APPLICATION OF MIXING AND DEPOSITION DATA OF BROWNIAN PARTICLES IN A MODEL ALVEOLUS TO HUMAN ALVEOLI

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Air-mixing in an expanding and contracting vessel influences the deposition of Brownian particles. In the present work, by using a previously developed technique to measure mixing and deposition of aerosol particles in a balloon (Otani, et al., 1990), extensive data were collected to cover a wide range of balloon expansion and contraction conditions. The results were examined in terms of dimensionless parameters derived from the convective diffusion equation to predict the behavior of submicron particles in actual human alveoli. It was found that (1) convective mixing in the balloon is characterized by the Reynolds number at the balloon throat; (2) for Re > 5, the mixing is determined by fluid convection; (3) for Re < 5, Brownian diffusion enhances the mixing; and (4) when dimensionless duration of balloon expansion and contraction is longer than $5 \times 10^{-4}$, the mixing does not influence particle deposition and the deposition efficiency can be predicted by diffusional deposition of particles in a spherical vessel with constant volume.

Introduction

Most submicron particles inhaled in the human lung are transported to the periphery of the human airway, where these particles are transferred to the residual air as a result of mixing between tidal air and the residual air and are deposited on the alveolar wall. To evaluate the effects on health of these submicron particles, the transferred and deposited fractions in the alveolar region must be accurately estimated. In the previous paper, an alveolus was modeled by a distensible balloon and the behavior of Brownian particles in the model alveolus was studied by using a newly developed experimental technique. As a result,
the behavior of Brownian particles in an expanding/contracting balloon was found to be characterized by two parameters, namely mixing volume $V_{\text{mix}}$ and deposition coefficient $\eta$ per unit breath, which were obtained from the following equations.

$$V_{\text{mix}} = \frac{R_n(V_0 + V_L)(1 - f_{1/n}^1)}{f_{1/n}^1(1 - f_{1/n}^1)}$$  \hspace{1cm} (1)$$

$$\eta = 1 - \left\{ f_{1/n}^1 + \frac{R_n(1 - f_{1/n}^1)}{1 - f_{1/n}^1} \right\}$$  \hspace{1cm} (2)$$

where $R_n$ is the number fraction of aerosol particles flowing out from the balloon during $n$ breaths to the initially trapped particles in the balloon, the initial balloon volume $V_0$, the balloon-throat volume $V_L$, and the residual fraction of particles in the balloon after $n$ breaths $f_{1/n}$.

Since the previous experiments were conducted by using a balloon with diameter in cm order, the question arises as to how we can apply these results to predict the behavior of Brownian particles in actual human alveoli, whose diameter is in the order of several hundred micrometers. Therefore, the goal for this work was to answer this question. However, because of experimental limitations we could not conduct experiments using a model alveolus of the same size as the actual one. In the present work, extensive data were collected to cover a wide range of balloon expansion/contraction conditions, and by applying the similarity law for the motion of Brownian particles their behavior in the actual alveolar region of the human lung was predicted.

1. Experimental

The experimental apparatus and procedures are the same as used in the previous work. In the present experiments, latex balloons of various dimensions, as shown in Table 1, were used. For all balloons, the ratio of the balloon-throat diameter to the balloon diameter was 0.75 and that of throat length to the balloon diameter was 0.65. These ratios are consistent with the model used by Cinkotai. The volume of the balloon $V(t)$ was varied at a constant rate as follows.

During expansion ($0 \leq t \leq \tau/2$)$$V(t) = V_0 + 2\Delta Vt/\tau$$  \hspace{1cm} (3)$$

During contraction ($\tau/2 \leq t \leq \tau$)$$V(t) = V_0 - 2\Delta V(t - \tau)/\tau$$  \hspace{1cm} (4)$$

where $\Delta V$ is the expansion/contraction volume and $\tau$ is the duration of balloon expansion/contraction. The experimental conditions are shown in Table 2. The range of $\Delta V$ used in the experiments is comparable to the ventilation fraction in the actual alveoli; $\Delta V = 3V_0$ for ventilation during heavy exercise and $\Delta V = 3V_0$ for maximum ventilation.

| Table 1. Dimensions of model alveolus |
|-----------------|-----|-----|-----|
| Diameter of balloon*1, $d_0$ [mm] | 10.0 | 20.0 | 22.0 |
| Diameter of balloon throat*2, $d_1$ [mm] | 7.5 | 15.0 | 16.5 |
| Length of balloon throat*2, [mm] | 6.5 | 13.0 | 14.3 |

*1 size of moulded latex balloon
*2 size of glass tube (these dimensions do not change during balloon expansion and contraction)

| Table 2. Experimental conditions |
|-----------------|-----|
| Test aerosol particles | 0.03–0.2 μm (DMA-classified NaCl monodisperse uncharged particles) |
| Duration of balloon expansion/contraction, $\tau$ | 6–120 s |
| Expansion/contraction volume of balloon, $\Delta V$ | 0.5–15.6 cm$^3$ ($\Delta V = V_0$–3$V_0$) |
| Number of balloon expansion/contractions, $n$ | 5–10 |

2. Similarity Law of Brownian Particle Motion

In an expanding/contracting balloon, influent clear air and residual aerosol in the balloon are mixed by convective mixing and/or Brownian diffusion. However, when turbulent mixing is negligible (this is the case for the alveolar region where Reynolds number is in the order of $10^{-4}$ to $10^{-3}$), the behavior of Brownian particles in the expanding/contracting balloon can be characterized by the following convective diffusion equation.

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = D \nabla^2 C$$  \hspace{1cm} (5)$$

For the diffusion of point particles, the boundary condition is

$$C = 0 \quad \text{at} \quad r = R(t)$$  \hspace{1cm} (6)$$

In Eqs. (5) and (6), $C$ is the aerosol concentration, $\mathbf{v}$ the fluid velocity, $D$ the diffusivity, $t$ the time, $\nabla$ is the differential operator, $r$ the radial distance, and $R(t)$ the radius of the balloon, which is a function of time. In solving Eqs. (5) and (6), Yu et al. introduced a quasisteady state assumption. First, they numerically solved the flow field, $\mathbf{v}$, with boundary condition of $v_r = dR(t)/dt$ at $r = R(t)$, and then obtained a numerical solution of Eq. (5) with the calculated flow field. If we follow their procedure, the above equations can be written in dimensionless form as
\[
\frac{\partial C^*}{\partial t^*} + Pe \mathbf{v}^* C^* = \nabla^2 C^* \tag{7}
\]

and

\[
C^* = 0 \quad \text{at} \quad r^* = 1 \tag{8}
\]

The dimensionless quantities appearing in Eqs. (7) and (8) are

\[
C^* = C(C_0), \quad t^* = tD/R(t)^2, \quad \mathbf{v}^* = \mathbf{v}/U,
\]

\[
Pe = UR(t)/D, \quad \nabla^2 = R(t)\nabla, \quad r^* = r/R(t) \tag{9}
\]

The fluid velocity \( \mathbf{v}^* \) in an expanding/contracting balloon is a function of balloon wall movement (the balloon shape and the rate of balloon expansion and contraction) as well as of Reynolds number. If we denote all the influences of the balloon wall movement on the flow field by \( \Omega \), the following functional relationship for \( C^* \) is expected.

\[
C^* = f(t^*, x^*, Pe, Re, \Omega) \tag{10}
\]

where \( x^* \) is the location in the balloon.

Since the mixing volume and deposition coefficient are determined by the concentration distribution in the balloon as a solution of Eq. (10) at \( r^* = t^* \), they are expressed by the following functional relationship.

\[
V_{\text{mix}} = f_1(t^*, Pe, Re, \Omega) \tag{11}
\]

\[
\eta = f_2(t^*, Pe, Re, \Omega) \tag{12}
\]

3. Results and Discussion

3.1 Mixing Volume

In a limiting case where particle diffusion is negligible, Eq. (11) can be reduced to

\[
V_{\text{mix}} = f_1(t^*, Re, \Omega) \tag{13}
\]

The above equation suggests that for a given \( t^* \) and \( \Omega \), \( V_{\text{mix}} \) is a function of \( Re \) alone. Further, because the flow field in the balloon is considered to be largely influenced by the flow entering the balloon through the throat, the mixing volumes \( V_{\text{mix}} \) are plotted against Reynolds number at the balloon throat in Fig. 1. The figure clearly shows that the mixing volumes for both \( \Delta V = V_0 \) and \( 3V_0 \) are functions of \( Re \) alone when \( Re \) is larger than about 5. Below \( Re = 5 \), \( V_{\text{mix}} \) increases with decreasing particle size, i.e., it is not a single-value function of \( Re \). Therefore, Fig. 1 demonstrates that the mixing in the balloon is characterized by a single parameter \( Re \) when \( Re > 5 \), whereas for \( Re < 5 \) we must account for other parameters, i.e., \( Pe \) for Brownian diffusion. Incidentally, the difference in \( V_{\text{mix}} \) due to \( \Delta V \) is attributed to the difference in duration of balloon expansion and contraction, i.e., the difference in \( t^* \).

\( V_{\text{mix}} \) for various initial balloon volumes and different \( \Delta V \) are converted to \( V_{\text{mix}}^* \), which is a mixing volume per unit time and unit initial volume of the balloon. \( V_{\text{mix}}^* \) for \( d_m = 0.1 \mu m \) is shown in Fig. 2 as a function of \( Re \). This figure shows that \( V_{\text{mix}}^* \) is a single-value function of \( Re \), being independent of \( \Delta V \) and \( t^* \).

Because all the balloons used in the present experiments are geometrically similar, one may question the necessity of choosing \( Re \) at the balloon throat to characterize mixing in the balloon. To answer this question, an additional experiment was carried out with a dissimilar balloon, i.e., a balloon with the same throat dimension but different initial volume. The measured \( V_{\text{mix}}^* \) for the dissimilar balloon is shown by the solid squares in Fig. 2. It is seen from the figure that the data fall on the same experimental curve, and we may conclude that for \( Re > 5 \) the convective mixing in the expanding/contracting balloon with various initial balloon volumes and various durations of balloon expansion/contraction is characterized by \( Re \) alone at the balloon throat, because the flow field in the balloon is determined by the flow entering the balloon through the throat.
3.2 Deposition coefficient

Here, we consider the limiting case where fluid convection has no influence on particle deposition. In Eq. (12), letting $Re \to 0$ we have

$$\eta = f_2(\tau^*, \Omega)$$

Equation (14)

**Figure 3** shows the deposition coefficient per breath as a function of the dimensionless duration of balloon expansion and contraction $\tau^*$. In calculating $\tau^*$, the radius of the balloon, $R(t)$, was chosen to be the time-averaged radius of the balloon, i.e., the balloon radius at $t = \tau/4$. In the figure, solid lines are the predicted curves for diffusional deposition in a spherical vessel with constant volume (see Appendix A). As seen from the figure, when the dimensionless duration of balloon expansion and contraction is larger than about $\tau^* = 5 \times 10^{-4}$ the experimental data agree with the predicted curve, suggesting that we need not include either the influence of mixing or the movement of the boundary to predict particle deposition. The deviation between the predicted curve and the experimental data in the region of $\tau^* < 5 \times 10^{-4}$ is attributed to the mixing, which lowers the concentration gradient at the wall surface.

3.3 Prediction of Particle Behavior in Human Alveoli

In evaluating the effect on health of inhaled particles, it is necessary to know what fraction of inhaled particles is transferred to the residual lung air rather than being expelled in the expired air. In this section, based on the results shown in Figs. 2 and 3, the fraction of particles transferred from the tidal air to the residual alveolar air during a single breath is estimated.

The dimensions of the human lung alveoli and the range of dimensionless parameters for the maximal and heavy breathing conditions are compared with those for the present model in Table 3. It is seen that $Re$ and $\tau^*$ for actual human alveoli are far from the critical $Re$ and $\tau^*$ at which convective mixing comes to play an important role in aerosol mixing and deposition. Therefore, in predicting the behavior of Brownian particles in actual human alveoli, we may take only the Brownian diffusion of particles into account, excluding the influence of convective air mixing and the movement of the alveolar wall.

Since Brownian diffusion is the only transport mechanism of aerosol particles in the human alveolar region and the movement of the alveolar wall does not affect particle deposition, the behavior of aerosol particles in human alveoli can be characterized by the unsteady-state diffusion equation for a spherical vessel of constant volume (see Appendix B). The problem of estimating fractional recovery of particles from an alveolus during a single breath may be reduced to the problem of calculating the fraction of particles that diffuse from the aerosol cloud entering an alveolus to the residual alveolar air while the alveolus is held at time-averaged volume during a breath. In the reduced problem, the change in ventilation volume of air is reflected by the change in the ratio of aerosol core radius to that of the alveolus, $k$. The calculation was carried out by assuming that the residual alveolar air initially resides in the spherical part of the alveolus. **Figure 4** shows the calculated concentration distribution change with dimensionless time $\tau^*$ respectively for $\Delta V = V_0$ ($k = 0.69$) and $3V_0$ ($k = 0.84$). Since the fraction of aerosol particles that are not expelled in the exhaled air is equal to the fraction of particles that diffuse into the layer of clean air, it is obtained by integrating the concentration from $\tau^* = 0$ to $\tau^*$ (see Eq. (B-3)). For instance, $d_a = 0.1 \mu m$ gives $\tau^* = 0.3$ for the heavy breathing condition ($\Delta V = V_0$) and $\tau^* = 0.2$ for maximum ventilation ($\Delta V = 3V_0$). For these $\tau^*$ values, the fractions of particles expelled in the expired air are respectively 0.04 and 0.20. This model calculation therefore suggests that, at a fixed breathing frequency, the larger the tidal volume, the greater is the fraction.

![Table 3. Comparison of characteristic dimensions and dimensionless parameters for the present model alveolus and those for the actual human alveolus.](image)

<table>
<thead>
<tr>
<th>Model</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter, $d_a$ [mm]</td>
<td>10–22</td>
</tr>
<tr>
<td>Throat diameter, $d_t$ [mm]</td>
<td>7.5–16.5</td>
</tr>
<tr>
<td>Breathing cycle, $\tau$ [s]</td>
<td>6–120</td>
</tr>
<tr>
<td>Air velocity in throat, $U$ [cm/s]</td>
<td>0.07–1.3</td>
</tr>
<tr>
<td>Dimensionless breathing cycle, $\tau^*$ [-]</td>
<td>$3 \times 10^{-8}$–$1 \times 10^{-7}$</td>
</tr>
<tr>
<td>Reynolds number, $Re$ [-]</td>
<td>0.3–20</td>
</tr>
<tr>
<td>Peclet number, $Pe$ [-]</td>
<td>$2 \times 10^3$–$2 \times 10^5$</td>
</tr>
</tbody>
</table>

* These values are taken from Cinkotai and Yu et al.
of aerosol particles entering an alveolus that is recovered from it.

Conclusion

From the experiments conducted in the present work it was found that (1) mixing between the tidal air and residual aerosol in an expanding/contracting balloon is characterized by the Reynolds number alone at the balloon throat when the Reynolds number is larger than about 5; (2) for \( Re < 5 \) particle diffusion enhances the mixing; and (3) when the dimensionless breathing cycle of the balloon expansion and contraction is longer than \( 5 \times 10^{-4} \), flow convection does not affect particle deposition and the deposited fraction is predicted by the solution for diffusional deposition in a sphere with constant volume.

Calculations of the deposition efficiency of Brownian particles in actual human alveoli based on the present experimental findings showed that, at a fixed breathing frequency, the larger the tidal volume the less is the fraction of aerosol particles deposited in the alveoli.

Appendix A  Deposition coefficient per breath

Under the condition where neither the flow field nor movement of the balloon wall affect aerosol behavior, the expanding/contracting balloon may be regarded as a spherical vessel of fixed size, the volume of which is equal to the average balloon volume (i.e., \( V_0 + AV/2 \)). Further, for simplicity, consider particle diffusion from a layer of aerosol into a core of clean air (the volume is \( AV/2 - V_k \)) as shown in Fig. A-1. Letting \( v = 0 \) in Eq. (5), the basic diffusion equation is given by the following equation.

\[
\frac{\partial C^*}{\partial t^*} = D \left( \frac{\partial^2 C^*}{\partial r^2} + \frac{2}{r} \frac{\partial C^*}{\partial r} \right)
\]

\[\text{(A-1)}\]

Fig. A-1. Model for Brownian particle deposition in a balloon

The initial and boundary conditions are

\[
C^* = 0 \quad \text{for} \quad 0 \leq r < r_c \\
C^* = 0 \quad \text{for} \quad r_c \leq r < r_b \\
C^* = 0 \quad \text{at} \quad r = r_b \quad \text{for} \quad t^* > 0
\]

\[\text{at} \quad t^* = 0 \quad \text{(A-2)}\]

\[\text{at} \quad t^* = 0 \quad \text{(A-3)}\]

where

\[
r_c = \frac{3}{4n} \left( \frac{V_0 + \frac{AV}{2}}{2} \right)^{1/3}
\]

\[\text{(A-4)}\]

\[
r_b = \frac{3}{4n} \left( \frac{AV}{2} - V_k \right)^{1/3}
\]

\[\text{(A-5)}\]

Introducing the dimensionless quantities given in Eq. (9) and letting \( R(t) = r_c \), Eqs. (A-1)-(A-3) are rewritten in dimensionless form as follows.

\[
\frac{\partial C^*}{\partial t^*} = \frac{2}{r^*} \frac{\partial C^*}{\partial r^*} + \frac{\partial^2 C^*}{\partial r^2}
\]

\[\text{(A-5)}\]

\[
C^* = 0 \quad \text{for} \quad 0 \leq r^* < \kappa
\]

\[\text{at} \quad t^* = 0 \quad \text{(A-6)}\]

\[
C^* = 1 \quad \text{for} \quad \kappa \leq r^* \leq 1
\]

\[\text{at} \quad t^* = 0 \quad \text{(A-7)}\]

where

\[
\kappa = \left( \frac{AV/2 - V_k}{V_0 + AV/2} \right)^{1/3}
\]

\[\text{(A-8)}\]

Eq. (A-5) can be solved analytically and the solution is

\[
C^* = \sum_{m=1}^{\kappa} A_m \exp(-m^2\pi^2 r^*) \sin(m\pi r^*)
\]

\[\text{at} \quad t^* = 0 \quad \text{(A-9)}\]

\[
A_m = \frac{2}{m^2\pi^2} \left( -m^2\cos(m\pi) + \kappa m^2\cos(m\pi) - \sin(m\pi) + \sin(m\pi) \right)
\]

\[\text{(A-9)}\]

From the above solution, deposition coefficient \( \eta \) per breath is obtained by performing the following integration.

\[
\eta = 1 - \int_0^1 C^* \cdot 4\pi r^2 dr^*
\]

\[\text{(A-10)}\]

\[
\text{(A-10)}\]

Appendix B  Recovery of aerosol from an alveolus

The recovery of inhaled aerosol particles that enter an alveolus during a single breath can be obtained by considering the particle diffusion from a core of aerosol to the surrounding layer of clean air, which is the reverse case of Appendix A. The concentration distribution in the balloon is obtained as a solution of Eq. (A-5) by using the following initial conditions.

\[
C^* = 1 \quad \text{for} \quad 0 \leq r^* < \kappa
\]

\[\text{at} \quad t^* = 0 \quad \text{(B-1)}\]

\[
C^* = 0 \quad \text{for} \quad \kappa \leq r^* \leq 1
\]

\[\text{at} \quad t^* = 0 \quad \text{(B-2)}\]
where $\kappa$ is the ratio of average aerosol core radius to that of the alveolus with average volume. Because it is assumed that clean air is initially present in the spherical part of the alveolus, letting $V_L = 0$ in Eq. (A-8), we have

$$\kappa = \left( \frac{A V / 2}{V_0 + A V / 2} \right)^{1/3}$$

The solution of the above equation is

$$C^* = \sum_{m=1}^{\infty} B_m \exp(-m^2 \pi^2 r^*) \frac{\sin(m \pi r^*)}{r^*}$$

$$B_m = \frac{2}{m^2 \pi^2} \left[ -k \sin(m \pi r^*) + \sin(m \pi r^*) \right]$$

Since particles that diffuse into the clean air layer are not expelled during balloon contraction, the recovery of aerosol particles during a single breath is calculated by the following equation.

$$R_n = \frac{\int_{r^*}^{\infty} C_{r^*}^{*} \Delta n r^* dr^*}{(4/3)\pi r^3}$$

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### Nomenclature

- $C$ = Particle concentration [m$^{-3}$]
- $C^*$ = Dimensionless particle concentration (= $C/C_0$) [-]
- $C_0$ = Initial particle concentration [m$^{-3}$]
- $D$ = Brownian diffusivity [m$^2$/s]
- $d_b$ = Diameter of balloon [mm]
- $d_t$ = Diameter of balloon throat [mm]
- $d_p$ = Particle diameter [m]
- $f_r$ = Fraction of residual particles in a balloon after $n$ breaths [-]
- $n$ = Number of breaths [-]
- $P_e$ = Peclet number (= $U R(t)/D$) [-]
- $R(t)$ = Radius of balloon at time $t$ [m]
- $R_o$ = Fractional recovery of aerosol during $n$ breaths [-]
- $Re$ = Reynolds number at balloon throat (= $\rho U d_t/\mu$) [-]

$r$ = radial distance [m]
$r^*$ = Dimensionless radial distance (= $r/R(t)$) [-]
$r_0$ = Radius of balloon with time-averaged volume during a breath [m]
$r_b$ = Radius of inner spherical core with time-averaged volume during a breath [m]
$t$ = Time [s]
$t^*$ = Dimensionless time (= $t D/r_b^2$) [-]
$U$ = Airflow velocity at balloon throat [m/s]
$V_0$ = Initial balloon volume [cm$^3$]
$V_L$ = Volume of balloon throat [cm$^3$]
$V_{mix}$ = Mixing volume between aerosol and clean air in balloon per breath [cm$^3$]
$V_{mix}'$ = Mixing volume between aerosol and clean air in balloon per unit time and unit initial volume (= $V_{mix}(V_0 t)$) [cm$^3$/min$^{-1}$]
$\Delta V$ = Expansion/contraction volume of balloon [cm$^3$]
$e$ = Fluid velocity vector [m/s]
$e^*$ = Dimensionless fluid velocity vector (= $e/U$) [-]
$v_r$ = Radial component of fluid velocity vector [m/s]
$x^*$ = Coordinate system [m]
$\eta$ = Deposition coefficient [-]
$\kappa$ = Ratio of inner spherical core radius to balloon radius (= $r_b/r_0$) [-]
$\mu$ = Fluid viscosity [Pa·s]
$\rho_f$ = Fluid density [kg/m$^3$]
$\tau$ = Duration of balloon expansion and contraction [s]
$\tau^*$ = Dimensionless duration of balloon expansion and contraction (= $\tau D/r_b^2$) [-]
$\Omega$ = Influence of balloon wall movement on the flow field [-]

### Literature Cited