STABILITY CRITERION FOR THE SURFACE OF THE DEPOSIT BED OF SOLID-LIQUID FLOWS IN HORIZONTAL PIPES

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During the transport of coarse particles through a pipeline, if unexpected accidents such as sudden pump stoppage due to failure of the power supply occurs, particles in the pipeline settle and form a deposit bed in the pipe bottom. In such cases, the settled particles must be removed prior to resuming operation. There are several ways of removing particles deposited in pipelines. One of them uses fluid drag force. In this paper we propose a new criterion: the fluid velocity at which dunes are formed on the surface of the deposit bed, taking into account both the shortening of removal time and the avoidance of pipe blockage. The stability criterion for the surface of the deposit bed in pipelines was theoretically and experimentally studied by using the linear stability theory.

Introduction

Solid-liquid flows are commonly encountered in several fields of industry, such as chemical processing and the mining of coal and other ores. The hydraulic transport of coarse particles has recently received considerable attention because of the reduction achieved in crushing and dewatering costs.^{8,10)}

During the transport of coarse particles through a pipeline, if unexpected accidents such as sudden pump stoppage due to failure of the power supply occurs, particles in the pipeline settle and form a deposit bed in the pipe bottom. In such cases, the settled particles must be removed prior to resuming operation. There are several ways of removing particles deposited in pipelines. One of them uses fluid drag force. The advantage of this method lies in the ability to resume transport without requiring the dismantling of the

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pipeline. Previously, the fluid velocity at which some particles on the surface of the deposit bed begin to move (the so-called critical tractive velocity) has been used,^{1,7)} because if the fluid velocity is excessive the particles are soon fluidized and cause pipe blockage. However, this method is not economical, because it requires a long period to remove the particles.

In this paper we propose a new criterion: the fluid velocity at which dunes are formed on the surface of the deposit bed, taking into account both the shortening of removal time and the avoidance of pipe blockage. As dunes are generally called sand waves in the field of civil engineering,¹²⁾ dunes formed on the surface of the deposit bed are regarded as a kind of wave. This means that the critical condition for forming dunes can be examined as the bound of stability of the bed surface. Therefore, the objective of the present study is to examine theoretically and experimentally the stability criterion for the surface of the deposit bed in a pipeline.

1. Experimental Apparatus and Procedures

Figure 1 shows a schematic diagram of the experimental apparatus used in this study. It consists of a mixing tank, a pump with a variable-speed drive, an electromagnetic flowmeter, horizontal test sections, a differential pressure transducer, a strain meter, an A/D converter and a personal computer (NEC PC9801VM). The test sections are 13 m-long transparent acrylic pipes of 39.7 mm and 49.7 mm i.d. to allow visual observation of particle behavior. Each test section is covered with a quadrilateral acrylic water jacket to eliminate the refraction of light by the curvature of the circular pipes.

Measurement of bed height was made with a ruler attached beside the water jacket. The pressure loss was measured by a strain-gauge differential-pressure transducer with a capacity of 10 kPa. Readings from the differential-pressure transducer were transmitted through the strain meter and the A/D converter to the personal computer. The formation of particle waves, that is, the stability criterion for the surface of the deposit bed, was decided by visual observation. The wave length of the dune was measured with the ruler. The wave velocity of the dune was obtained by measuring the time to pass through the 0.5-m test section.

The experimental procedure is as follows. First, the critical tractive condition is kept for about 20 minutes in order to flatten the bed surface, and then the bed height is measured. Secondly, the fluid velocity is increased until the formation of dunes is observed, and then the measurements of fluid velocity, pressure loss, velocity of dunes and wave length are made.

The particle sizes covered in this study were 0.67, 1.23, 2.18 and 2.24 mm; other properties are listed

Fig. 1. Schematic diagram of experimental apparatus.

Table 1. Properties of particles

Materials	d_s [mm]	S [—]	V_t [m/s]	R_{cp} [—]	C_D [—]
Quartz Sand	0.67	2.65	0.10	75	1.68
Crushed Rock	1.23	2.74	0.15	177	1.12
	2.18	2.74	0.16	443	1.92
Glass Bead	2.24	2.49	0.28	506	0.56

in Table 1. Here, the particle sizes are equivalent diameters of spheres.

2. Experimental Results

Figures 2 and 3 show examples of the relationship between the dimensionless bed height, $H (=H_b/D)$, and Froude number, $Fr (=u_0^2/gD_{e0})$, at the bound of stability of the bed surface (refer to Fig. 4). Solid lines indicate the calculated results from Eq. (28), and dashed lines indicate the calculated results from Eq. (30), which is based on the critical tractive velocity given by Eq. (31). It can be seen that Fr shows a gentle increase with increase of H .

Figures 5 and 6 show examples of the relationship between dimensionless bed height, H , and the hydraulic gradient, I . Solid lines indicate the calculated results from Eq. (37). The hydraulic gradient I increases with increase of H .

Figures 7 and 8 show examples of the relationship between H and the dimensionless wave number, k , given by

$$k = 2\pi D_e / \lambda_m \quad (1)$$

The typical tendency is a decrease of k with increase of H . This is owing to the change of dune shape from triangular to flattened with increase of H . This change of dune shape means that the length of particle waves becomes large with increase of H . As can be seen from Eq. (1), k is decreased by the increase of H , because the increase of H causes the decrease of D_e and the increase of λ_m .

Figures 9 and 10 show examples of the relationship

Fig. 2

Fig. 2, 3. Relationship between dimensionless bed height H and Froude Number Fr at the bound of stability of the bed surface.

Fig. 3

Fig. 4. Dunes formed at the surface of deposit bed: $D=49.7$ mm, $d_s=2.24$ mm, $H=0.4$.

between H and the dimensionless dune velocity, C_r . Solid lines indicate the calculated results from Eq. (26). C_r shows a gentle increase with increase of H .

3. Theoretical Study of Stability Criterion

3.1 Basic equations

Figure 11 shows a schematic diagram of flow with a deposit bed. A_w and A_b show the effective cross-sectional areas of the fluid flow and the deposit bed, respectively. As the fluid velocity increases, some particles on the surface of the deposit bed begin to move at a certain fluid velocity u_c , which has been generally called the critical tractive velocity. Further increase of fluid velocity causes the formation of dunes at a certain fluid velocity u_0 . This state of flow is schematically shown in Fig. 11 by the dashed line.

Let us start with the equation of motion and continuity for fluid, and the equation of continuity for solid particles. Here, the fluid flow in the deposit bed is quite small and is assumed to be negligible. Furthermore, it is assumed that the flow is one-dimensional and that the porosity in the deposit bed ε is constant.

Fluid:

$$u \frac{\partial u}{\partial x} = -\frac{1}{\rho A_w} \frac{\partial (P A_w)}{\partial x} - \frac{4\tau}{D_e} \quad (2)$$

$$\partial(u A_w)/\partial x = 0 \quad (3)$$

Particles:

$$\frac{\partial A_s}{\partial t} + \frac{1}{(1-\varepsilon)} \frac{\partial q_B}{\partial x} = 0 \quad (4)$$

where D_e is the hydraulic mean equivalent diameter defined by

$$4A_w/(P_s + P_w) = D_e \quad (5)$$

τ is the averaged shear stress over the cross section of the fluid flow, given by

$$\tau = (\tau_s P_s + \tau_w P_w)/(P_s + P_w) \quad (6)$$

In the analysis of bed formation in open channel flows, the equation for sediment transport rate is generally used instead of that for motion of particles,³⁾ because the equation of continuity is expressed in terms of sediment transport rate.

In Eq. (3), the term $(\partial A_w/\partial t)$ should be considered. In this study, after Kuroki *et al.*, it was assumed for simplicity of analysis that it was negligible, taking into account that the velocities of dunes were relatively small and the fluid flow above dunes could be considered to be a quasi-steady flow.

Variables in Eqs. (2)–(5) are assumed to be the sum of the time-averaged value and the fluctuating one due to the minute disturbance given to the surface of the deposit bed, and are expressed by

$$\left. \begin{aligned} A_s &= A_{w0} \tilde{A}_s, & A_w &= A_{w0}(1 + \tilde{A}_w) \\ u &= u_0(1 + \tilde{u}), & \tau &= \tau_0(1 + \tilde{\tau}) \\ P &= P_0(1 + \tilde{P}), & q_B &= q_{B0}(1 + \tilde{q}_B) \\ D_e &= D_{e0}(1 + \tilde{D}_e) \end{aligned} \right\} \quad (7)$$

Subscript 0 and superscript $\sim$ refer to the time-averaged value and the fluctuating one due to the disturbance, respectively.

In this study, the linear stability theory is assumed to be applied to the dune formation, because it has already been confirmed by Tatsumi *et al.*⁹⁾ that the generation of unstable waves could be analyzed by the linear stability theory.

Fig. 5

Fig. 6

Fig. 5, 6. Relationship between dimensionless bed height H and hydraulic gradient I at the bound of stability of the bed surface.

Fig. 7

Fig. 8

Fig. 7, 8. Relationship between dimensionless bed height H and dimensionless wave number k .

Fig. 9

Fig. 10

Fig. 9, 10. Relationship between dimensionless bed height H and dimensionless velocity of dunes.

Fig. 11. Schematic diagram of flow with a deposit bed.

Substituting Eq. (7) for Eqs. (2)–(5), and neglecting the terms of high degree, the following equations are obtained.

$$Fr(\partial \tilde{u}/\partial \bar{x}) + a_1(\partial \tilde{P}/\partial \bar{x}) + a_1(\partial \tilde{A}_w/\partial \bar{x}) - a_2 \tilde{D}_e - a_2 \tilde{P} + a_3 \tilde{\tau} = 0 \quad (8)$$

where

$$Fr = u_0^2/(gD_{e0}), \quad a_1 = P_0/(\rho g D_{e0}), \\ a_2 = P_x/(\rho g), \quad a_3 = 4\tau_0/(\rho g D_{e0}) \\ \bar{x} = x/D_{e0}$$

$$(\partial \tilde{u}/\partial \bar{x}) + (\partial \tilde{A}_w/\partial \bar{x}) = 0 \quad (9)$$

$$(\partial \tilde{A}_s/\partial \bar{\tau}) + A_r(\partial \tilde{q}_B/\partial \bar{x}) = 0 \quad (10)$$

$$\tilde{A}_w - \tilde{D}_e = 0 \quad (11)$$

Since P_s and P_w are of course affected by the disturbance, it is not proper to assume that $P_s + P_w$ is constant. However, it is assumed for simplicity of analysis that the change of P_s and P_w due to the disturbance is negligibly small.

The formation of dunes causes fluctuations in the fluid velocity and the hydraulic mean diameter. In this case, the Froude number is defined by means of the time-averaged values, which are denoted by subscript 0.

In Eq. (10), $\bar{t}$ and A_r are the dimensionless time and the dimensionless sediment rate, expressed by

$$\bar{t} = (u_0 D_{e0} / A_0) t \quad (12)$$

$$A_r = q_{B0} / [(1 - \varepsilon) u_0 D_{e0}^2] \quad (13)$$

Unknown quantities in Eqs. (8)–(11) are $\tilde{A}_s$, $\tilde{A}_w$, $\tilde{u}$, $\tilde{\tau}$, $\tilde{P}$, $\tilde{q}_B$ and $\tilde{D}_e$. Therefore, three further equations are necessary to achieve closure of the system.

The first equation is derived from the condition of total cross-sectional area of the pipe.

$$A_w + A_s + A_b = A = \pi D^2 / 4 \quad (14)$$

The second equation is one for the pressure loss. Here, we use the next equation proposed by Newitt *et al.*⁵⁾

$$(\partial P / \partial x) = (\partial P / \partial x)_w [1 + KC_v (S - 1) g D / V_m^2] \quad (15)$$

Strictly speaking, this equation is applicable to flow with a moving bed, and not applicable to flow with a deposit bed. Flow with a deposit bed, however, would be regarded as a special case of flow with the moving bed when the velocity of the moving bed becomes zero. We therefore assumed the use of Eq. (15).

The third equation is one for sediment transport rate. In this study, the next equation is assumed by taking into account the one proposed by Kuroki *et al.*³⁾ for open-channel flows.

$$q_B / g d_s^3 (S - 1) A_b = \phi [\tau_* (x - \delta)] \quad (16)$$

$$\tau_* = \tau / \rho g d_s (S - 1) \quad (17)$$

where δ is generally termed “lag distance” and was first introduced by Kennedy.²⁾ It is well-known that the lag distance plays an important role in estimating the sediment transport rate, and is defined as the one by which the local sediment transport rate lags the local fluid velocity at the flow state of the deposit bed.

Substituting Eq. (7) for Eqs. (14)–(16), and neglecting the terms of high degree, the following equations are obtained.

$$\tilde{A}_w + \tilde{A}_s = 0 \quad (18)$$

$$(\partial \tilde{P} / \partial \bar{x}) - a_4 \tilde{u} - a_5 \tilde{P} = 0 \quad (19)$$

$$\tilde{q}_B - a_6 \tilde{\tau} (\bar{x} - \bar{\delta}) = 0 \quad (20)$$

where

$$a_4 = 2 D_{e0} (P_x - P_{wx}) / P_0, \quad a_5 = D_{e0} P_x / P_0,$$

$$a_6 = \frac{\phi D_{e0} \tau_0}{\rho q_{B0}} \sqrt{\frac{d_s A_b}{g(S - 1)}}, \quad \bar{\delta} = \delta / D_{e0}$$

3.2 Analysis of stability criterion

Let us consider the minute disturbance given to the surface of the deposit bed, expressed by

$$\tilde{A}_s = \sum_{n=1}^{\infty} \hat{A}_{sn} \exp\{nik(\bar{x} - c\bar{t})\} \quad (21)$$

where $\hat{A}_{sn}$ is the amplitude of the disturbance. To analyze the stability of the surface of the deposit bed, all the disturbances expressed in Eq. (21) must be investigated. It has been confirmed, however, that the generation of unstable waves can be analyzed solely by the disturbance of $n=1$.⁹⁾ Therefore, we use the next equation in this analysis.

$$\tilde{A}_s = \hat{A}_s \exp\{ik(\bar{x} - c\bar{t})\} \quad (22)$$

c is the dimensionless complex velocity of dunes, expressed by

$$c = C_r + iC_i \quad (23)$$

where C_r is the dimensionless velocity of dunes, and the product of k and C_i is termed the dimensionless amplification degree of the disturbance.

As can be seen from Eq. (22),

If $kC_i > 0$, the disturbance is amplified (unstable).

If $kC_i = 0$, the bound of stability is attained (neutral).

If $kC_i < 0$, the disturbance is damped (stable).

Fluctuated quantities induced by the disturbance given by Eq. (22) are expressed as follows:

$$\left. \begin{aligned} \tilde{A}_w &= \hat{A}_w \exp\{ik(\bar{x} - c\bar{t})\} \\ \tilde{u} &= \hat{u} \exp\{ik(\bar{x} - c\bar{t})\} \\ \tilde{\tau} &= \hat{\tau} \exp\{ik(\bar{x} - c\bar{t})\} \\ \tilde{P} &= \hat{P} \exp\{ik(\bar{x} - c\bar{t})\} \\ \tilde{q}_B &= \hat{q}_B \exp\{ik(\bar{x} - c\bar{t})\} \\ \tilde{D}_e &= \hat{D}_e \exp\{ik(\bar{x} - c\bar{t})\} \end{aligned} \right\} \quad (24)$$

Substituting Eq. (24) for Eqs. (8)–(11) and Eqs. (14)–(16), the following simultaneous equations are obtained.

$$\begin{pmatrix} ikFr & ika_1 - a_2 & ika_1 & -a_2 & a_3 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -c & A_r \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ -a_4 & ik - a_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_6(1 - ik\bar{\delta}) & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{P} \\ \hat{A}_w \\ \hat{D}_e \\ \hat{\tau} \\ \hat{A}_s \\ \hat{q}_B \end{pmatrix} = 0 \quad (25)$$

To have a solution of non-zero for each quantities, it is necessary that the determinant of the matrix in Eq. (25) is always zero. Considering this condition, C_r and kC_i are derived as follows:

$$C_r = \frac{A_r a_6 \{ [(k^2 + a_5^2)Fr - a_4(a_1 a_5 - a_2)]k^2 + (a_1 a_4 + a_2)k^2 + a_2 a_5(a_4 + a_5) \}}{a_3(k^2 + a_5^2)} \quad (26)$$

$$kC_i = \frac{a_6 A_r k^2 \{ [(a_1 a_4 + a_2)k^2 + a_2 a_5(a_4 + a_5)]\bar{\delta} + (a_1 a_5 - a_2)a_4 - (k^2 + a_5^2)Fr \}}{a_3(k^2 + a_5^2)} \quad (27)$$

The stability criterion is given as $kC_i = 0$. Hence, we finally obtain the following expression for the Froude number, which gives the stability criterion for the surface of the deposit bed.

$$Fr = \frac{[(a_1 a_4 + a_2)k^2 + a_2 a_5(a_4 + a_5)]\bar{\delta} + (a_1 a_5 - a_2)a_4}{(k^2 + a_5^2)} \quad (28)$$

As can be seen from Eq. (28), the value of lag distance is necessary for calculating Froude number, Fr . Unfortunately, the formula for lag distance has not yet been theoretically obtained, so we tried to obtain an empirical formula for lag distance. We found by trial and error that the following expression gives good agreement with measurements in the range of the present work.

$$\bar{\delta} = \alpha \{ 1 + \beta [(D - H_b)/P_s]^m / \exp(\tau_*) \} \quad (29)$$

where

$$\alpha = a(d_s/D)^{-0.26}$$

$$\beta = b(d_s/D)^{1.07}$$

The coefficients a , b and index m were obtained by using least-square fittings; These are listed in **Table 2**.

The calculated results for Fr by use of Eq. (29) are shown in Figs. 2 and 3 by solid lines. It can be seen that the calculated results agree well with the experimental ones.

The dashed lines indicate the calculated results for Fr_c , based on the critical tractive velocity,¹⁾ expressed by

$$Fr_c = u_c^2 / g D_{e0} \quad (30)$$

$$u_c = \{ 4g(S-1)d_s / 3c_1^2 c_2 [d_s / (D(1-H))]^{2/n'} \}^{1/2} \quad (31)$$

$$c_1 = (2n' + 1)(2 + n') / 2n'^2 \quad (32)$$

$$c_2 = n' 2^{(2/n' + 2)} / [(1 + n')(2 + n')] \quad (33)$$

According to Kao *et al.*,¹⁾ $n' = 6.3$. As can be seen in these figures, Fr is approximately 1.2–3.0 times Fr_c . This means that the method based on Fr which was proposed in this study allows a reduction in the period of removing particles to about 10–70 percent in

Table 2. Constants of coefficients a , b and index m

d_s [mm]	a	b	m
0.67	1.69	0.32	2.7
1.23	1.87	0.15	1.9
2.18	2.72	0.07	1.9
2.24	3.03	0.06	2.5

comparison with that using the previous method, because the sediment transport rate is approximately proportional to the square of the fluid velocity.

It is well-known that the friction coefficient at the surface of the deposit bed, f_k , becomes large when dunes are formed.⁴⁾ In this case, f_k is expressed as the sum of the friction coefficient at the surface of the deposit bed without the formation of dunes, f_s , and that of dune shape, that is,

$$f_k = f_s + f_d \quad (34)$$

In the previous study,⁶⁾ f_s was given by

$$f_s = 8 / \{ 2.5 \ln[11(D - H_b)/(2d_s)] \}^2 \quad (35)$$

f_d is the additional friction coefficient due to the formation of dunes. As dunes consist of individual particles, it can be considered that f_d is affected by drag coefficient C_D of particles. Taking this into account, we obtained the following empirical formula, given by

$$f_d = 0.26(1 - H_b/D) / C_D + 3.63(d_s/D) \quad (36)$$

Then the hydraulic gradient I is expressed by

$$I = \frac{4\tau}{D} \frac{1}{\rho g} = f_k \frac{u^2}{2g} \frac{1}{D} \quad (37)$$

The calculated results for hydraulic gradient I from Eq. (37) by use of Eq. (34) are shown in Figs. 5 and 6 by solid lines. Agreement is nearly satisfactory.

As can be seen from Eq. (26), the value of the sediment transport rate is necessary for calculating the dimensionless dune velocity. There are many theoretical and experimental formulas for sediment transport rate in open-channel flows.¹¹⁾ On the other hand, unfortunately, no formula for sediment transport rate in pipeline flows has yet been theoretically obtained. Therefore, we obtained the following empirical formula for it by trial and error, taking into account that the sediment transport rate in open-channel flows is a function of dimensionless shear stress, τ_* .

$$\phi = 10^{[-4.87(\tau_* - \tau_{*c})^{0.2}(H_b/D)^n] - 0.18} \quad (38)$$

where

$$n = 6.3(H_b/D - 0.3)^2 + 3$$

$$\tau_{*c} = 0.1$$

The calculated results for C_r by use of Eq. (38) are shown in Figs. 9 and 10 by solid lines. The calculated results are in good agreement with the experimental ones.

Conclusions

The stability criterion for the surface of the deposit bed was theoretically and experimentally investigated. The conclusions obtained in this study are summarized as follows.

(1) The Froude number and the hydraulic gradient at the bound of stability increased with increase of dimensionless bed height.

(2) A mathematical model for investigating the bound of stability of the bed surface was proposed. The equation to give the Froude number at the bound of stability was obtained as Eq. (28).

(3) The calculated results of Froude number, hydraulic gradient and dimensionless dune velocity were compared with experimental ones, and agreement was nearly satisfactory. Hence the model proposed in this study was confirmed to be reasonable.

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Nomenclature

A, A_b, A_s, A_w	= cross-sectional areas of pipe, particle bed, dunes and fluid flows, respectively	[m ²]
a, b	= coefficients	[—]
C_D	= drag coefficient of a single particle	[—]
C_i	= imaginary part of c	[—]
C_r	= real part of c (= dimensionless dune velocity = V_G/u_*)	[—]
C_v	= delivered solid concentration by volume	[—]
c	= dimensionless complex velocity of dunes (= $C_r + iC_i$)	[—]
c_1, c_2	= function of n'	[—]
D	= pipe diameter	[m]
D_e	= equivalent pipe diameter for A_w	[m]
d_s	= particle diameter	[m]
Fr	= Froude number (= u_0^2/gD_{e0})	[—]
Fr_c	= Froude number (= u_c^2/gD_{e0})	[—]
f_d, f_k, f_s	= friction coefficients for dune shape, bed surface with dunes, and bed surface without dunes, respectively	[—]
g	= gravitational acceleration	[m/s ²]
H	= dimensionless bed height	[—]
H_b	= bed height	[m]
I	= hydraulic gradient	[—]
K	= coefficient	[—]
k	= dimensionless wave number	[—]
n	= index in Eq. (38)	[—]
n'	= constant	[—]

m	= index in Eq. (29)	[—]
P	= pressure	[Pa]
P_s	= breadth of bed surface	[m]
P_w	= perimeter of pipe wall	[m]
P_{wx}	= pressure loss for clear water flows	[Pa/m]
P_x	= pressure loss for solid-liquid flows	[Pa/m]
q_B	= sediment transport rate	[m ³ /s]
R_{ep}	= particle Reynolds number	[—]
S	= specific gravity of particles	[—]
t	= time	[s]
u	= fluid velocity	[m/s]
u_c	= critical tractive velocity	[m/s]
u_*	= friction velocity (= $(f_k/8)^{1/2}u$)	[m/s]
V_G	= velocity of dunes	[m/s]
V_m	= mean velocity of mixture flows over cross-sectional area of the pipe	[m/s]
V_t	= terminal settling velocity of a single particle	[m/s]
x	= length in flow direction	[m]
α, β	= coefficients	[—]
ε	= porosity in particle bed	[—]
δ	= lag distance	[m]
ρ	= density of fluid	[kg/m ³]
λ_m	= length of particle waves	[m]
τ	= averaged shear stress over the section of fluid flow (see Eq. (6))	[Pa]
τ_s, τ_w	= shear stress at bed surface and pipe wall, respectively	[Pa]
τ_*	= dimensionless shear stress	[—]
τ_{*c}	= dimensionless critical shear stress	[—]
ϕ	= function of sediment transport rate	[—]

<Subscript>

0 = time-averaged value

<Superscripts>

$\sim$ = fluctuated value due to minute disturbance

$\wedge$ = amplitude

— = dimensionless

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