A DYNAMIC MODEL OF CAPILLARY SUCTION APPARATUS

D.J. LEE*

Department of Chemical Engineering, National Taiwan University, Taipei, Taiwan, 10617, ROC

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A model for the dynamic behaviour of the wet front radius and the liquid saturation in a capillary suction apparatus (CSA) is developed. Governing equations based on mass/momentum balance are derived and solved numerically. The calculation shows that the effects of the initial conditions on system dynamics vanish rapidly, and that the system then evolves along a slow manifold which is independent of the initial conditions chosen. It is also noted that, after sufficient time, the log-log plot of the dimensionless wet front radius versus dimensionless time can be approximated by straight lines and the liquid saturation under the inner cylinder can be taken as constant with error less than 5%. The increase in solids concentration and/or the averaged specific resistance of cake increases the capillary suction time (CST) significantly. The proper range of experimental conditions is suggested. A rapid method based on the model for estimating the averaged specific resistance of cake is proposed and compared with experimental results.

Introduction

Capillary suction apparatus (abbr. CSA) has been widely used since first developed by Gale and Baskerville in 1967[1]. A CSA is composed of two plastic plates, a Whatman No. 17 chromatography paper, a stainless steel cylindrical column, several electrodes serving as sensors, and a timer. Sludge is poured into the column and the time the filtrate needs to travel between two concentric circles is called the capillary suction time (abbr. CST). Usually, a long CST means a large cake specific resistance[2]. Since the CSA test is much easier to perform and is more time-saving than the conventional filtration test, attention has been attracted to theoretical modelling of the fluid flow through the cake and the filter paper to achieve satisfactory data analysis.

There exist two ways of describing the liquid movement in a CSA[3]; the piston-like approach, which treats the liquid flow in the filter paper as a displacement process[6, 13, 15-18], and the diffusion-like approach, which treats the process as a diffusion process with diffusivity as a concentration-dependent property[0-12]. It is noted that the models developed from the piston-like approach might seriously overestimate the liquid invasion volume and cause errors in estimating the specific resistance of the cake[13]. Besides, from the physical point of view it is unrealistic to assume that the liquid saturation will change discontinuously from unity to zero at the moving front.

By assuming a power law dependence of diffusivity $D = D_s r^n$, where $D$ lumps the effects of permeability, relative permeability and capillary suction pressure, and $D_s$ is the value of $D$ for dry filter paper, the effects of various liquids and the parameters of CSA on CST are investigated[10]. A pseudo-steady state model is proposed. Based on the diffusion-like approach, CSA is utilized for estimating the averaged specific resistance of cake[11]. The relation between capillary suction pressure and liquid saturation is constructed. Various slurries are tested and the predicted specific resistance agrees well with the experimental results.

Although the diffusion-like approach is adequate in analysing the fluid flow behaviour in CSA, there exist several experimental observations that form the basis of data analysis with no theoretical justification. Two among the others are: (1) the dimensionless wet-front radius versus time data, which can be approximated by straight lines on a log-log scale for various liquid/CSA combinations[10]; and (2) the liquid saturation under the inner cylinder, which approaches a constant value when the wet-front radius is large whether the cake is compressible or not[11]. These observations directly relate to the validity of treating the cake formation process in CSA as a constant-pressure filtration process, which is the essential step in determining the averaged specific resistance of cake[11]. It is wondered whether these characteristics hold for most slurries (and CSA) in actual applications. The range of experimental conditions for valid CSA tests must also be determined. A dynamic model is required to obtain more insight into the process.

In this report, based on the mass/momentum balance and the concept of a diffusion-like approach, a dynamic model for capillary suction apparatus is developed. The effects of slurry concentration, cake specific resistance, CSA parameters, and liquid substance on CST are analysed. The limitations of using CSA in
determining slurry characteristics are also discussed. A rapid method for estimating the averaged specific resistance of cake is proposed and compared with experimental results.

1. Analysis

From the injection of the slurry into the inner cylinder, filtrate first wets the filter paper under the inner cylinder and then spreads out. During that process, cake forms on the paper continuously, the amount being proportional to the volume of filtrate if no sedimentation occurs.

1.1 Mass balance

Take the filter paper under the inner cylinder as the control volume (Fig. 1). The filtrate will flow across the cake and into the top surface of the control volume A. Meanwhile, the liquid will also diffuse out through the periphery boundary B. Therefore, the dynamic behaviour of liquid saturation under the inner cylinder is governed by the mass balance of liquid in the control volume. The mass balance for the liquid contained in the control volume can then be stated as:

\[
\frac{d}{dt} \left( \pi R_0^2 \delta s_0 \right) = \pi R_0^2 \delta v - 2 \pi R_0 \delta q |_{r=R_0},
\]

where \( v \) and \( q \) are the flux of filtrate flowing from the top surface into the control volume, and the flux of filtrate diffusing out of the control volume through the periphery boundary, respectively. The definitions are:

\[
v = \frac{1}{A} \frac{dV}{dt}
\]

and

\[
q = -\varepsilon \delta D \nabla s.
\]

From the diffusion-like approach, the liquid saturation profile is shown as\(^{10}\):

\[
s = s_0 \quad 0 \leq r \leq R_0,
\]

\[
s = s_0 \left( \frac{R-r}{R-R_0} \right)^{1/n} \quad R_0 \leq r \leq R.
\]

Therefore, the liquid invasion volume can be found as:

\[
V = \pi R_0^2 \delta s_0 G(y),
\]

where

\[
G(y) = 1 + \frac{2n}{n+1} y + \frac{2n^2}{(n+1)(2n+1)} y^2
\]

\[
y = \frac{R}{R_0} - 1.
\]

Substituting Eqs. (2), (3) and (4) into Eq. (1), the result is:

\[
[1 - G(y)] \frac{d\delta s_0}{dt} = s_0 \frac{dG(y)}{dt} \left( \frac{2D_0 s_0^{n+1}}{nR_0(R-R_0)} \right).
\]

1.2 Force balance

The force needed to drive the filtrate through the cake is provided by the capillary suction pressure of the filter paper under the inner cylinder, neglecting the liquid head\(^{13}\), or equivalently,

\[
\Delta P_{	ext{cap}} = \frac{P_c(s_0)}{1} \alpha_{av} C_0 V \nu \mu,
\]

where \( \alpha_{av} \) is the averaged specific resistance of the cake. Substituting Eqs. (2a), (4) into Eq. (7), the result is:

\[
P_c(s_0) = C_0 \mu \alpha_{av} \delta v^2 s_0 G(y) \left( \frac{2D_0 s_0^{n+1}}{nR_0(R-R_0)} \right).
\]

The function \( P_c(s) \) for the water/Whatman No. 17 filter paper system has been shown to correlate well as\(^{13}\):

\[
P_c(s) = P_{cd}(1-s)^m,
\]

where \( P_{cd} \) and \( m \) are 62,200 Pa and 1.64, respectively. The physical properties and other quantities needed in calculations are listed in Table 1.

1.3 Formulations

Combining Eqs. (6) and (8), the following dimensionless governing equations can be obtained:

\[
\frac{d\delta s_0^*}{d\tau} = \frac{2P^*}{\beta G(y)} \frac{4s_0^{n+2}}{n^2 y^2}
\]

and

\[
\frac{dy}{d\tau} = \frac{P^* (1 - G(y))}{\beta s_0^* G(y) F(y) + 2G(y) s_0^*} \frac{nF(y) y}{n^2 F(y)}.
\]

The quantities appearing in Eqs. (10) and (11) are defined as follows.

\[
P^* = \frac{P_c}{P_{ref}}, \quad P_{ref} = \frac{D_1 \mu \delta^3}{R_0^2},
\]

\[
\beta = C_0 \alpha_{av} \delta^3, \quad \tau = \frac{D_1 t}{R_0^2}.
\]

Table 1. Quantities used in this work

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_0 )</td>
<td>0.535 cm</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.137 cm</td>
</tr>
<tr>
<td>( n )</td>
<td>0.73</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1 ( \times ) 10(^{-3} ) m(^2)/s</td>
</tr>
<tr>
<td>( D_1 )</td>
<td>0.71 \times 10(^{-4} ) m(^2)/s</td>
</tr>
<tr>
<td>( P_{cd} )</td>
<td>62,200 Pa</td>
</tr>
<tr>
<td>( m )</td>
<td>1.64</td>
</tr>
</tbody>
</table>
and
\[
F(y) = \frac{dG(y)}{dy} = \frac{2n}{n+1} + \frac{4n^2}{(n+1)(2n+1)} y. \tag{12}
\]

Eqs. (10) and (11) are two simultaneously nonlinear ordinary differential equations in which only two parameters, \( P_{ef} \) and \( \beta \), exist. When the liquid used and the CSA parameters are fixed, parameter \( \beta \), which is the product of the slurry concentration, the cake specific resistance, and the square of the paper thickness, will determine the system behaviour. The remaining problem is to choose proper initial conditions for completing the analysis.

The initial conditions, i.e., the wet-front radius and the liquid saturation \( s_0 \) at some \( \tau \), require knowledge of the fluid flow near the liquid injection. Actually, since the liquid cannot be injected instantly and the filter paper under the inner cylinder must be only partially wet at the injection point, it is hard to determine the liquid saturation and wet front radius near \( \tau = 0 \). Closer observation shows that there exists a singular point at \( y = 0 \) in Eqs. (10) and (11). In practice, the system dynamic behaviour at longer time (>5-10 s) is of interest. Therefore, if the effect of initial conditions on system dynamics vanishes rapidly, the basic characteristics will not be affected by the choice of initial conditions. In such a case, any values except those with \( y = 0 \) can be used. This point will be discussed further in the following sections.

1.4 Pseudo-steady state approximation

It is observed in experiments that the liquid saturation under the inner cylinder will approach a constant value when \( y \) is large. Under that condition, the derivative term in Eq. (10) vanishes and a simplified solution of Eqs. (10) and (11) can be found by setting \( ds^2_0/d\tau = 0 \). The result is
\[
\frac{2n^3}{3(n+1)(2n+1)} y^3 + \frac{n^2}{2(n+1)} y^2 - n_0^2 = 0. \tag{13}
\]

Eq. (13) is consistent with Eq. (10) in ref. 10, which is derived from an integral method based on pseudo-steady state approximation. Therefore, the result developed in ref. 10 is a special case of this work.

1.5 Numerical solution

Since no analytical solution is available for Eqs. (10) and (11), numerical integration is necessary. In this work, Gear’s method in the IMSL library is employed as the integration tool. The maximum relative error is set as \( 1 \times 10^{-6} \). All numerical works are performed on a VAX 6520 computer.

2. Results and Discussion

2.1 General

Figure 2 shows the calculation results with \( \beta = 10^7 \). Two sets of initial conditions are used to demonstrate the effects of initial conditions on system dynamics. It is shown that the calculated liquid saturations will coincide with each other when \( \tau \) is larger than about 1. After that, the liquid saturation will vary very slowly and, in practice, can be taken as a constant with error less than 5% when \( \tau \) ranges from 1 to \( 10^6 \). It is noted that the liquid saturation \( s_0 \) will approach zero at infinity time limit.

To obtain more insight into the dynamic characteristics of Eqs. (10) and (11), the effect of the initial conditions on system behaviour for CSA with \( \beta = 10^7 \) is summarized in Fig. 3. The calculation results with other \( \beta \) values all behave similarly. It is shown that no matter what initial conditions are chosen, the liquid saturation versus wet-front radius curves will all converge rapidly, usually for \( \tau < 0.3 \), to the locus A-B; i.e., curve A-B is the slow manifold of the solution surface of Eqs. (10) and (11) onto which all states on the phase plane are
attracted\(^2\). \(^3\)

Therefore, in numerical practice, any initial conditions except those with \(y = 0\) can be used. However, the liquid saturation under the inner cylinder should have some value between 0 and 1 by definition. Also, at the beginning of experiments, the wet front radius should be close to the boundary of the inner cylinder. Therefore, in the following study the initial conditions are set as \(s_0 = 1\) and \(y = 0.1\) at \(\tau = 0\) if not otherwise mentioned.

### 2.2 Effects of \(\beta\)

The effects of \(\beta\) values on wet-front radius and the liquid saturation \(s_0\) versus time relation are shown in Figs. 4 and 5. In most CSA applications, the \(\beta\) values will range from about \(10^3\) to \(10^7\) and the range of \(y\) value will usually be limited to less than about 15. Therefore, for the purpose of comparison, the range studied in Figs. 4 and 5 are general enough to cover most areas of interest.

It is clear that an increase in \(\beta\) will cause the CST to increase significantly. Take \(y = 5\) as an example. The CST for \(\beta = 10^3\), \(10^5\), \(10^7\), and \(10^{11}\) are \(\tau = 150\), 440, 3980 and 52,500, respectively. From the Figure, it is also clear that the log \(y\) versus log \(\tau\) curves can be approximated by straight lines with slopes depending on the range taken for correlation. In several calculations which extend the \(y\) value to larger than about 100, it is found that the slope of the log \(y\) versus log \(\tau\) curve approaches 0.33, which is the limiting case of Eq. (13) when \(y \to \infty\).

It is noted that when \(\beta\) is larger than \(10^{12}\), the curves show that the reflection point in the low \(y\) range and the CST’s might not increase when \(\beta\) increases. This inconsistency is caused by the initial conditions. Only the portions which can be taken as straight lines merit further discussion.

The liquid saturation \(s_0\) shown in Fig. 5 clearly indicates that a increase in \(\beta\) value will cause \(s_{0,\infty}\) to decrease. When \(\beta\) is less than \(10^4\), the curves will coincide with the line described by Eq. (13) with \(s_0 = 1\), which is the case with no cake formation. Since the maximum relative error was about 10 %, caused mainly by the individual quality variation between filter papers\(^4\), the \(\beta\) value of the sludge used in CSA tests should be larger than about \(10^5\) for meaningful data analysis.

It is also clear from the results that for cases with \(\beta\) larger than about \(10^{12}\) the liquid saturation will be very low and hard to differentiate. Besides, the CST’s in these cases are too large for practical usage. Therefore, it is suggested that the appropriate range of experimental conditions should be those with \(\beta\) ranging from about \(10^5\) to \(10^{11}\). The \(\beta\)’s of celite, CaCO\(_3\), kaolin, bentonite, and Cu (OH)\(_2\) slurries studied in ref. 11 are all within this range.

### 2.3 Comparisons

For checking purposes, the calculation results are compared with the experimental data in Fig. 6. The data are extracted from previous work\(^5\). It is clear that the wet-front dynamic data fitted well with the theory.

By fitting the experimental data with the calculation results shown in Fig. 4 and 5, the averaged specific resistance of cake can be estimated rapidly. For the sake of rapid estimation, the dimensionless capillary suction time is plotted against log \(\beta\) at several radial positions in Fig. 7. Several experimental results are also shown in the figure. The log \(\beta\) values for CaCO\(_3\) (2.71 wt%), kaolin (6.46 wt%), and bentonite slurry (1.37 wt%) are found as 6.9, 7.7, and 10.2, respectively. The averaged specific resistance of data in ref. 11 with viscosity near 1 cP are calculated and are shown in Table 2. The agreement between the estimated specific resistance and the vacuum filtration test results is fair.

### Conclusions

A dynamic model that relates the dynamic behaviour of the wet-front radius and liquid saturation in capillary suction apparatus (CSA) is developed. Governing equations (Eqs. (10) and (11)) based on the mass/force balance are derived and are solved by Gear’s method.
with relative error less than $1 \times 10^{-6}$. The parameters involved are the reference pressure $P_{ref}$, which includes the properties of liquid and filter paper, and $\beta$, which is the product of solid concentration, averaged specific resistance, and the square of paper thickness.

The calculation results show that the effects of the initial conditions on system dynamics will vanish rapidly and after that, the system evolves slowly along the slow manifold, which is independent of the initial conditions. When time is large, the log-log plot of the dimensionless wet-front radius $y$ versus $\tau$ can be approximated as straight lines and the liquid saturation under the inner cylinder can be taken as constant with error less than 5\%.

When $\beta$ increases, the CST will increase significantly. When $\beta$ is low ($< 10^3$), the CSA tests will be insensitive to the slurry used. When $\beta$ is very high ($> 10^{11}$), the CST will be too long for practical usage. The proper range of $\beta$ is thought to range from $10^3$ to $10^{11}$. A rapid method for estimating the specific resistance based on the CST at several radial positions is proposed. The estimated values agree well with the experimental results.

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Nomenclature

$A$ = cell area [m$^2$]
$C_0$ = solids concentration [kg/m$^3$]
$D$ = effective diffusivity [m$^2$/s]
$D_r$ = reference effective diffusivity [m$^2$/s]
$F$ = function in Eq. (12) [-]
$G$ = function defined in Eq. (5a) [-]
$L$ = cake thickness [m]
$m$ = index in Eq. (9) [-]
$n$ = index [-]
$p^*$ = dimensionless pressure [-]
$P_c$ = capillary suction pressure [Pa]

$P_{ref}$ = reference pressure defined in Eq. (12) [Pa]
$\Delta P_{cake}$ = pressure drop across the cake [Pa]
$q$ = diffusion flux [m$^3$/m$^2$-s]
$R_0$ = cylinder inner radius [m]
$R$ = equivalent radius of the front [m]
$r$ = radial coordinate [m]
$s$ = liquid saturation [-]
$s_0$ = liquid saturation under inner cylinder [-]
$t$ = time [s]
$V$ = liquid invasion volume [m$^3$]
$v$ = superficial velocity [m$/^3$/m$^2$-s]
$y = R/R_0 - 1$ [-]

$\alpha_{ref}$ = averaged specific resistance [m/kg]
$\beta$ = parameter defined in Eq. (12) [-]
$\varepsilon$ = wet filter paper porosity [-]
$\delta$ = wet filter paper thickness [m]
$\mu$ = liquid viscosity [Pa-s]
$\tau$ = dimensionless time [-]

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