EXPERIMENTAL AND MODELING STUDIES ON THE TRANSIENT BEHAVIOR OF A SIMULATED COUNTERCURRENT ADSORBER

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This paper presents a dynamic model which includes axial dispersion and interphase mass transfer for a simulated countercurrent adsorption separation process, together with a numerical technique based on the method of orthogonal collocation for the solution of the governing partial differential equations. It is shown that the experimental transient behavior may be predicted by modeling the system in terms of an equivalent true countercurrent process provided that the inherent limitation of this approach is noted. This approach nonetheless provides a handy tool for predicting the time required for the system to approach quasi-steady state.

Introduction

Without the actual movement of solid adsorbent, the intrinsic advantage of a continuous countercurrent contacting scheme has been realized in chromatographic separation processes by the use of a principle known as 'Simulated Moving Bed (SMB)'. Commercial processes based on this principle, in which effective countercurrent operation is achieved by switching the fluid inlet and outlet points at intervals through a fixed bed, have been developed for the separation of several hydrocarbon isomers since the 1960's. However, recent work has focused on new applications, in the area of downstream processing of bioproducts where chromatographic separation processes are conventionally carried out in a batchwise fashion. Experimental demonstrations of the use of SMB technology to continuously separate or purify amino acids, enzymes, tripeptides, proteins and disaccharides produced by fermentation have been reported. Considerable advances have also been made during the last few years in theoretical analyses that provide guidance in the design and optimization of SMB processes. Understandably, most of these studies have been concerned with the steady-state behavior of the process. However, knowledge of the transient behavior is also important with respect to start-up and system response to disturbances in operating conditions. Hence, the main goal of the present work is to develop a dynamic model to account for the transient pattern of a simulated countercurrent process.

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Recently, an overview of the modeling of simulated countercurrent systems has been given by Ruthven and Ching.\textsuperscript{8)} Two distinct approaches have been followed. One can either simulate the sequential nature of the experimental system directly or represent the experimental system in terms of an equivalent true countercurrent process. Further classification is possible by considering whether the bed elements are represented by an equilibrium-stage model or a continuous-flow model. In a previous paper, Ching and Ruthven\textsuperscript{9)} reported that the general features of the transient behavior of a simulated countercurrent process could be satisfactorily predicted by considering an equivalent true countercurrent process. Although the use of an equivalent countercurrent process may be expected to provide a realistic representation of the transient behavior, such an approach has not yet become widely adopted. However, it should noted that this approach has proved valuable as a means of simulating the steady-state behavior of SMB processes.\textsuperscript{8)} In the present study, we therefore set out to model the transient behavior of a simulated countercurrent process by considering an equivalent true countercurrent process. Mathematical treatment of the equivalent true countercurrent process in this study is different from that in the previous work.\textsuperscript{9)} Instead of using the equilibrium theory, the equivalent true countercurrent process is treated according to a dispersed plug-flow model which includes interphase mass transfer resistance. A numerical solution technique based on the method of orthogonal collocation is developed for the dynamic model with linear equilibrium relations. It should be noted that the computational procedure may be readily extended to systems in which the equilibrium relationship is non-linear.\textsuperscript{10) } Table 1 summarizes the various approaches for the modeling of the transient behavior of simulated countercurrent systems.

1. Mathematical Model

We consider an isothermal column through which the adsorbent phase moves at a velocity of $u$ countercurrently against the fluid phase flowing with an interstitial velocity of $v$. A schematic diagram of the countercurrent column is shown in Fig. 1. Assuming plug flow of solid, axially dispersed plug flow of fluid and linear mass transfer rate expression, the basic differential equations describing the column dynamics of component $n$ are

\begin{equation}
\frac{\partial c_n}{\partial t} = D_L \frac{\partial^2 c_n}{\partial z^2} - v \frac{\partial c_n}{\partial z} \left( 1 - \frac{e}{e} \right) k (q_n^* - q_n) \\
\frac{\partial q_n}{\partial t} = u \frac{\partial q_n}{\partial z} + k (q_n^* - q_n)
\end{equation}

Table 1. Summary of various approaches for the modeling of the transient behavior of SMB processes

<table>
<thead>
<tr>
<th>Method</th>
<th>Reference</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct simulation</td>
<td>11,12</td>
<td>Each column is considered as a cascade of ideal mixing stages.</td>
</tr>
<tr>
<td>Equilibrium stage model</td>
<td>2,13-16</td>
<td>Numerical solution of plug flow model with linear driving force rate expression.</td>
</tr>
<tr>
<td>Continuous flow model</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Equivalent countercurrent process | 9 | Each column is treated as a cascade of ideal mixing stages. |
| Equilibrium stage model | | Numerical solution of dispersed plug-flow model with linear driving force rate expression. |
| Continuous flow model | This work | |

Fig. 1. Schematic diagram of a continuous countercurrent adsorption column

where $q_n^*$ is the solid-phase concentration which is in equilibrium with the fluid-phase concentration. For a linear system, Eq. (2) describes the equilibrium relationship:

\begin{equation}
q_n^* = K c_n
\end{equation}

The initial and boundary conditions are represented by the following equations:

at $t = 0$ for all $z$, $c_n = q_n = 0$

\begin{equation}
D_L \frac{\partial c_n}{\partial z} = - v \left( c_n \bigg|_{z=0} - c_n \bigg|_{z=0+} \right)
\end{equation}

at $z = 0$,

\begin{equation}
\frac{\partial q_n}{\partial z} = 0
\end{equation}
\[ \frac{\partial c_n}{\partial z} = 0 \]

at \( z = L \),

\[ q_n = q_{n,p} \]  \hspace{1cm} (5)

With known values of the dispersion, kinetic, and equilibrium parameters, the solution of Eqs. (1)–(5) gives the transient concentration profile of component \( n \). The method of orthogonal collocation was used to solve the set of equations. Details of the formulation of the equations to describe the equivalent countercurrent process are given in the Appendix.

2. Experimental

2.1 Measurement of model parameters

Two aqueous binary mixtures, fructose-dextran (MW 9400) and fructose-raffinose, were selected as model systems for experimental studies of the simulated countercurrent process using silica gel (pore diameter = 66 Å, particle size = 22 μm) as adsorbent. Equilibrium and kinetic parameters for each component at 25°C were determined from pulse chromatographic experiments performed on an independent column (30 cm × 1.1 cm i.d.) packed with the same adsorbent. The experimental procedure is similar to those described by Ching and Ruthven\(^{17}\) and will not be repeated here. Pulse chromatographic experiments were also performed on four of the eight columns that made up the simulated countercurrent unit to measure the bed voidage and the dispersion characteristics. Blue dextran and D₂O were injected into the columns as tracers and the response peaks were analyzed to yield the average bed voidage and the dispersion coefficient respectively.

2.2 Simulated countercurrent operation

The general arrangement of the experimental system is shown schematically in Fig. 2. A set of eight identical jacketed columns, each 47.5 cm × 1.4 cm i.d., was packed with silica gel. The columns were connected in series through solenoid valves to form a closed loop. In addition, each column was equipped with two inlet lines (feed and eluent) and three outlet lines (extract, raffinate, and transfer) which were controlled by solenoid valves. The valves were operated in an appropriate sequence by a programmable logic controller. The experimental unit was configured in the 'Sorbet' mode with two columns in each section. Simulated countercurrent operation was achieved by advancing the feed, eluent extract, raffinate and recirculation points by one column, at fixed time intervals, in the direction of fluid flow.

The operation of the experimental unit may be conveniently understood by considering the equivalent true countercurrent process, as sketched in Fig. 3. To achieve separation of a binary mixture, i.e., a net flow of the more strongly adsorbed species (A) toward the extract point and a net flow of the less strongly adsorbed species (B) toward the raffinate point, it is necessary that the flow rates of components A and B in the individual sections be in the direction indicated in Fig. 3. Consideration of these flow conditions together with a knowledge of the equilibrium distribution coefficients for each component allows one to set the experimental flow rates throughout the system.\(^{18}\) The transient behavior of the unit was studied under three sets of operating conditions, summarized in Table 2. The equivalent countercurrent flow rates (S and D) are related to the actual eluent flow and the switch time by:

![Fig. 2. Schematic diagram showing the physical arrangement of columns and switch valves in the simulated countercurrent unit](image-url)
Table 3. Equilibrium and kinetic parameters at 25°C

<table>
<thead>
<tr>
<th></th>
<th>$K$</th>
<th>$k$ (min$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fructose</td>
<td>0.69</td>
<td>5.52</td>
</tr>
<tr>
<td>Raffinose</td>
<td>0.56</td>
<td>3.42</td>
</tr>
<tr>
<td>Dextran</td>
<td>0.13</td>
<td>0.60</td>
</tr>
</tbody>
</table>

inlet and outlet lines were established. At zero time the feed solution was introduced at the inlet of column 5. The flow rates of feed and eluent were controlled by HPLC pumps while those of extract and raffinate were controlled by peristaltic pumps. All runs were carried out at 25°C. Liquid samples were withdrawn from a sampling point located at the outlet of each column and analyzed by liquid chromatography. All the samples were taken as close as possible to the middle of the switching interval. In addition, cumulative samples of extract and raffinate streams were collected and analyzed in the same way. Operation was continued for several cycles until the concentration profile showed no significant change between successive cycles.

3. Results and Discussion

The response peaks obtained from pulse chromatographic experiments carried out over a range of liquid velocities with an independent column were analyzed according to the method of moments to yield the equilibrium and kinetic parameters.$^{1,2}$ Table 3 summarizes the results for the three carbohydrates. In addition to the equilibrium and kinetic parameters, knowledge of the bed voidage as well as the dispersion characteristics of the simulated countercurrent unit is required in order to calculate theoretical concentration profiles from the dispersion model. The bed voidage was determined from pulse chromatographic experiments performed on four representative columns of the countercurrent unit using blue dextran, a molecule too large to penetrate the adsorbent pores, as tracer. The average value of the bed voidage was found to be 0.45. Similarly, pulse chromatographic experiments carried out on the four columns using D$_2$O as tracer measured axial mixing in the columns as well as in the hold-up of the column end-sections and the pipes and valves between the columns. We assume that all these mixings can be accounted for by an effective axial dispersion coefficient $D_L$ which depends on liquid velocity. The dispersion coefficient obtained from moment analysis is given approximately by $D_L = 0.46v$. For the countercurrent unit where the effective liquid flow rate in each section is $(u + v)$, $D_L = 0.16(u + v)$.

Experimental concentration profiles measured at selected times for runs 1 and 2 are shown in Figs. 4.
and 5 respectively, together with theoretical profiles calculated according to the dispersed plug-flow model. These profiles are obtained from the solution of Eqs. (1)–(5), formulated to describe the equivalent countercurrent process in Fig. 3 by considering the continuity of concentration between columns and the appropriate balances at the feed and recirculation points. It is evident that the simulated profiles provide a good representation of the experimentally observed behavior. Fig. 6 shows plots of experimental and theoretical concentrations of fructose in extract and dextran in raffinate for run 1 vs. time. It is evident that the rate of approach to steady state for dextran is higher than that for fructose. The same phenomenon may be observed in a fixed-bed adsorber where a component with a small $K$ value causes an early breakthrough when the bed is subjected to a step change in concentration. Figure 7 illustrates the results obtained from run 2. It is clear that the difference in the rates of approach to steady state for fructose and raffinose is negligible as the difference in the $K$ values of these two components is small.

Since the performance of a continuous separation process is ultimately measured in terms of steady-state product purity, product recovery and product concentration, it is of great importance to examine the steady-state concentration profiles obtained in this study. Table 4 shows the values of these experimental and theoretical performance parameters for the three runs. It is evident that the experimental
Table 4. Summary of experimental and theoretical performance parameters for extract and raffinate products (values calculated from theoretical model are given in brackets)

<table>
<thead>
<tr>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 2a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extract product (% w/v)</td>
<td>$c_f = 4.79 (4.83)$</td>
<td>$c_f = 4.07 (4.42)$</td>
</tr>
<tr>
<td>$c_d = 0.14 (0.15)$</td>
<td>$c_d = 0.85 (0.63)$</td>
<td>$c_d = 0.34 (0.23)$</td>
</tr>
<tr>
<td>Purity</td>
<td>0.97 (0.97)</td>
<td>0.83 (0.88)</td>
</tr>
<tr>
<td>Recovery</td>
<td>0.96 (0.97)</td>
<td>0.81 (0.88)</td>
</tr>
<tr>
<td>Conc. ratio</td>
<td>0.96 (0.97)</td>
<td>0.81 (0.88)</td>
</tr>
<tr>
<td>Raffinate product (% w/v)</td>
<td>$c_f = 0.18 (0.17)$</td>
<td>$c_f = 0.84 (0.58)$</td>
</tr>
<tr>
<td>$c_d = 4.89 (4.85)$</td>
<td>$c_d = 4.16 (4.37)$</td>
<td>$c_d = 2.89 (2.89)$</td>
</tr>
<tr>
<td>Purity</td>
<td>0.96 (0.97)</td>
<td>0.83 (0.88)</td>
</tr>
<tr>
<td>Recovery</td>
<td>0.98 (0.97)</td>
<td>0.83 (0.87)</td>
</tr>
<tr>
<td>Conc. ratio</td>
<td>0.98 (0.97)</td>
<td>0.83 (0.87)</td>
</tr>
</tbody>
</table>

Purity = $c_i/(c_1 + c_2)$; for extract stream: $c_1 = c_f$, $c_2 = c_d$ or $c_d$; for raffinate stream: $c_1 = c_d$ or $c_d$, $c_2 = c_f$. Recovery for extract stream = (extract flow x $c_f$)/amount of $f$ in feed, recovery for raffinate stream = (raffinate flow x $c_d$ or $c_d$)/amount of $d$ or $r$ in feed. Conc. ratio = $c_i$/concentration of 1 in feed; for extract stream: $c_i = c_f$; for raffinate stream: $c_i = c_d$ or $c_d$.

Purities and recoveries of extract and raffinate for all three runs are in good agreement with theoretical values. The extract and raffinate products for run 1 are seen to be quite pure. The flow conditions for run 1 were selected so as to ensure high product concentration. With a slight adjustment of flow conditions it is possible to achieve a complete separation for this fructose-dextran mixture, which has a large separation factor (ratio of $K_f$ to $K_d$). The extract and raffinate products for run 2 are, however, somewhat impure. To improve the system performance, a new set of flow conditions was selected, as shown under run 2a in Table 2. As a result, higher product purity and recovery are obtained at the expense of greater product dilution, as shown in Table 4. Further enhancement of product purity by optimizing flow conditions may prove difficult as the separation factor for this particular mixture is rather small. For such 'difficult' separation problems it is more profitable to enhance performance by increasing the adsorbent selectivity rather than the system efficiency. The effect of the modified flow conditions on the transient behavior of the system is shown in Fig. 7. The transient time required by run 2a to reach quasi-steady state, starting from a clean bed, is shorter than that required by run 2. It is worthwhile mentioning that, instead of starting from a clean bed, the transient time may be substantially reduced by pre-loading the system in accordance with the final steady-state concentration profiles.\(^{11}\)

How well a simulated countercurrent system approaches true countercurrent behavior depends mainly on the degree of subdivision of the adsorbent bed. Obviously, if the bed were divided into very small subsections with the switch interval reduced correspondingly, one would have a perfect analogue of the countercurrent flow. Since the cost and complexity of a simulated countercurrent system increase with the number of subdivisions, it is of great practical interest to determine the minimum degree of subdivision needed for a simulated countercurrent unit to give an adequate approach to continuous countercurrent operation. This problem has been considered by Hidajat et al.\(^{19}\) and Morbidelli and co-workers\(^{20,21}\) who presented theoretical analyses of the effects of subdividing the adsorbent bed. Hidajat et al.\(^{19}\) found that there is very little difference between the behavior of a simulated countercurrent system with four or more columns per section and a true continuous countercurrent process. Even with only two columns per section the deviation from true countercurrent behavior is modest. Similar conclusions are given by Morbidelli and co-workers.\(^{20,21}\) The good agreement between experimental data obtained from the simulated countercurrent unit with two columns per section and theoretical predictions calculated from the equivalent true countercurrent model confirms these theoretical analyses and is consistent with the observations of Hashimoto et al.,\(^{16}\) who also compared experimental data with theoretical predictions.

Some additional comments are needed to interpret more closely the results discussed so far. Due to the sequential nature of operation the equivalent true countercurrent model cannot be expected to account fully for the behavior of the simulated countercurrent unit. Hidajat et al.\(^{19}\) reported that theoretical profiles calculated from the equivalent true countercurrent model were in good agreement with experimental profiles measured at the midpoint of the switch interval. This observation is confirmed in the present work and it should therefore be noted that the equivalent true countercurrent model cannot predict the progression of concentration profiles within a switch interval. From the modeling point of view it may be argued that the direct
simulation approach, which simulates the precise nature of the transient profile at any instant, is more appropriate than the idealized equivalent countercurrent model, for modeling the transient behavior of simulated countercurrent systems. Nonetheless, there are several advantages in using the equivalent true countercurrent model to describe simulated countercurrent systems. For example, the use of the equivalent true countercurrent model is mathematically less complicated than the direct simulation approach. Also, the equivalent true countercurrent model serves as a common basis for theoretical analysis of simulated countercurrent processes which may use different equipment designs to effect the continuous countercurrent movement. In addition to the advantages mentioned above, the use of the equivalent true countercurrent model in this study represents a unified approach to the analysis of simulated countercurrent systems since the operating conditions of the experimental unit were also determined from the equivalent true countercurrent model.

Conclusion

The transient behavior of a simulated countercurrent system similar to the 'SorbeX' process has been modeled by considering an equivalent true countercurrent process. Predicted concentration profiles are in good agreement with experimental profiles measured at the midpoint of the switch interval. The model with parameters estimated from independent experiments provides a useful method for estimating the time required to approach steady state under different operating conditions. It is shown that the rate of approach to quasi-steady state and process performance varied with flow conditions. A reduction in transient time was achieved by changing the flow conditions but this led to higher product dilution.

Appendix

The method of orthogonal collocation was used to obtain the solution of Eqs. (1)–(5). For each column we have an equation of type (1) boundary conditions equations which can be written in dimensionless time and length:

\[
\frac{\partial c_x}{\partial T} = \frac{1}{Pe_\infty} \frac{\partial^2 c_x}{\partial x^2} - \frac{1}{x} \frac{\partial c_x}{\partial x} - St(q_x^* - q_x)
\]  

\[
\frac{\partial q_x}{\partial T} = \frac{1}{\beta} \frac{\partial q_x}{\partial x} + \frac{St}{\beta} (q_x^* - q_x)
\]  

\[
\frac{\partial c_x}{\partial x} \bigg|_{x=0} = -Pe(c_{x,1} - c_{x,0})
\]  

\[
\frac{\partial q_x}{\partial x} \bigg|_{x=0} = 0
\]  

\[
\frac{\partial c_x}{\partial x} \bigg|_{x=1} = 0
\]  

\[
\frac{\partial q_x}{\partial x} \bigg|_{x=0} = 0
\]  

Eq. (7) may be reduced to a set of ordinary differential equations by the method of orthogonal collocation. 

\[
\frac{dc_{x,j}}{dT} = \frac{1}{Pe_x} \sum_{i=1}^{M+2} B_{x,i} c_{x,i}(t) - \frac{1}{x} \sum_{i=1}^{M+2} A_{x,i} c_{x,i}(t) - St(K_{c,j}(t) - q_{x,j}(t))
\]  

\[
\frac{dq_{x,j}}{dT} = \frac{1}{\beta} \sum_{i=1}^{M+2} A_{x,i} q_{x,i}(t) + \frac{St}{\beta} (K_{c,j}(t) - q_{x,j}(t))
\]  

where \( j = 2, \ldots, M + 1 \) and \( M \) = number of collocation points. At \( x = 0, c_{x,1} = c_{x,0} \) and \( c_{x,2} = c_{c,1} \), hence Eq. (8) becomes:

\[
\sum_{i=1}^{M+2} A_{x,i} c_{x,i}(t) = 0
\]  

and Eqs. (9)–(11) become:

\[
\sum_{i=1}^{M+2} A_{x,i} c_{x,i}(t) = 0
\]  

\[
q_{x,M+2} = q_{x,p}
\]  

Rearranging Eqs. (14) and (15) to express \( c_{c,1} \) and \( c_{c,M+2} \) in terms of the internal collocation points \( c_{c,2}, \ldots, c_{c,M+1} \):

\[
c_{c,1} = \frac{1}{M+1} \sum_{i=2}^{M+1} (R_{3M+2} - R_{3A_{c,1,i}}) c_{c,i}(t) - Pe R_{e} C_{c,p}
\]  

\[
c_{c,M+2} = \frac{1}{M+1} \sum_{i=2}^{M+1} (R_{3M+2} - R_{3A_{c,1,i}}) c_{c,i}(t) + Pe R_{e} C_{c,p}
\]  

and rewriting Eq. (16):

\[
q_{c,i} = \frac{1}{A_{c,1,i}} \sum_{i=2}^{M+1} A_{c,i} q_{c,i}(t) - \frac{A_{c,M+2}}{A_{c,1,i}} q_{c,p}
\]  

Now substituting \( C_{c,1} \) and \( C_{c,M+2} \) from Eqs. (18) and (19) into Eq. (12), we obtain the following equation:

\[
\frac{dc_{x,j}}{dT} = \sum_{i=1}^{M+1} \left( \frac{1}{Pe_x} B_{x,i} - \frac{1}{x} A_{x,i} \right) c_{x,i}(t)
\]  

\[
+ \left( \frac{1}{Pe_x} B_{x,1} - \frac{1}{x} A_{x,1} \right)(R_{3A_{c,1,i} - R_{3A_{c,1,i}}})
\]  

\[
+ \left( \frac{1}{Pe_x} B_{x,M+2} - \frac{1}{x} A_{x,M+2} \right)(R_{3A_{c,1,i} - R_{3A_{c,1,i}}})
\]  

\[
- Pe R_{e} \left[ \frac{1}{x} B_{x,1} - \frac{1}{x} A_{x,1} \right] c_{c,p}
\]  

\[
- St(K_{c,j}(t) - q_{x,j}(t))
\]  

and substituting \( q_{x,1} \) and \( q_{x,M+2} \) from Eqs. (20) and (17) into Eq. (13), we obtain:

\[
\frac{dq_{x,j}}{dT} = -\frac{1}{\beta} \sum_{i=1}^{M+2} \left( A_{x,i} - A_{x,1} A_{x,1} \right) q_{x,i}(t)
\]  

\[
+ \left( \frac{1}{\beta} A_{x,1} A_{x,1} + \frac{S_{d}}{\beta} (K_{c,j}(t) - q_{x,j}(t))
\]  

For each column in the 'SorbeX' configuration we therefore have
Eqs. (21) and (22) and Eqs. (19) and (20) linking columns ($p+1$), $p$, and ($p-1$):
\[ c_{d}(M+2) = c_{u,p+1} = \sum_{i=0}^{M+1} (R_{1}A_{1,i} - R_{2}A_{M+2,i})c_{d}(i) + PeR_{1}c_{u,p} \] 
\[ q_{u}(1) = q_{u,p-1} = -\frac{1}{A_{1,1}} \sum_{i=0}^{M+1} A_{1,i}q_{d}(i) - \frac{A_{1,m+2}}{A_{1,1}}q_{u,p} \]

For the system configuration considered here, we have eight columns and two components ($n=2$). Eight internal collocation points ($M=8$) were used for each column. Therefore in total we have (number of columns) x ($n x 2M = 256$) ordinary differential equations to be solved to obtain the unsteady-state concentration profiles. The computations were carried out on an IBM 3090 computer using the IMSL subroutine DGEAR to solve the resulting system of ordinary differential equations.

**Nomenclature**

- $a$ = cross-sectional area of an adsorption column [cm$^2$]
- $A_{1,i}$ = collocation coefficient for the gradient [--]
- $B_{1,i}$ = collocation coefficient for the Laplacian [--]
- $c$ = fluid-phase concentration [g/100 cm$^3$]
- $D$ = eluent flow rate [cm$^2$/min]
- $D_{L}$ = axial dispersion coefficient [cm$^2$/min]
- $E$ = extract flow rate [cm$^2$/min]
- $F$ = feed flow rate [cm$^2$/min]
- $k$ = overall mass transfer coefficient [min$^{-1}$]
- $K$ = distribution coefficient [--]
- $L$ = length of an adsorption column [cm]
- $M$ = number of collocation points [--]
- $Pe$ = Peclet number, $vL/D_{L}$ [--]
- $q$ = solid-phase concentration (particle volume basis) [g/100 cm$^3$]
- $q^{*}$ = solid-phase concentration in equilibrium with $c$ [g/100 cm$^3$]
- $R$ = raffinate flow rate [cm$^2$/min]
- $S$ = hypothetical solid recirculation rate [cm$^2$/min]
- $St$ = Stanton number, $kL/u$ [--]
- $t$ = time [min]
- $T$ = $1 - \epsilon u t/sL$ [--]
- $u$ = hypothetical solid velocity [cm/min]
- $v$ = hypothetical liquid interstitial velocity [cm/min]
- $x$ = axial distance [cm]
- $z$ = $A_{1,i}$ [--]
- $\alpha$ = $(1 - \epsilon u t)/x$ [--]
- $\beta$ = $(1 - \epsilon)/x$ [--]
- $\epsilon$ = bed voidage [--]

**Subscripts**

- $f$ = fructose
- $d$ = dextran
- $n$ = component index
- $p$ = column number
- $r$ = raffinose

**Literature Cited**