MOMENTUM, WITH HEAT AND MASS TRANSFER THROUGH GAS-LIQUID INTERFACE WITH ACCELERATED INTERFACIAL VELOCITY

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Effects of interfacial velocity on interfacial momentum, heat and mass transfer through a gas-liquid interface were quantitatively investigated. The rates of momentum, heat and mass transfer depend not only on the magnitude of the interfacial velocity but also on the change of acceleration in interfacial velocity.

In addition, by considering the interfacial tension gradient to be the cause of interfacial acceleration, the effect of the interfacial tension gradient on interfacial velocity, interfacial momentum, heat and mass transfer were also quantitatively investigated. Both the interfacial velocity and the rates of momentum, heat and mass transfer depend not only on the magnitude of the interfacial tension gradient but also on the change of interfacial tension.

Introduction

The interfacial velocity is an extremely important factor in any discussion of momentum, heat and mass transfer across an interface between different phases. For this reason, the authors previously investigated quantitatively the effects of interfacial velocity and interfacial tension gradient on momentum, heat and mass transfer through a fluid-fluid interface¹⁾. In that study, the interfacial velocity was considered for a constant flow speed in the flow direction. In the case when interfacial velocity is caused by the interfacial tension gradient (Marangoni effect), when the interfacial tension gradient is inversely proportional to the square root of the flow direction coordinate x , there is an interfacial velocity with a constant flow speed in the flow direction. However, when the interfacial tension gradient is not inversely proportional to the square root of x , or when there is a change in the quantity of liquid due to condensation or evaporation, or when there is a weir, etc., the interfacial velocity in the flow direction changes.

In this study, a quantitative investigation of the effects of interfacial velocity on momentum, heat and mass transfer through a gas-liquid interface was carried out. In addition, by considering the interfacial tension gradient to be the cause of interfacial acceleration (Marangoni effect), it is possible to clarify the effects of interfacial tension gradient on interfacial velocity, momentum, heat and mass transfer through a gas-liquid interface.

1. Analysis

A simple two-dimensional configuration for a semi-infinite gas-liquid interface is shown in Fig. 1. The

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assumptions made are: (a) free stream conditions in liquid phase are uniform; (b) the solute concentration and temperature difference between the free stream and interface are low enough that physical properties of both phases can be regarded as constant, and the Rayleigh effect, which depends on density difference, can be neglected; (c) heat and mass transfer rates are controlled by the resistances of the liquid phase; (d) the mass transfer rate is low enough that the normal velocity at the interface can be regarded as negligible; (e) the interface can be considered to be flat; and (f) the shear stress of the gas phase can be ignored. Then when $x = 0$, $u_s = u_\infty$. Two-dimensional laminar boundary layer equations can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u \frac{\partial W}{\partial x} + v \frac{\partial W}{\partial y} = D \frac{\partial^2 W}{\partial y^2} \quad (4)$$

The boundary conditions are,

$$y = 0; \quad u = u_s(x) \quad \text{or} \quad \mu \frac{\partial u}{\partial y} = -\frac{\partial \sigma}{\partial x}, \quad v = 0 \quad (5)$$

$$T = T_s = \text{constant}, \quad W = W_s = \text{constant}$$

$$y = \infty; \quad u = u_\infty = \text{constant} \quad (6)$$

$$T = T_\infty = \text{constant}, \quad W = W_\infty = \text{constant}$$

Numerical analysis of Eqs. (1)-(6) was carried out. (i) When $u_s \approx u_\infty$, the three-equation local non-similarity

Fig. 1 Simple two-dimensional configuration

(a) Experimental results for sodium dodecyl sulfate solution

(b) Experimental results for sodium dodecylbenzene sulfonate solution

Fig. 2 Interfacial velocity distribution²⁾

model^{3,4)} was used (Appendix 1). (ii) When $u_s \gg u_\infty$, the similarity model was used (Appendix 2).

2. Results and Discussion

2.1 Comparison with previous experimental results

Imaishi, *et al.*²⁾ investigated experimentally the interfacial acceleration phenomena prevailing along a horizontal gas-liquid interface of surfactant solutions upstream of a weir. The size of gas-liquid interface was 10 cm width and 46.5 cm length. Aqueous solutions of four surfactants (sodium dodecyl sulfate, sodium dodecylbenzene sulfonate, n-hexanoic acid and n-heptanoic acid) were used for testing. They measured the velocity distributions both along the interface and in liquid using

(a) Experimental results for sodium dodecyl sulfate solution

(b) Experimental results for sodium dodecylbenzene sulfonate solution

Fig. 3 Velocity distribution: comparison with previous experimental results of Imaishi, *et al.*²⁾

a laser doppler velocimeter, and the interfacial tension distributions according to the Wilhelmy method. Considering the experimental results of the interfacial velocity distribution along the interface, shown in Fig. 2, as the boundary conditions for Eq. (5), the velocity distribution inside a liquid phase and the interfacial tension distribution were calculated. Figures 3 and 4 show a comparison of numerical results obtained by the three-equation local non-similarity model with experimental results. In these figures, the plots are experimental results and solid lines are numerical results. In these figures, the numerical results are in good agreement with the experimental results by Imaishi, *et al.*²⁾. It was confirmed that the existence of an extremely gentle interfacial tension gradient causes an interfacial acceleration phenomenon.

2.2 Effect of interfacial velocity on momentum, heat and mass transfer through the interface

In section 2.1, the interfacial velocity distribution obtained by Imaishi *et al.*²⁾ were used for the boundary conditions. To generalize the discussion, the following changes in interfacial velocity at a gas-liquid interface were used for the boundary condition in Eq. (5) (Appendix 1):

$$\frac{u_s(x)}{u_\infty} = 1 + c_1 \left(\frac{x}{L} \right)^m \quad (7)$$

The effect of interfacial velocity on the friction coefficient, Cf_N , and the Nusselt number, Nu_x , are respec-

(a) Experimental results for sodium dodecyl sulfate solution

(b) Experimental results for sodium dodecylbenzene sulfonate solution

Fig. 4 Interfacial tension distribution: comparison with previous experimental results of Imaishi, *et al.*²⁾

Fig. 5 Effect of interfacial velocity on friction coefficient

tively shown in Figs. 5 and 6. When $u_s \gg u_\infty$, the following asymptotic solutions can be obtained (Appendix 2):
when $Pr = 1 \sim 10^3$,

Fig. 6 Effect of interfacial velocity on Nusselt number

Table 1. The value of $F''(0)$ and $\Phi'(0)$

	$m = 0$	$m = 1/3$	$m = 1/2$	$m = 2/3$	$m = 1$	$m = 2$
	$n = 1/2$	$n = 1$	$n = 5/4$	$n = 3/2$	$n = 2$	$n = 7/2$
$F''(0)$	0.4438	0.5880	0.6290	0.6593	0.7071	0.7785
$-\Phi'(0)$	$Pr = 1$	0.4438	0.4260	0.4209	0.4172	0.4115
	$Pr = 10$	1.680	1.653	1.646	1.641	1.632
	$Pr = 10^2$	5.545	5.516	5.507	5.500	5.491
	$Pr = 10^3$	17.75	17.72	17.71	17.70	17.69
$-\frac{\Phi'(0)}{Pr^{1/2}}$	$Pr = 1$	0.4438	0.4315	0.4209	0.4172	0.4115
	$Pr = 10$	0.5313	0.5256	0.5205	0.5189	0.5161
	$Pr = 10^2$	0.5545	0.5525	0.5507	0.5500	0.5491
	$Pr = 10^3$	0.5613	0.5607	0.5600	0.5597	0.5594
$Pr \gg 1$		$1/\sqrt{\pi} = 0.5642$				

$$\frac{1}{2} C_{f_N} Re_x^{1/2} = \sqrt{m+1} |F''(0)| \left(\frac{u_s}{u_\infty} \right)^{1/2} \quad (8)$$

$$Nu_x Re_x^{-1/2} = -\sqrt{m+1} \Phi'(0) \left(\frac{u_s}{u_\infty} \right)^{1/2} \quad (9)$$

when $Pr \gg 1$,

$$Nu_x Re_x^{-1/2} = \sqrt{m+1} \frac{1}{\sqrt{\pi}} Pr^{1/2} \left(\frac{u_s}{u_\infty} \right)^{1/2} \quad (10)$$

The values of $F''(0)$ and $-\Phi'(0)$ are shown in Table 1. From the above results, the following can be deduced:

(a) The friction coefficient and Nusselt number depend not only on the magnitude of the interfacial velocity but also on the change in its acceleration, i.e. on the value of m .

Fig. 7 Velocity distribution

(b) The friction coefficient and Nusselt number increase with a rise in the value of u_s/u_∞ . In particular, when $u_s \gg u_\infty$ the friction coefficient and the Nusselt number are proportional to the square root of the value of u_s/u_∞ .

2.3 Effect of interfacial tension gradient on interfacial velocity, momentum, heat and mass transfer

To clarify the effect of the interfacial tension gradient on the interfacial velocity, momentum, heat and mass transfer through a gas-liquid interface, the following changes in interfacial tension at a gas-liquid interface was used for the boundary condition in Eq. (5) (Appendix 1):

$$\frac{Ma_x}{Re_x^2} Re_x^{1/2} = c_2 \left(\frac{x}{L} \right)^{n-1/2} \quad (11)$$

$$\sigma \propto x^n \quad (11)'$$

The dimensionless numbers are defined as follows:

$$\frac{Ma_x}{Re_x} = \frac{(\partial\sigma/\partial x)}{\mu u_\infty/x} = \frac{\text{Interfacial tension gradient driven force}}{\text{Viscous force}} \quad (12)$$

$$\frac{Ma_x}{Re_x^2} = \frac{(\partial\sigma/\partial x)}{\rho u_\infty^2} = \frac{\text{Interfacial tension gradient driven force}}{\text{Inertia force}} \quad (13)$$

The velocity distribution in a liquid phase are shown in Fig. 7. It can be seen from Fig. 7 that the velocity distri-

Fig. 8 Effect of interfacial tension gradient on interfacial velocity

Table 2. Distributions of σ , $\partial\sigma/\partial x$, Ma_x , $(Ma_x/Re_x^2) Re_x^{1/2}$ and the degree of acceleration of u_s

σ	∞	x^n	$\frac{3m+1}{x^2}$
$\frac{\partial\sigma}{\partial x}$	∞	x^{n-1}	$\frac{3m-1}{x^2}$
Ma_x	∞	$x^n + 1$	$\frac{3(m+1)}{x^2}$
$\frac{Ma_x}{Re_x^2} Re_x^{1/2}$	∞	$x^{n-1/2}$	$\frac{3m}{x^2}$
u_s	∞	$\frac{2n-1}{x^3}$	x^m

bution is affected by the degree of acceleration of an interfacial tension, i.e. by the value of n . With the same value of $(Ma_x/Re_x^2) Re_x^{1/2}$, the value of $(\partial u/\partial \eta)_{\eta=0}$, which is the velocity gradient along y axis, increases as n increases.

Figure 8 shows the effect of the interfacial tension gradient on interfacial velocity. When $u_s \gg u_\infty$, the following can be obtained from Eq. (5) as shown in Appendix 2:

$$Re_s = \left[\sqrt{\frac{2(n+1)}{3}} |F''(0)| \right]^{-2/3} Ma_x^{2/3} \quad (14)$$

$$\frac{u_s}{u_\infty} = \left[\sqrt{\frac{2(n+1)}{3}} |F''(0)| \right]^{-2/3} \left(\frac{Ma_x}{Re_x^2} Re_x^{1/2} \right)^{2/3} \quad (15)$$

$$u_s = \left[\sqrt{\frac{2(n+1)}{3}} |F''(0)| \right]^{-2/3} \left\{ \frac{(\partial\sigma/\partial x) x^{1/2}}{(\rho\mu)^{1/2}} \right\}^{2/3} \quad (16)$$

From the above results, the following can be deduced:

(a) The interfacial velocity depends not only on the magnitude of the interfacial tension gradient but also on its distribution in the flow direction. At any given Marangoni number the interfacial velocity decreases as n increases.

Fig. 9 Effect of interfacial tension gradient on friction coefficient

(b) The interfacial velocity increases with a rise in the value of $(Ma_x / Re_x^2) Re_x^{1/2}$. In particular, when $u_s \gg u_\infty$, the interfacial velocity is proportional to the 2/3 power of $(Ma_x / Re_x^2) Re_x^{1/2}$.

(c) If the relationship with respect to x is known for any one of the interfacial tension, the interfacial tension gradient, the Marangoni number, the value of $(Ma_x / Re_x^2) Re_x^{1/2}$, or the interfacial velocity, then the relations for the others are also known as shown in Table 2.

The effect of interfacial tension gradient on the friction coefficient and Nusselt number are, respectively, shown in Figs. 9 and 10. The friction coefficient can be obtained from Eq. (5) (Appendix 1):

$$\frac{1}{2} C_{f_N} Re_x^{1/2} \left| 1 - \frac{u_s}{u_\infty} \right| = \frac{Ma_x}{Re_x^2} Re_x^{1/2} \quad (17)$$

When $u_s \gg u_\infty$, the following equation for the Nusselt number can be obtained from Eqs. (9) and (15) (Appendix 2):

$$Nu_x Re_x^{-1/2} = -\Phi'(0) \left\{ \frac{2(n+1)}{3} \right\}^{1/3} \times |F''(0)|^{-1/3} \left(\frac{Ma_x}{Re_x^2} Re_x^{1/2} \right)^{1/3} \quad (18)$$

When $Pr \gg 1$,

$$Nu_x Re_x^{-1/2} = \frac{1}{\sqrt{\pi}} Pr^{1/2} \left\{ \frac{2(n+1)}{3} \right\}^{1/3} \times |F''(0)|^{-1/3} \left(\frac{Ma_x}{Re_x^2} Re_x^{1/2} \right)^{1/3} \quad (19)$$

Where $F''(0)$, $-\Phi'(0)$ for given values of n are also shown in Table 1. From Figs. 9 and 10, the following can be deduced:

(a) The interfacial tension gradient accelerates the rate of momentum, heat and mass transfer through a gas-liquid interface.

Fig. 10 Effect of interfacial tension gradient on Nusselt number

(b) The rate of momentum, heat and mass transfer through a gas-liquid interface is independent of the change in the interfacial tension in flow direction.

(c) The effect of interfacial tension gradient on the rate of heat and mass transfer can be estimated by the case in which interfacial velocity is constant ($n = 1/2$).

(d) When $(Ma_x / Re_x^2) Re_x^{1/2} > 10$, Eq. (18) can be applied.

Conclusion

An analysis of the effects of interfacial velocity and interfacial tension gradient on momentum, heat and mass transfer through a gas-liquid interface with acceleration of interfacial velocities in the flow direction led to the following conclusion:

(a) The rate of momentum, heat and mass transfer through a gas-liquid interface depends not only the magnitude of the interfacial velocity but also on the change of the interfacial acceleration in interfacial velocity.

(b) When the interfacial velocity is due to the interfacial tension gradient, the interfacial velocity depends not only on the magnitude of the interfacial tension gradient but also on the change of interfacial tension in the flow direction.

(c) The interfacial velocity and interfacial tension gradient have a very important influence on momentum, heat and mass transfer through a gas-liquid interface.

Appendix 1 The three-equation local non-similarity model

Stream function and similarity parameters are defined as:

$$\phi = \sqrt{V u_\infty x} f(\xi, \eta) \quad (A-1)$$

$$\eta = y \sqrt{\frac{u_\infty}{\nu x}} \quad (A-2)$$

$$\xi = c_1^{1/m} \left(\frac{x}{L} \right) \text{ or } c_2^{1/n} \left(\frac{x}{L} \right) \quad (A-3)$$

$$\phi = \frac{T - T_s}{T_\infty - T_s} \quad (\text{for heat transfer}) \quad (A-4)$$

$$\phi = \frac{W - W_s}{W_s - W_\infty} \quad (\text{for mass transfer}) \quad (\text{A-5})$$

By substituting Eqs. (A-1)-(A-5) into Eqs. (1)-(6), reduced equations, auxiliary equations and boundary conditions are obtained as follows.

$$f''' + \frac{1}{2} f f'' = \xi (f' g' - f'' g) \quad (\text{A-6})$$

$$g''' + \frac{1}{2} (f'' g + f g'') = f g'' - f'' g + \xi \{ (g')^2 - g g'' + f' h' - f'' h \} \quad (\text{A-7})$$

$$h''' + \frac{1}{2} (f'' h + 2 g g'' + f h'') = 2 \xi (g')^2 - g g'' + f' h' - f'' h + \xi (3 g' h' - g h'' - 2 g' h) \quad (\text{A-8})$$

$$\frac{\phi''}{Pr} + \frac{1}{2} f \phi' = \xi (f' \theta - \theta' g) \quad (\text{A-9})$$

$$\frac{\phi''}{Pr} + \frac{1}{2} (g \phi' + f \theta') = f' \theta - \phi' g + \xi (f' \chi - \theta' g' - \phi' h - g \theta') \quad (\text{A-10})$$

$$\frac{\chi''}{Pr} + \frac{1}{2} (f \chi' + 2 g \theta' + \phi' h) = 2 (f' \chi + \theta' g' - \phi' h - g \theta') + \xi (2 g' \chi - g \chi' + \theta' h' - 2 h \theta') \quad (\text{A-11})$$

Boundary conditions:

i) For comparison with experimental results, and for calculating the effect of interfacial velocity on momentum, heat and mass transfer,

$$\eta = 0; \quad f(\xi, 0) = 0, \quad f'(\xi, 0) = \frac{u_s}{u_\infty} \quad (\text{A-12})$$

$$g(\xi, 0) = 0, \quad g'(\xi, 0) = \frac{\partial}{\partial \xi} \left(\frac{u_s}{u_\infty} \right) \quad (\text{A-13})$$

$$h(\xi, 0) = 0, \quad h'(\xi, 0) = \frac{\partial^2}{\partial \xi^2} \left(\frac{u_s}{u_\infty} \right) \quad (\text{A-14})$$

$$\phi(\xi, 0) = 1, \quad \theta(\xi, 0) = \chi(\xi, 0) = 0 \quad (\text{A-15})$$

$$\eta = \infty; \quad f'(\xi, \infty) = 1, \quad g'(\xi, \infty) = h'(\xi, \infty) = 0 \quad (\text{A-16})$$

$$\phi(\xi, \infty) = \theta(\xi, \infty) = \chi(\xi, \infty) = 0 \quad (\text{A-17})$$

ii) For calculating the effect of interfacial tension gradient on interfacial velocity, momentum, heat and mass transfer,

$$\eta = 0; \quad f(\xi, 0) = 0, \quad f''(\xi, 0) = -\frac{Ma_x Re_x^{1/2}}{Re_x^2} \quad (\text{A-18})$$

$$g(\xi, 0) = 0, \quad g''(\xi, 0) = -\frac{\partial}{\partial \xi} \left(\frac{Ma_x Re_x^{1/2}}{Re_x^2} \right) \quad (\text{A-19})$$

$$h(\xi, 0) = 0, \quad h''(\xi, 0) = -\frac{\partial^2}{\partial \xi^2} \left(\frac{Ma_x Re_x^{1/2}}{Re_x^2} \right) \quad (\text{A-20})$$

$$\phi(\xi, 0) = 1, \quad \theta(\xi, 0) = \chi(\xi, 0) = 0 \quad (\text{A-21})$$

$$\eta = 0; \quad f'(\xi, \infty) = 1, \quad g'(\xi, \infty) = h'(\xi, \infty) = 0 \quad (\text{A-22})$$

$$\phi(\xi, \infty) = \theta(\xi, \infty) = \chi(\xi, \infty) = 0 \quad (\text{A-23})$$

where

$$g = \frac{\partial f}{\partial \xi}, \quad g' = \frac{\partial f'}{\partial \xi}, \quad g'' = \frac{\partial f''}{\partial \xi}, \quad g''' = \frac{\partial f'''}{\partial \xi} \quad (\text{A-24})$$

$$h = \frac{\partial^2 f}{\partial \xi^2}, \quad h' = \frac{\partial^2 f'}{\partial \xi^2}, \quad h'' = \frac{\partial^2 f''}{\partial \xi^2}, \quad h''' = \frac{\partial^2 f'''}{\partial \xi^2} \quad (\text{A-25})$$

$$\theta = \frac{\partial \phi}{\partial \xi}, \quad \theta' = \frac{\partial \phi'}{\partial \xi}, \quad \theta'' = \frac{\partial \phi''}{\partial \xi} \quad (\text{A-26})$$

$$\chi = \frac{\partial^2 \phi}{\partial \xi^2}, \quad \chi' = \frac{\partial^2 \phi'}{\partial \xi^2}, \quad \chi'' = \frac{\partial^2 \phi''}{\partial \xi^2} \quad (\text{A-27})$$

The primes indicate differentiation with respect to η . From Eqs. (A-1)-(A-27), the following relationships can be obtained:

$$\{1 - f'(\xi, 0)\} \frac{1}{2} C f_N Re_x^{1/2} = f''(\xi, 0) = -\frac{Ma_x Re_x^{1/2}}{Re_x^2} \quad (\text{A-28})$$

$$Nu_x Re_x^{-1/2} = -\phi'(\xi, 0) \quad (\text{A-29})$$

Appendix 2 The similarity model

When $u_s \gg u_\infty$ and $(u_s/u_\infty) \propto x^m$, stream function and similarity parameters are defined as:

$$\Psi_s = \sqrt{\frac{v u_s x}{m+1}} F(\xi, \eta) \quad (\text{A-30})$$

$$\eta_s = y \sqrt{\frac{(m+1) u_s}{v x}} \quad (\text{A-31})$$

$$\Phi = \frac{T - T_\infty}{T_s - T_\infty} \quad (\text{for heat transfer}) \quad (\text{A-32})$$

$$\Phi = \frac{W - W_\infty}{W_s - W_\infty} \quad (\text{for mass transfer}) \quad (\text{A-33})$$

By substituting Eqs. (A-30)-(A-33) into Eqs. (1)-(6), reduced equations and boundary conditions are obtained as follows:

$$F''' + \frac{1}{2} F F'' - \frac{m}{m+1} (F')^2 = 0 \quad (\text{A-34})$$

$$\Phi'' + \frac{1}{2} Pr F \Phi' = 0 \quad (\text{A-35})$$

Boundary conditions:

$$\eta_s = 0; \quad F'(0) = 1 \quad \text{or} \quad \sqrt{m+1} |F''(0)| \left(\frac{u_s}{u_\infty} \right)^{3/2} = \frac{Ma_x Re_x^{1/2}}{Re_x^2} \quad (\text{A-36})$$

$$F(0) = 0, \quad \Phi(0) = 1$$

$$\eta_s = \infty; \quad F'(\infty) = 0, \quad \Phi(\infty) = 0 \quad (\text{A-37})$$

where the primes indicate differentiation with respect to η_s . From Eqs. (A-34)-(A-37), Eqs. (8) and (9) can be obtained. From Eq. (A-35), the following relationship can be obtained:

$$-\Phi'(0) = \frac{1}{\int_0^\infty \exp\left(-\frac{Pr}{2} \int_0^{\eta_s} F d\eta_s\right) d\eta_s} \quad (\text{A-38})$$

When $Pr \gg 1$, Eq. (A-38) becomes:

$$-\Phi'(0) = \frac{1}{\sqrt{\pi}} Pr^{1/2} \quad (\text{A-39})$$

From Eqs. (A-29) and (A-39), Eq. (10) can be obtained. From Eq. (A-36),

$$Re_s = [\sqrt{m+1} |F''(0)|]^{-2/3} Ma_x^{2/3} \quad (\text{A-40})$$

$$\frac{u_s}{u_\infty} = [\sqrt{m+1} |F''(0)|]^{-2/3} \left(\frac{Ma_x Re_x^{1/2}}{Re_x^2} \right)^{2/3} \quad (\text{A-41})$$

$$u_s = [\sqrt{m+1} |F''(0)|]^{-2/3} \left\{ \frac{(\partial \sigma / \partial x) x^{1/2}}{(\rho \mu)^{1/2}} \right\}^{2/3} \quad (\text{A-42})$$

when $\sigma \propto x^n$, from Eqs. (A-40)-(A-42),

$$u_s \propto x^{\frac{2n-1}{3}} \quad (\text{A-43})$$

$$m = \frac{2n-1}{3} \quad (\text{A-44})$$

From Eqs. (A-40)-(A-44), Eqs. (14)-(16) can be obtained.

When shear stress of gaseous phase is not ignored, Eqs.(14)-(16) become as follows:

$$Re_s = \left[\sqrt{\frac{2(n+1)}{3}} |F''(0)| \left\{ 1 + \left(\frac{\rho_g}{\rho_l} \right)^{1/2} \left(\frac{\mu_g}{\mu_l} \right)^{1/2} \right\} \right]^{-2/3} Ma_x^{2/3} \quad (\text{A-45})$$

$$\frac{u_s}{u_\infty} = \left[\sqrt{\frac{2(n+1)}{3}} |F''(0)| \left\{ 1 + \left(\frac{\rho_g}{\rho_l} \right)^{1/2} \left(\frac{\mu_g}{\mu_l} \right)^{1/2} \right\} \right]^{-2/3} \times \left(\frac{Ma_x}{Re_x^2} Re_x^{1/2} \right)^{2/3} \quad (\text{A-46})$$

$$u_s = \left[\sqrt{\frac{2(n+1)}{3}} |F''(0)| \left\{ 1 + \left(\frac{\rho_g}{\rho_l} \right)^{1/2} \left(\frac{\mu_g}{\mu_l} \right)^{1/2} \right\} \right]^{-2/3} \times \left\{ \frac{(\partial\sigma/\partial x) x^{1/2}}{(\rho_l \mu_l)^{1/2}} \right\}^{2/3} \quad (\text{A-47})$$

Nomenclature

c_1, c_2	= arbitrary constant defined by Eqs. (7) and (11)	[-]
Cf_N	= friction coefficient ($= \tau_s / \{\rho u_\infty (u_s - u_\infty) / 2\}$)	[-]
D	= diffusivity	[m ² /s]
F	= dimensionless stream function based on u_s	[-]
f	= dimensionless stream function based on u_∞	[-]
g	= ξ -derivative of f	[-]
h	= ξ -derivative of g	[-]
h_T	= heat transfer coefficient	[W/(m ² ·K)]
k	= mass transfer coefficient	[m/s]
L	= characteristic length	[m]
m	= constant	[-]
Ma_x	= Marangoni number ($= (\partial\sigma/\partial x) x^2 / (\mu\nu)$)	[-]
n	= constant	[-]
Nu_x	= Nusselt number ($= h_T x / \kappa, (1 - W_s) kx / D$)	[-]
Pr	= Prandtl number ($= \nu / \alpha, \nu / D$)	[-]
Re_s	= Reynolds number based on u_s ($= u_s x / \nu$)	[-]

Re_x	= Reynolds number based on u_∞ ($= u_\infty x / \nu$)	[-]
T	= Temperature	[K]
u	= velocity parallel to interface	[m/s]
v	= velocity normal to interface	[m/s]
W	= mass fraction of component	[-]
x	= coordinate parallel to interface	[m]
y	= coordinate normal to interface	[m]
α	= thermal diffusivity	[m ² /s]
η	= similarity parameter defined by Eq. (A-2)	[-]
η_s	= similarity parameter defined by Eq. (A-31)	[-]
θ	= ξ -derivative of ϕ	[-]
κ	= thermal conductivity	[W/(m·K)]
μ	= viscosity	[Pa·s]
ν	= kinematic viscosity	[m ² /s]
ξ	= transformed streamwise coordinate defined by Eq. (A-3)	[-]
ρ	= density	[kg/m ³]
σ	= interfacial tension	[N/m]
τ	= shear stress ($= -\mu (\partial u / \partial y)_{y=0}$)	[N/m ²]
Φ	= dimensionless temperature or concentration in case of $u_s \gg u_\infty$, defined by Eqs. (A-32) and (A-33)	[-]
ϕ	= dimensionless temperature or concentration in case of $u_s \approx u_\infty$, defined by Eqs. (A-4) and (A-5)	[-]
χ	= ξ -derivative of θ	[-]
Ψ	= stream function based on u_s	[-]
ψ	= stream function based on u_∞	[-]

<Subscripts>

g	= gas phase
l	= liquid phase
s	= interface or based on interfacial velocity
∞	= free stream

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