DISTRIBUTIONS OF PRESSURE AND FLOW RATE IN A HOLLOW-FIBER MEMBRANE MODULE FOR PLASMA SEPARATION

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It is attempted to make an advanced model for the flow of water without solid particles in a hollow-fiber membrane module for plasma separation including the tubular shell part, considering that part is similar to a packed column. On the basis of the model, distribution of pressure and flow rate in hollow-fibers and in the tubular shell are estimated. The availability of the model is confirmed by comparing the estimated distribution with results of measurement of the pressure loss in the shell and that in a hollow-fiber. Furthermore, the probability of reflux is confirmed and the position of reflux is estimated based on the model.

Introduction

A hollow-fiber membrane module for plasma separation is already on the market. Its performance is characterized by various factors, such as the properties of the membrane, the mechanics of permeation, and the flow characteristic in the module. Some research studies concerning the subject have been carried out. Bruining²⁾ made a model of the relationship between pressure loss and direction of flow based on overall mass balance. Belfort *et al.*¹⁾ estimated the friction factor of a hollow-fiber. Numerical analysis of flow in the module has been carried out by Kelsey *et al.*³⁾ and Park *et al.*⁴⁾ The researches described above are based mainly on a theoretical point of view. There are few researches based on experiments.

In the previous paper⁵⁾, experiments concerning the flow mechanics of water in a hollow-fiber membrane module on the market were carried out. On the basis of the results, it became clear that Hagen-Poiseuille law and D'Arcy's law are almost applicable to flow in a hollow-fiber and to permeation through a membrane, respectively. In addition, a simple model concerning the flow mechanism was proposed. Though the model was confirmed to be almost appropriate, the flow in a tubular shell part was not considered. The pressure in the shell is predicted to change in the axial direction, and the change will have important influence on the permeation. It is necessary to estimate the pressure loss in the shell.

In this paper, we attempt to make an advanced model of the flow in a module including a tubular shell part, considering that the part is similar to a packed

column. On the basis of the model, distribution of pressure and flow rate in hollow-fibers and in the tubular shell are estimated. The availability of the model is discussed, based on the results of the measurement of pressure loss in the shell.

1. Experimental

The plasma separator used (SULFLUX FS-05, made by Kaneka Corporation) was the same as that described in the previous paper⁵⁾. **Table 1** shows its specification and **Fig. 1** (a) module "U" is its outline. It consists of a bundle of hollow-fibers, a shell which contains the bundle and two caps, connected respectively to the shell at the entrance and at the exit. Liquid which permeates the membrane flows out from exit "S", called "shell side", while retentate liquid flows out from exit "R", called "retentate side". It was confirmed⁵⁾ that liquid stagnates for a long time in the entrance cap and that the velocity at the entrance is not uniform over the cross-section of the bundle of hollow-fibers. In addition, pressure loss in the caps cannot be negligible. It is desirable to exclude the influence of the caps since the purpose of this paper is to estimate the pressure and flow rate distributions in hollow-fibers and in the shell, as described above. Therefore, the devised module shown in **Fig. 1** (b), module "P", was used. It was connected with two circular pipes with the same inner diameter as that of the shell. A grid was put at the entrance pipe for rectification of flow. **Figure 2** shows the velocity distribution of liquid passing through the grid as measured by an electro-chemical method⁵⁾. It is confirmed that the distribution is almost flat and that the flow into the entrance was rectified with the grid. The experimental

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Table 1. Specification of plasma separator

Hollow-fiber material	polysulfone
inner diameter	3.40×10^{-4} m
thickness of membrane	5.00×10^{-5} m
diameter of pore in membrane	3.00×10^{-7} m
effective area (approximate)	1.79×10^{-4} m ² /fiber
number of fibers (approximate)	2800

Fig. 1 Outline of plasma separator (dimensions in mm)

Fig. 2 Velocity distribution at the entrance

apparatus is illustrated schematically in **Fig. 3**. It is almost the same as that described in the previous paper⁵⁾ except for the module. Test liquid was ion-exchanged water without solid particles. **Figure 4** shows the positions of pressure taps (0.3 mm diameter) schematically. Pressure loss between two taps was measured with a manometer. Pressure loss in the bundle of hollow-fibers, representative transmembrane pressure, and pressure loss in the shell were obtained by the following equations.

$$\Delta P = P_i - P_o \quad (1)$$

Fig. 3 Experimental apparatus

$$TMP = \frac{P_o + P_i}{2} - P_{s4} \quad (2)$$

$$\Delta P_{sj} = P_{sj} - P_{s4} \quad (3)$$

where j represents the number of the position of a pressure tap.

Table 2 shows the experimental conditions. Variable parameters are the entrance flow rate and the ratio of retentate flow rate to entrance flow rate.

2. An Advanced Model Concerning the Flow in the Module

Figure 5 shows the pressure loss in the bundle of hollow-fibers. The abscissa is the average flow rate between entrance and exit flow rates expressed as follows, and is regarded as a representative flow rate.

$$Q_a = \frac{Q_i + Q_o}{2} = \frac{(1 + F) Q_i}{2} \quad (4)$$

It is expected that the Hagen-Poiseuille law is almost applicable. The relationship between TMP and permeate flow rate at constant F value is shown in **Fig. 6**. TMP is proportional to permeate flow rate, and D'Arcy's law is almost applicable to permeation through the membrane. In the previous model⁵⁾, on the basis of the same results

Fig. 4 Positions of pressure taps (dimensions in mm)

as described above, it was assumed that the pressure gradient in the axial direction of a hollow-fiber took a constant value, which could be calculated by the following equation:

$$\frac{\Delta P}{L} = \frac{128 \mu}{\pi d_{fi}^4} \cdot \frac{Q_a}{N_f} \quad (5)$$

and the velocity of liquid which permeated through the membrane was represented as follows with constant pressure in the shell.

$$U_p(z) = K \{P(z) - P_{s4}\} \quad (6)$$

where z is the axial distance from the entrance of a hollow-fiber and K is a coefficient. Though the model could represent the flow in the module approximately, flow rate and pressure in the hollow-fibers and in the shell, which are not regarded as constant, cannot be estimated based on the model. To estimate them, it is necessary to make a model based on the following, more precise assumptions.

1. In the model, the entrance and the exit regions, where hollow-fibers are fixed with urethane resin, are excluded because liquid does not permeate and the flow rate does not vary in the regions. Assuming that Hagen-Poiseuille law is applicable, pressure loss in the regions is calculated by the following equations.

$$\Delta P_{ent} = \frac{128 \mu}{\pi d_{fi}^4} \cdot \frac{Q_i}{N_f} \cdot L_e \quad (7)$$

$$\Delta P_{ext} = \frac{128 \mu}{\pi d_{fi}^4} \cdot \frac{F Q_i}{N_f} \cdot L_e \quad (8)$$

where L_e is the length of the regions. In the following consideration, z is the axial distance from the edge of the entrance region.

2. Considering that the flow rate in a bundle of hollow-

Table 2. Experimental conditions

F [-]	0.5	0.6	0.7	0.8
Q_i [m ³ /s]				
1.25×10^{-6}		○		
1.67×10^{-6}		○		
2.08×10^{-6}	○	○	○	○
2.50×10^{-6}		○

Fig. 5 Relation between pressure loss and average flow rate

Fig. 6 Relation between transmembrane pressure and permeate flow rate

fibers is a function of z , the pressure gradient in a hollow-fiber is expressed as follows.

$$\frac{dP(z)}{dz} = -\frac{128 \mu}{\pi d_{fi}^4} \cdot \frac{Q(z)}{N_f} \quad (9)$$

3. Regarding P_s as a function of z , transmembrane pressure also becomes a function of z as follows.

$$TMP(z) = P(z) - P_s(z) \quad (10)$$

D'Arcy's law is applicable to permeate flow, and the driving force of permeation is TMP . The velocity of liquid that permeates through the membrane is, therefore, expressed as follows.

$$U_p(z) = K \cdot TMP(z) \quad (11)$$

where U_p is the velocity at the inner wall of a fiber.

4. It is very difficult to estimate the pressure distribution in the tubular shell theoretically. Because hollow-fibers are not arranged regularly in the shell, and it is impossible to decide the shape of the flow pass. However, from the viewpoint described above, the shell can be considered to be similar to a packed column. Thus the pressure

Fig. 7 Pressure distribution in the shell at various F values

loss in the shell is assumed to be represented by the following equation, which corresponds to the Kozeny-Carman equation.

$$\frac{dP_s(z)}{dz} = -\frac{2k\mu}{\pi \varepsilon r_h^2 D^2} Q_s(z) \quad (12)$$

where k is constant, and ε and r_h are void fraction and hydraulic radius, respectively, which are calculated as follows for the module used.

$$\varepsilon = 1 - \frac{N_f d_{fo}^2}{D^2} = 0.436 \quad (13)$$

$$r_h = \frac{\varepsilon D^2}{4 N_f d_{fo}} = 8.50 \times 10^{-5} \text{ m} \quad (14)$$

On the basis of the assumptions described above and the mass balance in a hollow-fiber and the shell, the following differential equations concerning $Q(z)$ and $Q_s(z)$ are derived.

$$\frac{dQ(z)}{dz} = -\pi d_{fi} N_f K \cdot TMP(z) \quad (15)$$

$$\frac{dQ_s(z)}{dz} = \pi d_{fi} N_f K \cdot TMP(z) \quad (16)$$

By substituting Eqs. (15) and (16) into Eqs. (9) and (12), differential equations concerning $P(z)$ and $P_s(z)$ are derived as follows.

$$\frac{d^2 P(z)}{dz^2} = A \cdot TMP(z) \quad (17)$$

$$\frac{d^2 P_s(z)}{dz^2} = -B \cdot TMP(z) \quad (18)$$

where

$$A = \frac{128 K \mu}{d_{fi}^3} \quad (19)$$

$$B = \frac{2 k \mu d_{fi} N_f K}{r_h^2 D^2 \varepsilon} \quad (20)$$

Eqs. (15)-(18) are solved under the following condition.

$$Q(0) = Q_i, Q_s(0) = 0 \quad (21)$$

The solutions of the equations are obtained as follows.

Fig. 8 Pressure distribution in the shell at various entrance flow rates

$$P(z) - P(0) = \frac{A}{A+B} \{TMP(0) (\cosh \sqrt{A+B} z - 1) - \frac{128 \mu Q_i}{\pi d_{fi}^4 \sqrt{A+B}} \sinh \sqrt{A+B} z\} - \frac{B}{A+B} \cdot \frac{128 \mu Q_i}{\pi d_{fi}^4} z \quad (22)$$

$$P_s(z) - P(0) = -\frac{B}{A+B} \{TMP(0) \cosh \sqrt{A+B} z - \frac{128 \mu Q_i}{\pi d_{fi}^4} \left(\frac{\sinh \sqrt{A+B} z}{\sqrt{A+B}} - z \right)\} - \frac{A}{A+B} \cdot TMP(0) \quad (23)$$

$$Q(z) = -\pi d_{fi} N_f K \cdot TMP(0) \frac{\sinh \sqrt{A+B} z}{\sqrt{A+B}} + \frac{Q_i}{A+B} (A \cosh \sqrt{A+B} z + B) \quad (24)$$

$$Q_s(z) = \pi d_{fi} N_f K \left\{ TMP(0) \frac{\sinh \sqrt{A+B} z}{\sqrt{A+B}} - \frac{128 \mu Q_i}{\pi d_{fi}^4 (A+B)} (\cosh \sqrt{A+B} z - 1) \right\} \quad (25)$$

These equations contain unknown quantities k , K , and $TMP(0)$. It might be better to use the value of TMP at the exit instead of $TMP(0)$, because TMP is predicted to be fixed first at the exit at the beginning of the permeation. But, in this paper, steady state is considered, and $TMP(0)$ is used in order to make the equations simple. In any case, it is impossible to calculate both $TMP(0)$ and TMP at the exit without experimental results. The value of $TMP(0)$, which is calculated by the following equation, is substituted into Eqs. (22)-(25).

$$TMP(0) = P_i - \Delta P_{ent} - P_{s0} \quad (26)$$

where $P_i - \Delta P_{ent}$ corresponds to the pressure in a hollow-fiber at $z=0$. k and K must satisfy the experimental conditions represented as follows.

$$Q(L) = F Q_i \quad (27)$$

$$Q_s(L) = (1-F) Q_i \quad (28)$$

where L is the length of a fiber excluding those in the entrance and the exit regions. At the same time, it is necessary that the following equation is satisfied.

$$P(L) - P(0) = P_o - P_i + (\Delta P_{ent} + \Delta P_{ext}) \quad (29)$$

$$P_s(L) - P_s(0) = P_{s4} - P_{s0} \quad (30)$$

where $(\Delta P_{ent} + \Delta P_{ext})$ corresponds to the pressure loss in the entrance and exit regions. The left-hand sides of the equations are calculated by Eqs. (22), (23) and the right-

Fig. 9 Pressure distribution in a fiber at various F values

Fig. 10 Pressure distribution in a fiber at various entrance flow rates

hand sides are the experimental results. The values that satisfy Eqs. (27), (28) and minimize the difference between the two sides of Eqs. (29), (30) were chosen as those of k and K . **Figures 7, 8, 9 and 10** compare the results of calculation and experiment. The lines and symbols show the results of calculation and experiment, respectively. The relationship between $P_s(z)$ and the experimental results P_{sj} is shown as follows.

$$P_s(jL/4) - P_s(0) = P_{sj} - P_{s0} \quad (31)$$

It appears that the two results agree well with each other. Considering that the values of k and K were decided based on the experimental results at $z=0$ and L , experimental results of P_s at other positions are represented well by the calculated results. Based on this comparison, the model is considered to be applicable to the flow in the module.

Calculated values of k and K for various conditions are listed in **Table 3**. K , which is a coefficient in Eq. (11), represents the permeability of the whole module and is expected to take a constant value for various conditions. k is also expected to take a constant value for the same packed column. Mean values of k and K in Table 3 are 2.2 and $3.97 \times 10^{-8} \text{ m}^2 \cdot \text{s/kg}$, respectively. When pressure distributions in hollow-fibers and in the shell are calculated by using the mean values, the mean error becomes about 10 %. Taking the complexity of the flow path in the shell and errors of measurement into account, the mean values can be regarded as representative constant values of k and K . All values of k in Table 3 are less than

Table 3. Values of k and K

Q_i [m^3/s]	F [-]	k [-]	K [$\text{m}^2 \cdot \text{s/kg}$]
2.08×10^{-6}	0.5	2.41	4.08×10^{-8}
2.08×10^{-6}	0.6	2.24	3.92×10^{-8}
2.08×10^{-6}	0.7	2.26	3.66×10^{-8}
2.08×10^{-6}	0.8	2.10	3.42×10^{-8}
1.25×10^{-6}	0.6	2.24	4.06×10^{-8}
1.67×10^{-6}	0.6	2.15	4.15×10^{-8}
2.50×10^{-6}	0.6	2.00	4.50×10^{-8}

Fig. 11 Distributions of pressure and flow rate in a module

3, though k takes about 5 or 6 for the flow in a packed column which consists of spheres. It is considered that the fluid in the shell flows parallel to the axis of hollow-fibers and that there are no obstacles interrupting the path of the fluid. The resistance to flow is, therefore, less than that in a packed column.

3. Pressure and Flow Rate Distribution in the Module

Figure 11 shows an example of the distributions of pressure and flow rate in the module under the condition $Q_i = 2.08 \times 10^{-6} \text{ m}^3/\text{s}$, $F = 0.6$, calculated by Eqs. (22)-(25). Both the pressure in hollow-fibers and that in the shell are shown as the difference from $P(0)$. P becomes smaller than P_s , i.e. TMP takes negative values, in the region $z/L > 0.714$. In that region, the direction of permeation changes. Liquid permeates from the shell into hollow-fibers through the membrane and Q increases while Q_s decreases. The change is called "reflux" in this paper. The reflux is predicted to have significant influence on the performance of the module. It is important to estimate the position where reflux occurs. Considered as described above, pressure loss in the shell cannot be neglected in the estimation of the position of reflux even though it is small compared with that in hollow-fibers. In addition, P decreases nonlinearly. Eq. (5) is, therefore, not accurate.

Distributions of TMP for all experimental condi-

Fig. 12 Distribution of *TMP* at various *F* values

tions are summarized in Figs. 12 and 13. Figure 12 shows the dependency on *F*. When *F* is large, flow rate and pressure loss in hollow-fibers becomes large while that in the shell becomes small. Therefore, with increase of *F*, *TMP* becomes easy to take negative values and the position of reflux moves away from the exit of fibers. Figure 13 shows the dependency on *Q_i*. *TMP* becomes 0 at almost the same position regardless of the value of *Q_i*. The reason can be explained as follows. When *F* is constant, it is expected that the ratio of *Q* to *Q_s* at each axial position is almost constant and that distributions of *TMP* for various values of *Q_i* are similar. According to that consideration, it is expected that reflux occurs at the same position at a constant *F* value. In addition, based on Figs. 12, 13, it is predicted that the dependency of *TMP* (0) on *Q_i* is greater than that on *F*.

As described above, the position where reflux occurs depends only on *F*. It is possible to estimate the position based on Fig. 12 for the range of *F* and *Q_i* values of the experimental condition, which is almost the same as that in practical use for plasma separation.

Conclusions

An advanced model of the flow in a hollow-fiber module including the shell was proposed and was confirmed to be applicable. On the basis of the model, distributions of pressure and flow rate both in the hollow-fibers and in the shell were estimated. The probability of reflux was confirmed and the position of reflux could be estimated. Pressure loss in the shell appears to be important for the estimation. In the next step, it is necessary for evaluation of the performance of the module to estimate the flow mechanics when filtration is carried out.

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Fig. 13 Distribution of *TMP* at various entrance flow rates

Nomenclature

<i>A</i>	= coefficient defined by Eq. (19)	[1/m ²]
<i>B</i>	= coefficient defined by Eq. (20)	[1/m ²]
<i>D</i>	= inner diameter of the shell	[m]
<i>d_{fi}</i>	= inner diameter of a fiber	[m]
<i>d_{fo}</i>	= outside diameter of a fiber	[m]
<i>F</i>	= ratio of retentate flow rate to entrance flow rate	[-]
<i>j</i>	= number representing the position of a pressure tap	[-]
<i>K</i>	= coefficient of permeability	[m ² ·s/kg]
<i>k</i>	= coefficient in Eq. (12)	[-]
<i>L</i>	= length of a fiber	[m]
<i>L_e</i>	= length of entrance or exit region	[m]
<i>N_f</i>	= total number of fibers	[-]
<i>P</i>	= pressure in a fiber	[Pa]
<i>P_i</i>	= pressure at the entrance	[Pa]
<i>P_o</i>	= pressure at the exit "R"	[Pa]
<i>P_s</i>	= pressure in the shell	[Pa]
<i>Q</i>	= flow rate in the bundle of fibers	[m ³ /s]
<i>Q_a</i>	= average between entrance and exit flow rates in the bundle of fibers	[m ³ /s]
<i>Q_i</i>	= entrance flow rate	[m ³ /s]
<i>Q_o</i>	= retentate flow rate	[m ³ /s]
<i>Q_s</i>	= flow rate in the shell	[m ³ /s]
<i>r</i>	= radial position at the cross-section of the bundle	[m]
<i>r_h</i>	= hydraulic radius	[m]
<i>TMP</i>	= transmembrane pressure	[Pa]
<i>U_i</i>	= cross average velocity at the entrance	[m/s]
<i>U_p</i>	= permeate velocity	[m/s]
<i>u_i</i>	= local velocity at the entrance	[m/s]
<i>z</i>	= axial distance along the bundle	[m]
ΔP	= pressure loss in a fiber	[Pa]
ΔP_{ent}	= pressure loss in the entrance region	[Pa]
ΔP_{ext}	= pressure loss in the exit region	[Pa]
ΔP_s	= pressure loss in the shell	[Pa]
ϵ	= void fraction	[-]
μ	= viscosity	[Pa·s]

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