SIZE CLASSIFICATION OF SUBMICRON POWDER BY AIR CYCLONE
AND THREE-DIMENSIONAL ANALYSIS

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Experimental and theoretical studies were made of the separation efficiency of an air cyclone separator. The results of numerical calculations of three-dimensional Navier-Stokes equations indicate that the flow field changes with circumferential angle. The upward and downward velocity components merge strongly near the entrance of the dust box. Large particles are collected on the upper wall, small particles on the conical wall.

It is also confirmed that particles with small inertia enter the dust box first, but finally exit from the cyclone because of the upward velocity component. The experimental partial separation efficiencies obtained agree well with the numerical calculations when the data are rearranged according to the particle inertia parameter.

Introduction

Cyclones are widely used for separating powders. Recently excellent performance with cut sizes below 0.5 μm was obtained by Jinoya et al.1) Optimization of cyclone dimensions in each part is, however, mainly determined by experiment, because the analysis of fluid flow and particle motions is very complicated. Numerical calculations of fluid flow and particle motions in the cyclone were conducted by Ayers et al.1) and Zhou et al.9), but their calculations were made assuming axi-symmetrical flow. To know the complicated flow field in the cyclone it is necessary to make numerical calculations in three dimensions.

In this paper, the three-dimensional Navier–Stokes equations are numerically solved and a theoretical estimation of the classification efficiency curve is compared with experimental results. The experimental partial separation efficiencies obtained by use of monodisperse latex particles and submicron powders agree well with the numerical calculations when the data are rearranged according to the particle inertia parameter.

1. Experimental Apparatus

The experimental apparatus used for the latex aerosol is shown in Fig. 1. The test particles are latex (Dp = 0.33–2.95 μm). The partial separation efficiency is determined from the count ratio of an aerosol counter (Kanomax, Model 3411).

Figure 2 shows the experimental apparatus for fine Kanto loam (JIS No. 11, Dp = 1.8 μm) as the test particles. A ring nozzle disperser10) (air jet type) made by Nisshin Flour Milling Company was used to disperse the agglomerated particles.

The particle size distribution at the cyclone inlet is measured by cascade impactor (Tokyo Dylec, LP-20) and liquid centrifuge (Shimazu, SA-CP3). Mass median diameters measured by the two methods are nearly equal and the particles are completely dispersed. The partial separation efficiency is determined by the mass of the filter side and the dust box, and the data of particle size distribution measured by the liquid

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Fig. 1. Experimental apparatus for latex aerosol

Fig. 2. Experimental apparatus for fine Kanto loam
centrifuge. The particle and air flow rates are 2 g/min and 0.148 m³/min respectively.

Figure 3 shows the notations of cyclone dimensions and Table 1 indicates the dimensions of cyclones of three types. Type A is the cyclone tested in the present work. Each of the dimensions is determined from Linoya's data. Types B and C in the table are just for reference. Type B is a standard type and type C is an ultra-mini cyclone.

2. Numerical Calculations

The three-dimensional Navier–Stokes equations and the equations of particle motions were numerically solved by digital computer. Table 2 summarizes the Navier–Stokes equations and equations of particle motion used in the numerical calculation. The general conservation equation in the 3-dimensional case is represented as follows:

\[
\frac{\partial}{\partial z} (ru) + \frac{\partial}{\partial r} (rv) + \frac{\partial}{\partial \theta} (rw) = \frac{\partial}{\partial z} \left( r \frac{\partial \phi}{\partial z} \right) + \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( r \frac{\partial \phi}{\partial \theta} \right) + S_\phi
\]

(1)

In the above equation, \( \phi \) is the general function of the conservation equation, \( S_\phi \) is the source term, \( \Gamma \) is the diffusion coefficient and \( u, v, w \) are fluid velocities of axial, radial and circumferential directions respectively. In the actual calculation, the cyclindrical coordinate is transformed to curvilinear coordinates \((\xi, \eta, \zeta)\), and the equation is expressed as Eq. (2).

\[
\frac{\partial}{\partial \xi} (G_1 \phi) + \frac{\partial}{\partial \eta} (G_2 \phi) + \frac{\partial}{\partial \zeta} (G_3 \phi)
\]

\[
= \frac{\partial}{\partial \xi} \left( D_\xi \frac{\partial \phi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( D_\eta \frac{\partial \phi}{\partial \eta} \right) + \frac{\partial}{\partial \zeta} \left( D_\zeta \frac{\partial \phi}{\partial \zeta} \right) + \left[ \frac{\partial}{\partial \xi} \left( D_1 \frac{\partial \phi}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left( D_2 \frac{\partial \phi}{\partial \eta} \right) + \frac{\partial}{\partial \zeta} \left( D_3 \frac{\partial \phi}{\partial \zeta} \right) \right] + J \cdot S_\phi
\]

(2)

The diffusion coefficients \( D_\xi, D_\eta, D_\zeta \) and \( D_1 - D_6 \) are given as follows:

\[
D_\xi = \frac{r \Gamma}{J} \left( a_1^2 + a_2^2 + a_3^2 \right)
\]

\[
D_\eta = \frac{r \Gamma}{J} \left( b_1^2 + b_2^2 + b_3^2 \right)
\]

\[
D_\zeta = \frac{r \Gamma}{J} \left( c_1^2 + c_2^2 + c_3^2 \right)
\]

\[
D_1 = \frac{r \Gamma}{J} \left( a_1 b_1 + a_2 b_2 + a_3 b_3 \right)
\]

\[
D_2 = \frac{r \Gamma}{J} \left( c_1 a_1 + c_2 a_2 + c_3 a_3 \right)
\]

\[
D_3 = \frac{r \Gamma}{J} \left( a_1 b_1 + a_2 b_2 + a_3 b_3 \right)
\]

(3)

Table 2. Equations of fluid and particle motion

\[
\frac{\partial}{\partial z} (ru) + \frac{\partial}{\partial r} (rv) + \frac{\partial}{\partial \theta} (rw) = \frac{\partial}{\partial z} \left( r \frac{\partial \phi}{\partial z} \right) + \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( r \frac{\partial \phi}{\partial \theta} \right) + S_\phi
\]

Table 1. Dimensions of cyclones of three types

<table>
<thead>
<tr>
<th>Type</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
<th>j</th>
<th>k</th>
<th>l</th>
<th>m</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.38</td>
<td>0.16</td>
<td>0.40</td>
<td>0.45</td>
<td>1.07</td>
<td>2.73</td>
<td>1.33</td>
<td>1.58</td>
<td>6.57</td>
<td>2.50</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0.60</td>
<td>0.20</td>
<td>0.50</td>
<td>0.40</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>0.85</td>
<td>4.0</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
<td>0.23</td>
<td>1.0</td>
<td>1.92</td>
<td>2.0</td>
<td>1.23</td>
<td>4.92</td>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Equations of fluid and particle motion

\[
\psi \frac{d^2 z}{dt^2} - r \psi \gamma = -C \frac{d \psi}{dt} - \psi \gamma
\]

\[
\mu \frac{d^2 z}{dt^2} + C \frac{d \psi}{dt} - \mu \gamma
\]

\[
\gamma = \frac{C \mu}{18 \mu D_u}, \quad \mu = \frac{C \mu}{18 \mu D_u}
\]
\[
D_4 = \frac{r^3}{J} \left( c_1 b_4 + c_2 b_2 + \frac{c_3 b_3}{r^2} \right)
\]
\[
D_5 = \frac{r^3}{J} \left( a_1 c_4 + a_2 c_2 + \frac{a_3 c_3}{r^2} \right)
\]
\[
D_6 = \frac{r^3}{J} \left( c_1 b_4 + c_2 b_2 + \frac{c_3 b_3}{r^2} \right)
\]
\[
\begin{align*}
 a_1 &= r \theta_z - r \theta_\eta, \\
 a_2 &= z \theta_\xi - z \theta_\zeta, \\
 a_3 &= z \theta_\xi - z \theta_\eta, \\
 b_1 &= r \theta_\zeta - r \theta_\eta, \\
 b_2 &= z \theta_\xi - z \theta_\zeta, \\
 b_3 &= z \theta_\xi - z \theta_\eta, \\
 c_1 &= r \theta_\zeta - r \theta_\eta, \\
 c_2 &= z \theta_\xi - z \theta_\eta, \\
 c_3 &= z \theta_\xi - z \theta_\eta
\end{align*}
\]
\]

(4)

In Eq. (2), J is the Jacobian matrix given by Eq. (5) and $G_1, G_2, G_3$ are the velocity components in $\xi, \eta, \zeta$ directions respectively.

\[
J = \begin{vmatrix}
\xi & \eta & \zeta \\
\theta_\xi & \theta_\eta & \theta_\zeta \\
G_1 & G_2 & G_3 \\
\end{vmatrix}
\]

(5)

\[
\begin{align*}
G_1 &= r u \theta_\zeta + r v \theta_\eta + w \theta_\zeta \\
G_2 &= r u \theta_\eta + r v \theta_\xi + w \theta_\xi \\
G_3 &= r u \theta_\zeta + r v \theta_\xi + w \theta_\xi
\end{align*}
\]

(6)

Table 3 summarizes the equations of fluid motion for turbulent flow. To calculate the flow field in the cyclone, the following calculational conditions are adapted.

1. Laminar flow ($Re = 3000, 5000, 10,000$).

2. Turbulent flow ($k-\varepsilon$ model$^6$)

Uniform velocity distribution is assumed at the cyclone inlet and zero velocity components are given at the cyclone wall. The boundary condition at the cyclone outlet of the exit tube is given by Eq. (7).

\[
\frac{\partial u}{\partial z} = 0, \quad \frac{\partial v}{\partial z} = 0, \quad v = 0
\]

(7)

In the case of turbulent flow, the assumption of local equilibrium of turbulence yields the following expression for the boundary value of turbulence energy $K$ and turbulent dissipation rate $\varepsilon$.

\[
K_{\text{wall}} = \frac{U^2}{C^2_{\mu}}
\]

(8)

\[
\varepsilon_{\text{wall}} = \frac{U^3}{\kappa_{\mu} \rho}, \quad \kappa = 0.4
\]

(9)

In Eq. (9), $y_p$ is the distance between the wall and the grid point, and the subscript "wall" refers to the grid point near the wall. The values of $K$ and $\varepsilon$ at the cyclone inlet plane are assumed as follows:

\[
K = 0.04 U_0^2
\]

(10)

In Eq. (11), $l_m$ is the mixing length and is assumed to be twenty percent of cyclone diameter. To calculate the diffusion and convective terms in Eq. (2), the exponential scheme$^5$ is used because it takes both central and upwind schemes into consideration. The set of simultaneous algebraic equations can be solved by successive underrelaxation, line-by-line iterative methods. The CPU time was about 25 h for each case using the HITAC-M680H at the Hiroshima University Computing Center.

A top view of the cyclone is shown in Fig. 4. The circumferential angles shown in this figure are used to show fluid velocity vectors and particle starting positions. The latter are numbered from 1 to 10 in Fig. 4.

Figure 5 shows the coordinate system in numerical calculation. Grid numbers in axial, radial and circumferential directions are 41, 31 and 29 respectively. The boundary-fitted curvilinear coordinate system developed by Thompson$^7$ is used for numerical calculation. The effect of grid shape on the fluid flow pattern in the cyclone is examined first.

Figure 6 shows another type of grid shape in the calculation. In this case, a conical cyclone wall is simulated as a stepwise shape. The calculated results of fluid velocity distribution are shown in Fig. 7. The calculated results of (a) and (b) correspond to the grid shapes in Figs. 5 and 6 respectively. The downward velocity components near the conical wall are large in case (a), but nearly zero velocity components are indicated in case (b). Because the boundary layer thickness near the conical wall is approximately equal to $1/\sqrt{Re}$, the calculated results of case (a) are more reasonable than those of case (b). The grid
system indicated in Fig. 5 is therefore used in the following calculations.

Fluid velocity vectors of different circumferential angles projected on a plane are shown in Figs. 8 and 9. In these figures, the flow Reynolds number in the cyclone is 5000. It is found that the flow field depends on the circumferential angle and that the assumption of axisymmetrical flow in the cyclone is not always correct. For example, in Fig. 8 the velocity vectors just below the exit tube are directed to the central axis, but in Fig. 9 the velocity vectors are directed in the opposite direction. The downward velocity components near the conical wall (region A) are large and the downflow (region A), and upflow (region B) merge strongly near the entrance of the dust box. It might be considered that a small particle collected on the cyclone wall moves downward near the wall, but near the entrance of dust box the particle may have a chance to move out to the exit tube due to the upward flow velocity components. It is also found that there are upward flow velocity components even in the upper part of the dust box.

Calculated results by use of $k-\varepsilon$ turbulence model are indicated in Figs. 10 and 11. It is also found that the flow field depends on the circumferential angle, and the same characteristics obtained from Figs. 8 and 9 are also found in these figures.

The calculated particle trajectories for different particle diameters projected on a plane ($\theta=0^\circ$) are shown in Fig. 12. It is assumed that the particles are collected when they touch the wall surface. The particle starting positions are kept constant at $\theta=47.2^\circ$. In the figures, the particle starting position and final position are numbered to distinguish each particle. The large particles ($D_p=2.4 \mu m$) are collected on the wall, but some small particles ($D_p=2.0 \mu m$) enter the dust box first, then move out of the cyclone because of the upward flow velocity component. If the particle diameter becomes much smaller ($D_p=0.8 \mu m$), the particles do not enter the dust box, but move around the exit tube and out of the cyclone.

The effect of the circumferential angle of particle starting position is shown in Fig. 13. The particle diameter is $1.5 \mu m$, and Figs. 13(a) and 13(b) indicate initial circumferential angles of $47.2^\circ$ and $87.7^\circ$ respectively. It is found that some particles enter the dust box, but move out of the cyclone when $\theta=47.2^\circ$.

On the other hand, most of the particles are collected
Fig. 7. Effect of grid shape on flow field of cyclone for different grid coordinate systems ($Re=5000, \theta=67.5^\circ$)

Fig. 8. Fluid velocity distribution (Curvilinear coordinates, $Re=5000$)

Fig. 9. Fluid velocity distribution for different circumferential angles (curvilinear coordinates, $Re=5000$)

Fig. 10. Fluid velocity distribution (curvilinear coordinates, turbulent flow)
at the upper part of the cyclone wall when $\theta = 87.7^\circ$.

3. Experimental Results

The numerical calculation does not take particle repulsion or re-entrainment into consideration. In the case of Kanto loam particles, the effect of particle repulsion or re-entrainment on the partial separation efficiency must be examined.

Figure 14 shows experimental results of partial separation efficiency with and without adhesive oil. The particles used were fine Kanto loam. Comparing the data for the high-inertia region, the partial separation efficiency with adhesive oil is higher than that in the case without adhesive oil. The numerical calculation shown in Fig. 12 indicates that some of the large particles collide with the upper cylindrical wall first. Thus the probability of a large particle's repulsion or re-entrainment is higher than that for small particles. From Fig. 14, particle repulsion or re-entrainment is the main reason for the deterioration of the partial separation efficiency in the high-inertia region.

Figure 15 shows the experimental data obtained from latex, Kanto loam and stearic acid\(^3\) compared with the numerical calculations. The solid line indicates calculated results when the flow field is laminar and the dotted line is the result when the flow field is turbulent ($k-\varepsilon$ model). The experimental flow Reynolds number is more than 40,000. The experimental data do not agree with the calculated results if laminar flow is assumed. The experimental data obtained agree well with the calculated results when the flow is turbulent.

The interaction between gas and particles is excluded in the numerical calculation. When the cyclone is used in a high loaded dust condition, a certain correction is necessary to apply this calculated result to the construction of a cyclone separator.

Conclusion

Experimental and theoretical studies of the separation efficiency of the air cyclone separator were made. The results of numerical calculations of three-dimensional Navier-Stokes equations indicate that the flow field changes with circumferential angle.
enter the dust box first, but finally they exit from the cyclone because of the upward velocity component.

The numerical results obtained agree with air cyclone characteristics, experimental or empirical previously obtained.

The experimental partial separation efficiencies obtained from latex and Kanto loam particles agree well with the numerical calculations when the data are rearranged according to the particle inertia parameter.

Nomenclature

- $a, b$ = height and width of cyclone inlet section [—]
- $a_c, b_l$ = constants defined by Eq. (4) [—]
- $C_p$ = drug coefficient of particle [—]
- $C$ = Cunningham’s slip correction factor [—]
- $d_l$ = exit tube diameter [cm]
- $D_p$ = particle diameter [$\mu$m]
- $D_k$ = mass median diameter measured by liquid centrifuge [cm]
- $D$ = cyclone diameter [cm]
- $D_1, D_2$ = diffusion coefficients defined by Eq. (3) [cm]
- $d_a, d_b$ = inlet diameter of dust box and diameter of dust box [cm]
- $g$ = gravity acceleration [cm/s²]
- $G$ = gravitational settling parameter [—]
- $G_1, G_2, G_3$ = dimensionless velocity components in $\xi, \eta, \zeta$ directions [—]
- $H$ = height of conical section [cm]
- $J$ = Jacobian matrix [—]
- $k, k_0$ = turbulence energy and dimensionless turbulence energy [cm³/s²] [—]
- $l$ = height of exit tube [cm]
- $L_m$ = mixing length [cm]
- $L$ = height of cylindrical section [cm]
- $p$ = pressure [—]
- $P$ = turbulent production term [—]
- $R_e$ = particle Reynolds number [—]
- $R_e = (D u_p) / \nu$ = flow Reynolds number [—]
- $r, z = coordinates$ of radial and axial directions [cm]
- $r = r (D), z = z (D), \theta = dimensionless coordinates of radial, axial and circumferential directions [—]
- $S_p$ = general function of source term [—]
- $u, \xi, \eta, \zeta$ = fluid velocities in axial, radial and circumferential directions [cm/s]
- $u_0$ = inlet velocity of cyclone [cm/s]
- $u_f$ = friction velocity [cm/s]
- $r_e$ = distance between wall and grid point [cm]
- $X_e, Z_e$ = height of dust box and total height of cyclone [cm]
- $\eta$ = partial separation efficiency [—]
- $\phi$ = general function of conservation equation [—]
- $\Gamma$ = diffusion coefficient [—]
- $\nu (=1 / R_e)$ = molecular viscosity [—]
- $v_i$ = turbulent viscosity [—]
- $\psi$ = inertia parameter [—]
- $\rho_p$ = particle density [g/cm³]
- $\mu$ = gas viscosity [g/cm·s]
- $\xi, \eta, \zeta$ = turbulent dissipation rate and dimensionless turbulent dissipation rate [cm³/s²] [—]
- $c_1, c_2, c_3, c_4$ = constants of turbulent model [—]
- $\xi, \eta, \zeta$ = curvilinear coordinate [—]
\( \kappa \) = constant defined by Eq. (9) [\( \text{[-]} \) ]

\( \sigma_D \) = constant defined by Eq. (11) [\( \text{[-]} \) ]

References