A VIRTUAL MASS AND A DRAG COEFFICIENTS FOR AN OSCILLATING PARTICLE

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There are many chemical processes in which particle motion in a fluid plays an important role, and there has been interest in the utilization of oscillatory flows in many promising applications.^{1,5)} The particle motions are usually evaluated by means of the steady-state fluid drag. However, the relative velocities between a fluid and a particle are not constant with respect to time in practical operations. Therefore, an understanding of the fluid force on a body moving with acceleration is essential for predicting the unsteady motion of a particle and for the design of oscillatory process equipment.⁷⁾ However, there are few usable data for estimating the fluid forces on an accelerating particle³⁾ over a wide range of Reynolds number. In the present study we report on a fluid force acting on a sphere oscillating sinusoidally in a fluid at rest, and the fluid force is evaluated as a virtual mass coefficient and a drag coefficient. And we propose equations for the virtual mass and drag coefficients of an oscillating sphere.

1. Experimental apparatus and method

The experimental method in the present study is almost the same as in the previous work.⁶⁾ A scotch yoke system and a force transducer were remade to be able to measure fluid forces noiselessly and accurately. The diameters of test spheres were 37.6 and 68.9 mm. Frequencies and amplitudes of

oscillation ranged from 0.5 to 2.4 Hz and 5 to 40 mm respectively. Kinematic viscosity was in the range of $10^{-6} - 6.5 \times 10^{-4} \text{ m}^2/\text{s}$.

We assume that the fluid force on an oscillating body can be expressed by the following equation.^{2,3,6)}

$$F = -M_0 C_m (dv/dt) - (1/2) \rho S_0 C_d v |v| \quad (1)$$

The virtual mass and drag coefficients in Eq. (1), C_m and C_d , which were averaged over one cycle of oscillation, were evaluated by using the Fourier transform^{2,3)} as follows.

$$C_m = \frac{6}{\pi^2 \rho a \omega^2 d^3} \int_0^{2\pi} F \cos \omega t d(\omega t) \quad (2)$$

$$C_d = \frac{3}{\pi \rho a^2 \omega^2 d^2} \int_0^{2\pi} F \sin \omega t d(\omega t) \quad (3)$$

The unfiltered fluid force signal was recorded in digitized form through use of an A/D converter. 512 data were measured in one cycle of oscillation, and C_m and C_d were calculated by the Complex-Fast-Fourier-Transform with the measured fluid force.

2. Results and Discussion

2.1 Virtual mass coefficient

Figure 1 shows the relation between C_m and a modified Reynolds number $d^2 \omega / \nu$. The dotted line (a) in this figure is the approximated analytical solution by Stokes⁴⁾. Although Minamizawa and Endoh³⁾ reported that the Stokes solution agreed with the experimental C_m when $a \omega d / \nu < 200$, depar-

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Fig. 1. Variation of virtual mass coefficients with modified Reynolds number. The dotted line (a) shows Stokes's theoretical relation. The solid line (b) shows the present correlation, Eq. (4).

tures of the Stokes solution from the present experimental values are evident in the range of $100 > d^2\omega/\nu$ and $1000 < d^2\omega/\nu$. The present experimental C_m can be correlated by the solid line (b) in Fig. 1, which is expressed by the following equation.

$$C_m = 0.7 + 15(d^2\omega/\nu)^{-0.75} \quad (4)$$

$$6 < d^2\omega/\nu < 4 \times 10^4 \quad 0.2 < a/d < 1.0$$

The values of C_m at the maximum acceleration of oscillation have also been studied, but they were almost the same as the average values of C_m shown in Fig. 1.

2.2 Drag coefficient

Figures 2 and 3 show results for drag coefficients. There was no simple relation between C_d and $d^2\omega/\nu$, but the following equations were found to well correlate the obtained data.

$$C_d = 23.4(d/a)(d^2\omega/\nu)^{-0.55} \quad (5)$$

$$1 < a\omega d/\nu < 200 \quad 0.2 < a/d < 1.0$$

$$C_d = 0.5 + 20(a^2\omega/\nu)^{-0.6} \quad (6)$$

$$200 < a\omega d/\nu < 4 \times 10^4 \quad 0.2 < a/d < 1.0$$

Minamizawa and Endoh proposed the following equations.³⁾

$$C_d = 45.4(d/a)(d^2\omega/\nu)^{-0.70} \quad (7)$$

$$1 < a\omega d/\nu < 20 \quad 0.07 < a/d < 1.85$$

$$C_d = 11(d/a)[\log(d^2\omega/\nu)]^{-2.5} \quad (8)$$

$$20 < a\omega d/\nu < 4000(a/d) \quad 0.07 < a/d < 0.56$$

Their equations could not well correlate the present experimental data. This may be due to the following reason. In their experiments, Minamizawa and Endoh used a filter to get a smooth signal. Although they corrected the phase lag of the measured force due to the filter, the effect of distortion of the measured force by the filter on the calculation of the virtual mass and drag coefficients may not be negligible. In the above equations, Eqs. (5)–(8), however, it is seen to be a common relation that C_d is proportional to d/a .

When a particle density is greater than a fluid

Fig. 2. Variation of drag coefficients in low Reynolds number region. The dotted line (a) shows Eq. (7). The solid line (b) shows the present correlation, Eq. (5).

Fig. 3. Correlation of drag coefficient in high Reynolds number region. The dotted line (a) shows Eq. (8). The solid line (b) shows the present result, Eq. (6).

Fig. 4. Drag coefficients at the maximum velocity of oscillation. The solid line (b) shows Eq. (9). The dotted line shows an equation by Minamizawa and Endoh.³⁾

density, the fluid drag due to the relative velocity between particle and fluid becomes important rather than the inertial force. **Fig. 4** shows the drag coefficients at the maximum velocity of oscillation, C_{dv} . The values of C_{dv} were evaluated by a summation of the Fourier coefficients until the 5th term. C_{dv} can be correlated by the following relation with the steady-state drag coefficient C_{ds} .

$$C_{dv} = C_{ds} + 12(a^2\omega/\nu)^{-0.46} \quad (9)$$

The values of C_{ds} were calculated from an equation

Fig. 5. Comparison of calculated force and measured one. Solid lines show measured forces. Dashed lines (1) show the calculated force by quasi-steady state method. Dotted lines (2) show the calculated force by using Eqs. (4)–(6).

by Turton and Levenspiel.⁸⁾ It must be noted that when $a^2\omega/\nu$ is small in value, C_{dv} is considerably greater than the steady-state one. The results by Minamizawa and Endoh are shown in Fig. 4 by a dotted line. Their results show extremely smaller values than the present results. This may be due to fewer data points per cycle of oscillation in their experiment.

2.3 Comparison with quasi-steady state method

To estimate the fluid force on an oscillating body, there is an approximated method that uses a steady-state drag coefficient and a theoretical virtual mass coefficient. Comparison of the measured force with the calculated one by this quasi-steady state (QSS) method is shown in **Fig. 5(a)** and **(b)**. F_D in the figure is a dimensionless force.

Shown in Fig. 5(a) is the force obtained at small Reynolds number. The calculated force by the QSS method is remarkably different from the measured force in terms of phase lag and magnitude of force

amplitude. The dotted line in the figure is the calculated force by using the correlation of virtual mass and drag coefficients, viz., Eqs. (4)–(6). This value almost agrees with the measured force.

Shown in Fig. 5(b) is the force obtained at high Reynolds number. The calculated force by the QSS method fitted with the measured one more closely compared to the case shown in Fig. 5(a).

We have compared the measured force with the calculated one under other conditions and found that the QSS method was usable only under the conditions of high Reynolds number ($a\omega d/\nu > 10,000$, $d^2\omega/\nu < 20,000$) and high amplitude ratio ($a/d > 0.7$). With decreasing Reynolds number, deviations of the calculated force from the measured one increase and the QSS method underestimates the magnitude of an actual fluid force.

Nomenclature

a	= amplitude of oscillation	[m]
C_d	= averaged drag coefficient over one cycle of oscillation	[—]
C_{ds}	= steady state drag coefficient	[—]
C_{dv}	= drag coefficient at maximum velocity of oscillation	[—]
C_m	= averaged virtual mass coefficient over one cycle of oscillation	[—]
d	= diameter of sphere	[m]
F	= fluid force	[N]
F_D	= dimensionless force = $2F/[\rho(a\omega)^2 S_0]$	[—]
f	= frequency of oscillation	[Hz]
M_0	= mass of displaced fluid	[kg]
S_0	= projected area of sphere	[m ²]
t	= time	[s]
v	= velocity of sphere	[m/s]
ρ	= fluid density	[kg/m ³]
ν	= kinematic viscosity of fluid	[m ² /s]
ω	= angular frequency of oscillation	[Hz]

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