GLOBAL ANALYSIS OF HEAT TRANSFER IN CZ CRYSTAL GROWTH OF OXIDE

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For the crystal growth of LiNbO_3 , a global analysis of heat transfer in an inductively heated CZ furnace was carried out by use of the finite element method. The effects of the temperature dependency of physical properties such as viscosity and surface tension of the melt, as well as the effect of processing parameters such as crystal rotation, on the flow and temperature fields in the furnace and on the melt/crystal interface shape were investigated. With the mathematical model developed in the present work, the interface inversion caused by crystal rotation was demonstrated theoretically. It is found that the effect of the temperature dependency of viscosity on CZ crystal growth is not significant under the conditions used here, and that the thermocapillary force strongly affects crystal growth. The critical crystal rotation rate at which the interface inversion occurs becomes much larger, and the RF coil current required to keep the crystal diameter constant becomes smaller, in comparison with those without thermocapillary flow in the melt.

Introduction

Oxide single crystals such as LiNbO_3 and LiTaO_3 are among the most important raw materials in modern acoustic-optical-electronics industries, and are utilized as the materials for acoustic transducers, optical modulators and optical frequency multipliers. The Czochralski (CZ) method is usually applied to the growth of such crystals. For the production of a perfect single crystal, it is important to acquire correct knowledge about the heat transfer mechanism and to control the heat transfer in the CZ equipment, because the crystal quality is closely related to the transport phenomena in the furnace⁶⁾.

One of the methods used to predict the phenomena in the CZ furnace is numerical simulation. Regarding the CZ growth system of semiconductor crystals such as silicon and GaAs, extensive studies^{1, 4, 7, 12, 15)} have recently been carried out, especially on the global analysis of heat transfer in the furnace, where one can predict the temperature distributions in the furnace and the crystal shapes (diameter, melt/crystal interface) under given furnace geometry and heater power. Yet, there have been few theoretical studies of oxide crystal growth.

Kobayashi^{9, 10)} first studied numerically the CZ crystal growth system of oxide, and suggested that the interface inversion is accompanied by a change of flow mode in the melt. He, however, emphasized the qualita-

tive aspects of the melt flow in very simplified circumstances, and did not provide practically important quantities such as the melt/crystal interface shape. Tsukada *et al.*^{14, 19, 20)} and Kopetsch¹¹⁾ computed the temperature and flow fields in the melt and crystal under the assumption that the crucible and the ambient wall are maintained at respective constant temperatures, taking into account the melt/crystal interface shape, and demonstrated theoretically an interface inversion in the CZ growth system of sapphire.

Derby *et al.*^{3, 16)} carried out a numerical simulation of the CZ system of GGG consisting of the crystal, the melt, the crucible and the heater, where the temperature of the heater is fixed. Moreover, their model was extended to an inductively heated CZ furnace²⁾, and the effects of pedestal heat transfer and of changing melt volume on the flow in the melt were examined without crystal rotation. However, in the strict sense their analysis was not a global one since not all elements of the CZ furnace were taken into account.

Recently, thermophysical properties of melts have been measured extensively^{8, 17, 18)}. These studies reveal that the viscosity of the oxide melt increases remarkably near the melting-point temperature.

The aim of this work is to develop a global analysis of an inductively heated CZ furnace for a LiNbO_3 crystal growth, and to reveal the effect of the temperature dependency of the physical properties of the melt, i.e.

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viscosity and surface tension, as well as the effect of processing parameters such as crystal rotation rate on the flow and temperature fields and the melt/crystal interface shape.

1. Theory

In the CZ crystal growth process for oxide, radio-frequency (RF) induction heating is frequently used. The AC electric current in the coil induces an eddy current in the metal crucible. The raw material of crystal in the crucible is melted by Joulean heating from the eddy current. In the global analysis of heat transfer in the inductively heated CZ furnace, the electromagnetic field in the system is first computed, then the power distribution in the crucible, and lastly the flow and temperature fields in the furnace.

1.1 Electromagnetic field

The following points are assumed in numerical simulation of the electromagnetic field. (1) The system is axially symmetric. (2) The media are linear, isotropic and stationary. (3) All materials in the system are non-magnetic and have no charge. (4) The displacement current is neglected. Under these assumptions, the governing equations, i.e. the Maxwell equations, are expressed as follows.

$$\text{rot}(\mathbf{B} / \mu_0) = \mathbf{J} \quad (1)$$

$$\text{rot}(\mathbf{E}) = -(\partial \mathbf{B} / \partial t) \quad (2)$$

$$\text{div}(\mathbf{B}) = 0 \quad (3)$$

$$\text{div}(\mathbf{E}) = 0 \quad (4)$$

$$\mathbf{j} = \sigma \mathbf{E} \quad (5)$$

where μ_0 is the permeability of the vacuum and σ is the electric conductivity, which has non-zero value only in the crucible.

Introducing the vector potential $\mathbf{A}$, defined by Eq. (6),

$$\mathbf{B} = \text{rot}(\mathbf{A}) \quad (6)$$

and assuming axial symmetry, Eqs. (1)-(5) are transformed into the following simple scalar equation.

$$\begin{aligned} & (\partial / \partial r) \{ (1/r) (\partial \Psi / \partial r) \} \\ & + (\partial / \partial z) \{ (1/r) (\partial \Psi / \partial z) \} \\ & = -\mu_0 J_\theta \end{aligned} \quad (7)$$

in which Ψ is the magnetic stream function defined by rA_θ , where A_θ is the azimuthal component of $\mathbf{A}$.

When the physical quantities such as vector potential and current density can be expressed as a time-harmonic function, i.e. $J_\theta = J_0 \cos(\omega t)$, the solution of Eq. (7) can be described by the following equation^{5, 21)}.

$$\Psi = \Psi_i(r, z) \cos(\omega t) + \Psi_o(r, z) \sin(\omega t) \quad (8)$$

where Ψ_i and Ψ_o are respectively the in-phase and out-of-phase components of Ψ .

By solving the equations obtained by substituting

Eq. (8) into Eq. (7), under the boundary conditions of $\Psi_i = \Psi_o = 0$ at the center line and the far field, the electromagnetic field in the CZ furnace is obtained.

The distributions of the heat generation rate per unit volume in the crucible are given by

$$Q = \sigma' \omega^2 (\Psi_i^2 + \Psi_o^2) / (2r^2). \quad (9)$$

1.2 Flow and temperature fields

In the global analysis developed here, the temperature distributions in the furnace, the flow pattern in the melt, the melt/crystal interface shape and the electric current in a RF coil necessary to keep the crystal diameter constant can be computed simultaneously if the furnace geometry and the temperature at the chamber wall are given as input data.

The following assumptions are made in the calculations. (1) The system is in a pseudo-steady state and is axially symmetric. (2) The flow is laminar and the fluid is incompressible. (3) Heat transfer in the crystal is dominated by conduction although, strictly speaking, energy transfer via radiation within the crystal should be considered in addition to conduction because the oxide crystal is optically semitransparent. In the melt, heat transfer is dominated by conduction and convection. (4) The crystal is pulled continuously from the melt at a constant rate. (5) Heat loss from the surface elements is due to radiation alone. (6) The melt/crystal interface is at the melting-point isotherm.

Under the above assumptions, the dimensionless forms of governing equations for flow and temperature fields are given as follows.

In the melt:

$$Pr \mathbf{v}_i^* \cdot \nabla T_i^* = \nabla \cdot \{ \kappa(T_i^*) \nabla T_i^* \} \quad (10)$$

$$\mathbf{v}_i^* \cdot \nabla \mathbf{v}_i^* = -\nabla p_i^* - \nabla \cdot \boldsymbol{\tau}_i^* + Gr(T_i^* - 1) \mathbf{e}_z \quad (11)$$

$$\nabla \cdot \mathbf{v}_i^* = 0 \quad (12)$$

In the crystal:

$$Pe \mathbf{e}_z \cdot \nabla T_s^* = \nabla \cdot \{ \kappa(T_s^*) \nabla T_s^* \} \quad (13)$$

In the crucible:

$$\nabla \cdot \{ \kappa(T_c^*) \nabla T_c^* \} + Q^* = 0 \quad (14)$$

Elsewhere:

$$\nabla \cdot \{ \kappa(T_i^*) \nabla T_i^* \} = 0 \quad (15)$$

where superscript (*) represents the dimensionless variable. In Eq. (14), Q^* is the dimensionless form of the heat generation rate given by Eq. (9).

The boundary conditions for the temperature field are expressed as follows.

At the melt/gas interface:

$$-\kappa(T_i^*) \mathbf{n} \cdot \nabla T_i^* = q_{rad, i}^* \quad (16-1)$$

At the melt/crystal interface:

$$\kappa(T_i^*) \mathbf{n} \cdot \nabla T_i^* - \kappa(T_s^*) \mathbf{n} \cdot \nabla T_s^* = PeSt \mathbf{n} \cdot \mathbf{e}_z \quad (16-2)$$

$$T_i^* = T_s^* = 1 \quad (16-3)$$

Fig. 1 Schematic diagram of the inductively heated CZ furnace

At the crystal side and top:

$$-\kappa(T_s^*)n \cdot \nabla T_s^* = q_{rad,s}^* \quad (16-4)$$

At the other exposed surfaces:

$$-\kappa(T_i^*)n \cdot \nabla T_i^* = q_{rad,i}^* \quad (16-5)$$

At the center line:

$$n \cdot \nabla T_i^* = n \cdot \nabla T_s^* = n \cdot \nabla T_i^* = 0 \quad (16-6,7,8)$$

In these equations, $q_{rad,k}^*$ ($k = 1, s$ and i) is heat loss per unit area on the boundary surfaces due to radiation, and is given by the following equation¹⁹⁾.

$$q_{rad,k}^* = R_k (T_k^{*4} - (\sum A_j \epsilon_j T_j^{*4} G_{jk}) / A_k / \epsilon_k) \quad (17)$$

where R_k is radiation number and G_{jk} is Gebhart's absorption factor, which is the fraction of emission from surface A_j that reaches A_k and is absorbed. In the estimation of G_{jk} , the view factors between each surface element included in the furnace should be calculated. Since in the global analysis there exist many surface elements, some of which are hidden by the other elements, the calculation of view factors frequently consumes a lot of CPU time. In the present work, we developed a method to calculate the view factors effectively by combining the analytical solutions.

The boundary conditions for the flow field are expressed as follows.

At both the side wall and the bottom of the crucible:

Table 1. Physical properties and processing parameters used in calculations

1. Physical properties

Electric conductivity	
crucible (σ_c) [$\Omega^{-1} \cdot m^{-1}$]	2.33×10^6
chamber (σ_{ch}) [$\Omega^{-1} \cdot m^{-1}$]	4.00×10^6
Melting point (T_m) [K]	1526
Heat of solidification (ΔH_s) [J·kg ⁻¹]	4.56×10^5
Thermal conductivity	
crystal (k_c) [W·m ⁻¹ ·K ⁻¹]	3.09
melt (k_l) [W·m ⁻¹ ·K ⁻¹]	3.09
crucible (k_c) [W·m ⁻¹ ·K ⁻¹]	90.0
insulator (k_{in}) [W·m ⁻¹ ·K ⁻¹]	0.400
radiation shield (k_{sh}) [W·m ⁻¹ ·K ⁻¹]	3.00
Density	
crystal (ρ_c) [kg·m ⁻³]	4.63×10^3
melt (ρ_l) [kg·m ⁻³]	3.67×10^3
Viscosity (μ_l) [mPa·s]	41.2
Kinematic viscosity (ν_l) [m ² ·s ⁻¹]	1.123×10^{-5}
Heat capacity	
crystal (C_{ps}) [J·kg ⁻¹ ·K ⁻¹]	1.02×10^3
melt (C_{pl}) [J·kg ⁻¹ ·K ⁻¹]	1.02×10^3
Emissivity	
crystal (ϵ_c) [-]	0.3
melt (ϵ_l) [-]	0.3
crucible (ϵ_c) [-]	0.2
insulator (ϵ_{in}) [-]	0.4
radiation shield (ϵ_{sh}) [-]	0.35
Thermal expansion coefficient (β) [K ⁻¹]	1.70×10^{-4}
Surface tension (γ) [N·m ⁻¹]	0.175
Temperature coefficient of surface tension ($-d\gamma/dT$) [N·m ⁻¹ ·K ⁻¹]	3.00×10^{-4}
Contact angle	
against crystal (θ_c) [deg]	17
against crucible (θ_c) [deg]	90

2. Processing parameters

Current frequency of RF coil (ω) [kHz]	7.00
Crucible inner radius (r_c) [m]	2.85×10^{-2}
Crucible height (z_c) [m]	6.00×10^{-2}
Crystal radius (r_s) [m]	1.23×10^{-2}
Crystal length (l_s) [m]	5.10×10^{-2}
Melt depth (l_l) [m]	3.90×10^{-2}
Chamber temperature (T_{ch}) [K]	305.0
Crystal rotation rate (n) [s ⁻¹]	0 - 1.32
Pull rate (V_s) [m·s ⁻¹]	2.8×10^{-7}

$$u_i^* = v_i^* = w_i^* = 0 \quad (18-1,2,3)$$

At the melt/gas interface:

$$n \cdot v_i^* = 0, \quad \tau_i^* : nt = Ma \nabla T^* \cdot t \quad (18-4,5)$$

$$\tau_i^* : ne_\theta = 0 \quad (18-6)$$

At the melt/crystal interface:

$$u_i^* = w_i^* = 0, \quad v_i^* = r^* Re_s \quad (18-7,8,9)$$

At the center line:

$$u_i^* = v_i^* = 0, \quad \tau_i^* : nt = 0 \quad (18-10,11,12)$$

The melt/crystal interface shape is determined so that Eq. (16-3) is satisfied; i.e. the interface coincides with the melting-point isotherm.

The melt/gas interface shape is calculated by solving the Young-Laplace equation.

1.3 Numerical procedure

To solve the above problem, the Galerkin finite element method is used. The calculation domain is discretized by isoparametric quadrilateral elements, where

Fig. 2 Contours of stream functions in the melt (left side) and isotherms in the furnace (right side). (a) shows the results without crystal rotation and (b) at $Re_s = 275$. The contour intervals are 0.1 [-] for the stream function and 15.26 [K] for the temperature

the total number of elements and nodal points are 4240 and 4374 in the electro-magnetic field, and 1095 and 4664 in the flow and temperature fields, respectively. In each element, magnetic stream functions are approximated with bilinear trial functions ψ as follows.

$$\Psi(r, z) = \Sigma \psi_j \Psi_j \quad (19-1)$$

Velocity vectors and temperature are approximated with the biquadratic trial function ϕ as follows.

$$\mathbf{v}^*(r^*, z^*) = \Sigma \phi_i \mathbf{v}_i^* \quad (19-2)$$

$$T^*(r^*, z^*) = \Sigma \phi_i T_i^* \quad (19-3)$$

The linear trial function χ_k , which is discontinuous between each element, is used to approximate pressure.

$$p^*(r^*, z^*) = \Sigma \chi_k p_k^* \quad (19-4)$$

By substituting Eqs. (19) into the governing equations and boundary conditions for the electro-magnetic field and the flow and temperature fields, the Galerkin procedure gives a set of algebraic equations. In the present work, the crystal diameter is fixed and the current density in the coil is obtained as one of the unknown variables, together with velocity vectors, pressure, temperature and the melt/crystal interface shape, by solving simultaneously the set of algebraic equations by use of the Newton-Raphson scheme. Since it is found from Eqs. (7) and (9) that Q^* is proportional to J_0^2 , Q^* can be expressed by hQ_0^* where Q_0^* represents the local heat generation rate calculated at a reference RF coil current density J_0^0 , and the coefficient $h (= J_0^2/J_0^{02})$ is a practical variable to be solved.

2. Results and Discussion

In the present work, the geometry of the CZ furnace shown in **Fig. 1** is adopted, where a LiNbO_3 single crystal of 25 mm diameter is pulled continuously at 1 mm/h from the melt in the Pt crucible (60 mm^{O.D.} × 60 mm^H × 1.5 mm^t). Physical properties and processing parameters used here are listed in **Table 1**. Since no reliable values of the physical properties are available for

Fig. 3 Contours of stream functions in the melt (left side) and isotherms in the furnace (right side) calculated with the melt viscosity dependent on temperature. (a) shows the results without crystal rotation and (b) at $Re_s = 275$. The contour intervals are 0.1 [-] for the stream function and 15.26 [K] for the temperature

LiNbO_3 crystal and melt, especially at high temperature, some are extrapolated from the values at lower temperature and some for different materials such as sapphire¹⁹⁾ are used. The temperature dependency of the melt viscosity is taken into account by fitting the values measured by Shigematsu *et al.*¹⁷⁾ with the following equation.

$$\mu = 3710 / (T - 1492.0) \quad [\text{mPa.s}] \quad (20)$$

Viscosity in Table 1 corresponds to that estimated at 1582 K by use of Eq. (20).

In Section 2.1 we describe the effects of the temperature dependency of viscosity, where the thermocapillary effect is ignored by setting $Ma = 0$ in the boundary condition. The effect of thermocapillary force is discussed in Section 2.2.

2.1 Effect of temperature dependency of the melt viscosity

Figure 2 shows the contours of stream functions in the melt, and isotherms in the melt, the crystal, the crucible and a part of the thermal insulator for $Gr = 4.67 \times 10^5$ and $Ma = 0$, where viscosity of the melt is assumed to be independent of temperature, i.e. $\mu = 41.2$ [mPa.s]. In **Fig. 2**, (a) is the case without crystal rotation ($Re_s = 0$) and (b) is at $Re_s = 275$. Since Pr of LiNbO_3 melt is relatively high, 13.6, the temperature field and the melt/crystal interface shape are strongly affected by the flow pattern in the melt. For (a) $Re_s = 0$, only the clockwise vortex caused by free convection is present in the melt where the maximum value of stream function is 1.11, and the melt/crystal interface is M-shaped. For (b) $Re_s = 275$, a counter-clockwise vortex due to crystal rotation exists under the crystal rod in addition to free convection, where the maximum value of the stream function is 0.980 and the minimum value is -0.076, and the melt/crystal interface becomes convex to the crystal.

Figure 3 shows the contours of the stream function in the melt and isotherms in the CZ furnace for (a) $Re_s = 0$ and (b) 275, where the melt viscosity depends on the temperature as described by Eq. (20). The maximum

Fig. 4 Effect of the crystal rotation rate on the melt/crystal interface shape

Fig. 5 Effect of the crystal rotation rate on the electric current in the RF coil and on the maximum temperature in the crucible

value of the stream function is 1.13 in (a) and 1.14 in (b), and the minimum value is -0.09 in (b). In comparison with the results in Fig. 2, there is little difference in qualitative aspects of the flow pattern or the temperature distributions in the CZ furnace, although the maximum value of stream function at $Re_s = 275$ is larger than that in Fig. 2 and the temperature distribution near the melt/crystal interface is slightly different between the two cases.

Figure 4 shows the effect of Re_s , i.e. the crystal rotation rate, on the melt/crystal interface shape, where Δz^* represents the difference between z^* on the interface and z^* at the crystal periphery. From the figure, the interface shape is strongly influenced by Re_s , i.e. the flow pattern in the melt. When the free convection is dominant (small Re_s), the melt/crystal interface is convex to the melt where Δz^* is negative. But it becomes convex to the crystal beyond a certain Re_s . This abrupt change in interface shape is due to the counter-clockwise vortex under the interface caused by crystal rotation as shown in Figs. 2 and 3. The effect of temperature dependency of the melt viscosity on the interface shape is small, except that the interface shape is more convex to the crystal at higher Re_s . Consequently, the critical Re_s at which interface inversion takes place is almost coincident in both cases, i.e. approximately 215. In a previous study¹⁹⁾ we demonstrated theoretically the interface inversion of Al_2O_3

Fig. 6 Contours of stream functions in the melt (left side) and isotherms in the furnace (right side). (a) shows the results without crystal rotation and (b) at $Re_s = 600$. The contour intervals for the stream functions and temperature are $0.2 [-]$ and $15.26 [K]$ respectively.

crystal growth system with a semi-global model where the crucible wall temperature was assumed to be constant. In the present work, the interface inversion was predicted theoretically by a global analysis.

Figure 5 shows the effect of the crystal rotation rate on the RF coil current necessary to keep the crystal diameter constant (25 mm) and on the maximum value of temperature in the crucible. From the figure it is found that current and crucible temperature increase with Re_s . Since the counter-clockwise flow under the crystal caused by crystal rotation, as shown in Figs. 2 (b) and 3 (b), suppresses heat transfer via convection from the crucible to the crystal, the crucible temperature, i.e. the current in the coil, must be increased to maintain the same heat loss from the crystal surface. The effect of temperature dependency of the melt viscosity was found to be rather small.

2.2 Effect of thermocapillary force

In addition to forced and free (buoyancy-driven) convections, the Marangoni (surface tension-driven) convection should be also taken into account in the flow analysis. The Marangoni convection is caused by two driving forces: thermocapillary and solutalcapillary forces. Recently it was reported¹³⁾ that the convective pattern of the $LiNbO_3$ melt changes according to the partial pressure of oxygen in the furnace gas atmosphere, and that this might be due to the variation in the oxidation number of the metal in the melt. Although the solutalcapillary force should be taken into account in such a case, only the effect of the thermocapillary force was investigated in the present work, because of the lack of data for the concentration coefficient of surface tension. The Marangoni number based on the thermocapillary force, Ma , in this analysis is 2.82×10^4 using the data listed in Table 1.

Figure 6 shows the contours of stream function and isotherms, where (a) shows the result at $Re_s = 0$ and (b) at 600. In this section, the viscosity is assumed to be constant as in the case shown in Fig. 2. For $Re_s = 0$, a clockwise flow caused by free and Marangoni convection exists in the melt pool, and the maximum value of

Fig. 7 Effect of crystal rotation rate on the melt/crystal interface shape when Marangoni convection is taken into account

stream function, 2.88, is much larger than the value of 1.11 without the Marangoni convection in Fig. 2. The stronger flow brings heat much more efficiently from crucible to crystal. The RF coil current to keep the crystal diameter constant (25 mm) and the maximum crucible temperature are much smaller (see Fig. 8). To transport nearly the same amount of heat at a smaller temperature difference, a larger melt/crystal area is needed and the interface deformation is increased.

Figure 7 shows the effect of crystal rotation rate on the shapes of the melt/crystal interface for $Ma = 2.82 \times 10^4$. In comparison with Fig. 4, the interface is more convex to the melt at $Re_s = 0$, and becomes more convex as Re_s increases from 0 to 350. As the forced flow is induced by crystal rotation, and the heat transfer at the melt/crystal interface becomes less efficient, the crucible temperature, i.e. RF coil current, must be increased, as shown in Fig. 8. But at $Re_s > 400$ the forced convection roll cell appears first near the center line and the melt with the higher temperature ascends toward the melt/crystal interface. Also, the melt/crystal interface becomes less convex to the melt as Re_s increases. Consequently, it becomes almost convex to the crystal beyond $Re_s = 500$. From the figure, the critical Re_s is inferred to exist between 500 and 600. This value is much larger than that without the Marangoni flow, $Re_s = 215$. Also, the effect of the strong Marangoni flow remains in the melt/crystal shape near the crystal periphery.

Conclusions

For LiNbO_3 crystal growth in an inductively heated CZ furnace, global analysis was carried out by use of the finite element method and the effect of temperature dependency of physical properties on the crystal growth was studied. The following conclusions were obtained.

- 1) By use of a mathematical model developed in the present work, interface inversion was theoretically demonstrated.
- 2) The effect of temperature dependency of viscosity of the melt on the melt/crystal interface and other charac-

Fig. 8 Effect of crystal rotation rate on the electric current in the RF coil and on the maximum temperature in the crucible with Marangoni convection

teristics of crystal growth are rather small under the calculation conditions used here.

3) Marangoni convection strongly affects CZ crystal growth. The critical Reynolds number becomes larger and the RF current becomes smaller in comparison with those without the Marangoni flow.

Nomenclature

A	= vector potential	[Wb·m ⁻¹]
$A_{j \text{ or } k}$	= area of surface element	[m ²]
A_θ	= azimuthal component of A	[Wb·m ⁻¹]
B	= magnetic flux density	[Wb]
C_p	= heat capacity	[J·kg ⁻¹ ·K ⁻¹]
E	= electric field strength	[V·m ⁻¹]
G_{jk}	= Gebhart's absorption factor	[-]
Gr	= Grashof number ($= r_c^3 \beta T_m g / \nu_l^3$)	[-]
g	= gravitational acceleration	[m·s ⁻²]
ΔH_s	= latent heat of solidification	[J·kg ⁻¹]
h	= parameter of volumetric heat generation rate	[-]
J	= current density	[A·m ⁻²]
J_0	= current density of RF coil	[A·m ⁻²]
J_θ	= azimuthal component of J	[A·m ⁻²]
k	= thermal conductivity	[W·m ⁻¹ ·K ⁻¹]
l_l	= melt length	[m]
l_s	= crystal length	[m]
Ma	= Marangoni number ($= -(d\gamma/dT) r_c T_m / \rho_l \nu_l^2$)	[-]
n	= rotation rate	[s ⁻¹]
Pe	= Peclet number ($= \rho_s C_{ps} V_s r_c / k_l$)	[-]
Pr	= Prandtl number ($= \mu_l C_{pl} / k_l$)	[-]
p	= pressure	[Pa]
p^*	= $(p - p_0 + \rho_l g z) r_c^2 / \rho_l \nu_l^2$	[-]
p_0	= ambient gas pressure	[Pa]
Q	= volumetric heat generation rate	[W·m ⁻³]
Q^*	= $Q r_c^2 / k_l T_m$	[-]
q_{rad}	= heat flux due to radiation	[W·m ⁻²]
q_{rad}^*	= $q_{rad} r_c / k_l T_m$	[-]
R	= radiation number ($= \epsilon \sigma r_c T_m^3 / k_l$)	[-]
Re_s	= Reynolds number ($= 2\pi r_c^2 n / \nu_l$)	[-]
r	= radial distance in cylindrical coordinates	[m]
r^*	= r/r_c	[-]
r_c	= crucible radius	[m]
r_s	= crystal radius	[m]
S_t	= Stefan number ($= \Delta H_s / C_{ps} T_m$)	[-]
T	= temperature	[K]
T^*	= T/T_m	[-]

T_{ch}	= chamber temperature	[K]
T_m	= melting point temperature	[K]
t	= time	[s]
u	= radial component of velocity vector	[m·s ⁻¹]
u^*	= ur_c / v_l	[-]
V_s	= crystal pull rate	[m·s ⁻¹]
v	= velocity vector	[m·s ⁻¹]
v^*	= vr_c / v_l	[-]
v	= azimuthal component of velocity vector	[m·s ⁻¹]
v^*	= vr_c / v_l	[-]
w	= axial component of velocity vector	[m·s ⁻¹]
w^*	= wr_c / v_l	[-]
z	= axial distance in cylindrical coordinates	[m]
z^*	= z / r_c	[-]
z_c	= crucible height	[m]
Δz	= difference between z on the melt/crystal interface and z at the crystal periphery	[m]
Δz^*	= $\Delta z / r_c$	[-]
β	= thermal expansion coefficient	[K ⁻¹]
γ	= surface tension	[N·m ⁻¹]
ε	= emissivity	[-]
θ	= contact angle	[deg]
μ	= viscosity	[Pa·s]
μ_0	= permeability of vacuum	[H·m ⁻¹]
ν	= kinematic viscosity	[m ² ·s ⁻¹]
κ	= ratio of thermal conductivity (= k / k_l)	[-]
ρ	= density	[kg·m ⁻³]
σ	= Stefan-Boltzman constant	[W·m ⁻² ·K ⁻⁴]
σ'	= electric conductivity	[Ω ⁻¹ ·m ⁻¹]
τ	= stress tensor	[Pa]
τ^*	= $\tau r_c^2 / \rho_l v_l$	[-]
ω	= frequency of RF current	[rad·s ⁻¹]
Ψ	= magnetic stream function	[Wb]
Ψ_i	= in-phase component of Ψ	[Wb]
Ψ_o	= out-of-phase component of Ψ	[Wb]
<Subscripts>		
c	= crucible	

i, j, k	= material
l	= melt
s	= crystal

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