measured by a Pitot tube. The angle α was determined from the relation:

$$\alpha = \tan^{-1}(V_r/V_h) \quad (19)$$

The α value from Eq. (19) coincided with the inclination angle of a short thread fitted to the wire immersed at point Z, within 20% error. This result indicates that if V_r and V_h are given as a function of operating conditions the flow angle α of bubbles leaving point Z can be predicted beforehand. The values of V_r are estimated from d_B and the liquid

Fig. 12. Comparison of Z/D_n values calculated by Eqs. (15) and (16) with those observed

Fig. 13. Liquid velocity profiles in the bubble penetration region

Fig. 14. Relationship between V_m/V_j and x/D_j

physical properties.^{14,15)} The relationship between d_B and the operating conditions in the present experimental systems could be expressed in the following form within 10% error:

$$d_B = \beta F_r^{-0.096} O_h^{-0.70} \quad (20)$$

where the value of empirical constant β is 9.58×10^{-6} m for methanol solutions, 6.81×10^{-6} m for ethanol solutions and 2.82×10^{-5} m for the liquids other than methanol and ethanol solutions. V_h values were correlated by Eq. (21) within 10% error.

$$V_h = \gamma F_r^{0.173} O_h^{0.781} (\sin \theta)^{-0.839} \quad (21)$$

The value of empirical constant γ in Eq. (21) is 18.10 m/s for methanol solutions, 15.60 m/s for ethanol solutions and 26.30 m/s for the liquids other than methanol and ethanol solutions.

Conclusions

For an inclined plunging liquid jet system using nozzle having different L_n/D_n values, the gas entrainment rate Q_g and flow pattern were studied experimentally for various liquids of low viscosity. When nozzles of L_n/D_n above $(L_n/D_n)_{tr}$, i.e., $16.5(\sin \theta)^{-1.23}$, were employed, Q_g was confirmed to be independent of L_n/D_n . A new correlation for Q_g was presented. The changes in Q_g could be related to those in jet shape before plunging, in liquid velocity distribution near the plunging point and in length of gas sheath formed. An empirical equation, useful for predicting the bubble penetration depth Z , was also presented. Furthermore, the flow behavior of entrained bubbles due to variation of operating conditions was found to be well explained by the changes in liquid velocity distribution and the bubble size in the submerged two-phase region.

Nomenclature

d_B	= equivalent diameter of gas bubble	[m]
D_j	= jet diameter at L_j	[m]
D_n	= nozzle diameter	[m]
F_r	= Froude number ($= V_n/\sqrt{gD_n}$)	[—]
$(F_r)_B$	= Froude number corresponding to bending point of jet	[—]
$(F_r)_1, (F_r)_2$	= transition Froude number at P_1 and P_2 , respectively	[—]
g	= acceleration of gravity	[m/s ²]
l_f	= length of gas sheath; see Fig. 1b	[m]
L_j	= length of liquid jet; see Fig. 1b	[m]
L_n	= nozzle length	[m]
O_h	= Ohnesorge number ($= \mu/\sqrt{\rho\sigma D_n}$)	[—]
Q_g	= volumetric flow rate of entrained gas	[m ³ /s]
Q_L	= volumetric flow rate of liquid through the nozzle	[m ³ /s]
Re	= Reynolds number ($= \rho V_n D_n/\mu$)	[—]
$(Re)_1, (Re)_2$	= transition Re number at P_1 and P_2 , respectively	[—]
V	= liquid velocity at any point vertical to axis where V_m exists	[m/s]
V_h	= horizontal velocity component at	

	point Z	[m/s]
V_j	= average jet velocity at L_j	[m/s]
V_m	= center-line velocity of submerged jet	[m/s]
V_n	= jet velocity at nozzle exit	[m/s]
$(V_n)_1, (V_n)_2$	= critical jet velocity corresponding to P_1 and P_2 , respectively	[m/s]
V_r	= rising velocity of bubble of d_B in size	[m/s]
x	= distance from bath surface	[m]
x_t	= transition distance from bath surface which V_m/V_j ratio becomes smaller than 1.0	[m]
y	= distance from x axis	[m]
Z	= bubble penetration depth	[m]
α	= average flow angle of bubbles leaving from point Z	[—]
μ	= viscosity of liquid	[Pa · s]
ρ, ρ_a	= density of liquid and air, respectively	[kg/m ³]
σ	= surface tension of liquid	[N/m]
ν	= kinematic viscosity of liquid	[m ² /s]
θ	= angle between liquid jet discharge and bath surface	

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