EFFECT OF AN ELECTRIC FIELD ON THE BEHAVIOR OF A DROP MOVING IN A QUIESCENT LIQUID

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A mathematical model for predicting the behavior of a dielectric liquid drop moving in a medium of another dielectric liquid in the presence of a uniform electric field has been developed. It is found that the terminal velocity of the drop is not strongly affected by the strength of electric field at small Reynolds numbers. A wake appears behind the drop due to the hybrid circulations induced by both the gravity and the electric force even at small Reynolds numbers and thus the drop is deformed asymmetrically about the equatorial plane.

Introduction

When a uniform electric field is applied to a dielectric liquid in which a drop of another dielectric liquid is suspended in a stationary state, circulations are induced both inside and outside the drop due to the tangential stress resulting from interaction between the electric field and charge built up at the drop/continuous phase interface. These circulations were first described by Taylor⁵⁾ who proposed criteria on the direction of the circulations and on the mode of drop deformation. Taylor's theory for steady electric fields was then extended by Torza *et al.*⁶⁾ to be valid for both steady and alternating electric fields. On the other hand, circulation developing in a drop moving under the influence of gravity has been investigated by many workers after the pioneering work by Hadamard and Rybczynski.

It can be readily inferred that hybrid circulations result when a drop of dielectric liquid moves through another such liquid by gravity in the presence of a uniform electric field. Chang *et al.*^{1,2)} computed the flow pattern around a drop translating parallel to the electric field at a small or intermediate Reynolds number and revealed that the separation angle on, and the wake length behind, the drop vary as a function of the applied field strength. They³⁾ also investigated theoretically the influence of such circulations on the rates of heat and mass transfer through the drop surface from the viewpoint of improving efficiencies of direct heat and mass exchangers with the aid of the circulations.

However, most of the previous theoretical studies of the electric induced circulations around stationary or translating drops have dealt exclusively with spherical or nearly spherical drops, although such drops may be deformed by the electric and hydrodynamic forces acting on their surfaces. The deformation of the drops can be increased considerably with an increasing electric field

strength. Recently Tsukada *et al.*⁷⁾ developed a numerical technique with which one can exactly predict not only the circulations but the deformation of a drop suspended in a stationary state in a dielectric liquid under the influence of a uniform electric field.

The first aim of the present work is to develop a mathematical model which can predict the behavior of a drop translating through a quiescent liquid in the presence of a uniform electric field by extending the model developed in the previous work.⁷⁾ The second is to investigate theoretically the relationship between the hybrid circulations and the drop deformation.

1 Theory

Figure 1 shows the cylindrical coordinate system used in the calculations, where a drop of liquid (2) moves through a quiescent liquid (1) under a uniform electric field E_0 parallel to the z -axis. We assume that the system is in steady state and axisymmetric. Another assumption we employ is that the drop and the continuous phase are leaky dielectric liquids⁷⁾.

Under the above assumptions, the dimensionless governing equations for the electric field are given by the following Laplace equations:

$$\nabla^2 V_1 = 0, \quad R \nabla^2 V_2 = 0 \quad (1-1, 2)$$

where the subscripts, 1 and 2, refer to the continuous phase and the drop, respectively.

The boundary conditions for Eq. (1) are given by the following equations.

At the drop surface:

$$V_1 = V_2 \quad (2)$$

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Fig. 1 Cylindrical coordinates

$$\frac{\partial V_1}{\partial n} = R \frac{\partial V_2}{\partial n} \quad (3)$$

At the outer boundary:

$$V_1 = E_0 r_o \cos \theta \quad (4)$$

At the center line:

$$\frac{\partial V_1}{\partial n} = \frac{\partial V_2}{\partial n} = 0 \quad (5)$$

The governing equations for the flow field, i.e. the momentum and continuity equations, in the dimensionless form are expressed as follows:

$$-\left(\frac{1}{Ca}\right) \bar{v}_1 \cdot \nabla \bar{v}_1 - \nabla p_1 - \nabla \cdot \bar{\tau}_1 = 0 \quad (6-1)$$

$$-\left(\frac{\alpha}{Ca}\right) \bar{v}_2 \cdot \nabla \bar{v}_2 - \nabla p_2 - \kappa \nabla \cdot \bar{\tau}_2 = 0 \quad (6-2)$$

$$\nabla \cdot \bar{v}_1 = 0, \nabla \cdot \bar{v}_2 = 0 \quad (7-1,2)$$

The above equations are subjected to the following boundary conditions.

At the drop surface:

$$\bar{\tau}_1 : \bar{n} \bar{t} - \kappa \bar{\tau}_2 : \bar{n} \bar{t} = (1/4\pi) (\bar{\tau}_1^E : \bar{n} \bar{t} - S \bar{\tau}_2^E : \bar{n} \bar{t}) \quad (8)$$

$$-p_1 + p_2 - \bar{\tau}_1 : \bar{n} \bar{n} + \kappa \bar{\tau}_2 : \bar{n} \bar{n} + (1/4\pi) (\bar{\tau}_1^E : \bar{n} \bar{n} - S \bar{\tau}_2^E : \bar{n} \bar{n}) - Bo z + \lambda + 2H = 0 \quad (9)$$

At the outer boundary:

$$u_1 = 0 \quad (10)$$

$$v_1 = V_t \quad (11)$$

At the center line:

$$\bar{n} \cdot \bar{v}_1 = \bar{\tau}_1 : \bar{n} \bar{t} = \bar{n} \cdot \bar{v}_2 = \bar{\tau}_2 : \bar{n} \bar{t} = 0 \quad (12)$$

Eqs. (8) and (9) show the tangential and normal stress balances on the drop surface, where $\bar{\tau}^E$ is the dimensionless Maxwell's stress tensor whose components are given as:

$$\tau_{lm}^E = -\frac{1}{2} |E|^2 \delta_{lm} + E_l E_m \quad (13)$$

The solution for the drop shape should satisfy Eq. (9) under the constraint for a fixed volume of the drop:

$$\int_0^\pi f^3 \sin \theta d\theta = \frac{1}{4} \quad (14)$$

The terminal velocity of the drop, V_t , is obtained from the constraint for the origin of the coordinates being at the mass center of the drop:

$$\int_0^\pi f^4 \sin \theta \cos \theta d\theta = 0 \quad (15)$$

If $Bo=0$ in Eq. (9) and $V_t=0$ in Eq. (11), the above governing equations and boundary conditions reduce to those for a stationary drop in a dielectric liquid under the influence of a uniform electric field⁷⁾.

The finite element method for solving free-boundary problems is applied to the set of above equations, Eqs.(1)-(15), to describe the electric and flow fields around a drop, the drop shape and the terminal velocity.

The electric potential and velocity vector are approximated by Lagrangian biquadratic polynomials, ϕ , defined over a mesh consisting of nine-noded quadrilateral elements:

$$V(r, z) = \sum \phi_i(r, z) V_i \quad (16)$$

$$\bar{v}(r, z) = \sum \phi_i(r, z) \bar{v}_i \quad (17)$$

The dynamic pressure is approximated by linear basis functions, ψ , which is discontinuous across each element:

$$p(r, z) = \sum \Psi_i(r, z) p_i \quad (18)$$

Since the drop shape is itself a part of the solution, it should also be expressed by the polynomials. The drop shape, f , shown in Fig. 1 is approximated by one-dimensional Lagrangian quadratic basis functions, χ , as follows.

$$f(\theta) = \sum \chi_i(\theta) f_i \quad (19)$$

Figure 2 shows the numerical mesh. Basically, the numbers of elements and nodes are 630 and 2,595, respectively. The outer boundary was set at 30 times the drop diameter far from the center of the drop. The discretization was confirmed to be enough to give no effect of mesh size on the solution.

Fig. 2 Finite element discretization

The application of the Galerkin weighted residual method to Eqs. (1)-(15) with the solution approximations, Eqs. (16)-(19), yields a set of nonlinear algebraic equations which are expressed by the following equation:

$$\bar{R}(\bar{x} : \bar{p}) = 0 \quad (20)$$

where $\bar{R}$ is a vector of residual equations, and $\bar{x}$ are vector unknowns, i.e. electric potential, velocity vector, pressure, drop shape, terminal velocity and so on. $\bar{p}$ represents the dimensionless parameters in Eqs. (1)-(15). We used Newton's method to solve Eq. (20).

2 Experiment

Figure 3 shows schematically the experimental apparatus, which was essentially the same as that used in the previous work⁷⁾ and consisted of a cell equipped with two electrodes and a high-voltage regulated D.C. power supply connected to one of the electrodes. A drop was injected into the continuous liquid through an injection needle from a syringe and then cut the two laser beams sequentially. Pulses from the photo-transistors started and stopped the universal counter. A D.C. high voltage was applied to the system before the drop was released from the needle. The terminal velocity of the drop was determined from the distance and the elapsed time between the two laser beams. For measurements of the drop shape and the drop volume, photographs of the drop were taken using a back light source. The cell was made of transparent acrylic resin, and it held a pair of parallel copper electrodes (150 x 150 x 5 mm). The distance between the electrodes was 155 mm, and the applied D.C. voltage was in the range of 0-25,000 V. Drop diameter was in the range of 1-5 mm. The whole experimental system was set in an air bath controlled at 303 K.

A silicone oil and a corn oil were used as the drop-forming liquid and the continuous phase, respectively. The physical properties of these liquids are listed in Table 1. It is difficult to measure accurately the electric conduc-

Fig. 3 Experimental apparatus

Table 1. Physical properties

	Corn oil	Silicone oil	Silicone oil/ Corn oil
conductivity σ [S m ⁻¹]	1.06×10 ⁻¹¹ 3.42×10 ⁻¹²	2.67×10 ⁻¹² 6.94×10 ⁻¹⁴	$R=0.252$ $R=0.020$
dielectric constant ϵ_r [-]	3.24	2.66	$S=0.82$
kinematic viscosity ν [m ² s ⁻¹]	4.61×10 ⁻⁵	1.78×10 ⁻⁵	$\kappa=0.39$
density ρ [kg m ⁻³]	9.14×10 ⁻²	9.41×10 ²	$\alpha=1.03$
interfacial tension γ [mN m ⁻¹]	1.45		

tivities of such dielectric liquids. The difficulty is particularly significant with the silicone oil, the conductivity of which is so small that it is strongly affected by the contaminants. For instance, the conductivity of the silicone oil used varied between 10⁻¹² and 10⁻¹⁴ [S m⁻¹] as shown in Table 1.

3 Results and Discussion

Figure 4 shows the relationship between the terminal velocity, V_p , and the volume-equivalent diameter, d_e , at various applied voltages, for silicone oil drops falling through the quiescent corn oil. In the figure, the lines and symbols show the calculated results and the experimental values, respectively. The result for $R=0.252$ and $V=20,000$ V is almost invisible since it falls on the line describing the results for $R=0.02$ and $V=15,000$ V. The terminal velocity of a drop of a given diameter decreases slightly as the applied voltage increases and the electric conductivity ratio of the drop to the continuous phase, R , decreases. The calculated results in the present analysis are in good agreement with the experimental ones.

The d_e - V_p relationship in Fig. 4 can be transformed into the relationship between the Reynolds number Re and the drag coefficient, C_D . The results are shown in Fig. 5, where the calculated results are for $R=0.02$. The broken line in Fig. 5 represents Hadamard-Rybczynski's analytical solution for zero applied voltage, and it is in a good agreement with the calculated results of the present analysis. Since Re here is relatively small (<1) as shown in Fig.

Fig. 4 Effect of the electric field on the terminal velocity of drops

Fig. 5 Reynolds number Re vs. drag coefficient C_D

5, it is inferred that the flow pattern around each drop is of a steady creeping flow and that its shape is kept spherical in the absence of the electric field.

Figure 6 shows the calculated equipotential lines, the velocity vectors and the drop shapes at various applied voltages for two values of R , where the drop diameter is fixed at 4.6 mm. It is shown that the drop becomes more oblate as the applied voltage increases and R decreases. Particularly, at a sufficiently high voltage, the drop is deformed asymmetrically about the equatorial plane to approximate a "spherical cap" or an "ellipsoidal cap" and a wake appears behind the drop in spite of the Reynolds number being so small (< 1). This is due to the interaction between the circulations caused by the gravity and the electric force. In the system with a drop of silicone oil moving through a corn oil considered here, the circulations from the pole to the equator are induced inside and outside the drop by the electric field, and the drop shape becomes oblate, according to Taylor's criterion⁵. Therefore, circulation by gravity is emphasized by Taylor's circulation at the front of the drop where circulation directions are the same, but a wake generates at the rear where the directions are opposite to each other. This difference between the front and the rear of the drop induces the asymmetric flow pattern around the drop and thus the asymmetric drop deformation about the equatorial plane.

Fig. 6 Effect of the electric field on the flow pattern around the drop and on the drop shape

Fig. 7 Effect of the electric field on the drop deformation

Figure 7 shows the relationship between the ratio of the vertical to the horizontal drop axes, a/b , and the volume-equivalent drop diameter, d_e , at various applied voltages. The plots and the lines in the figure represent the experimental and calculated values, respectively. The calculations are based on two alternative assumptions of R which correspond to two pairs of the measured electric conductivities listed in Table 1. The drop shape becomes more oblate as the drop diameter and the applied voltage increase and as R decreases. The experimental results are not in good agreement with any calculated ones. Particularly, the calculated results at high voltages predict much more oblate drop deformation than the experimental ones. This tendency is the same as that observed by the previous workers^{4,8}. One of the reasons for this discrepancy is the inaccuracy of the measurement of the electric conductivities of the liquids because of their indefinite contamination, as mentioned before. The drop deformation is more sensitive, than the terminal velocity, to the electric conductivities of the drop and continuous phases⁸.

Conclusions

Theoretical and experimental studies on the behavior of a dielectric drop moving through another dielectric liquid under a uniform electric field were carried out. A mathematical model which can deal with the above problem exactly was developed and the calculated results were compared with experimental ones. It was found that the terminal velocity of the drop is not strongly affected by the applied voltage. A wake appears behind the drop due to hybrid circulations and the drop is deformed asymmetrically about the equatorial plane even at small Reynolds numbers.

Nomenclature

a	= vertical length of drop	[m]
Bo	= Bond number $(=\rho_1 - \rho_2)gd_e^2/\gamma$	[-]
b	= horizontal length of drop	[m]
Ca	= Capillary number $(=\rho_1 v_1^2/\gamma d_e)$	[-]
C_D	= drag coefficient	[-]
	$(= 4gd_e(\rho_1 - \rho_2)/3V_1^2 \rho_1)$	[-]
d_e	= volume-equivalent diameter	[m]
E^*	= strength of electric field	[V.m ⁻¹]
E_0^*	= strength of electric field at outer boundary	[V.m ⁻¹]
E, E_0	= $E^*/(\gamma/4\pi \varepsilon_1 d_e)^{1/2}, E_0^*/(\gamma/4\pi \varepsilon_1 d_e)^{1/2}$	[-]
f^*	= radial distance to drop surface	[m]
f	= f/d_e	[-]
g	= gravitational acceleration	[m.s ⁻²]
H^*	= mean curvature	[m ⁻¹]
H	= H^*d_e	[-]
n	= normal unit vector	[-]
p^*	= dynamic pressure	[Pa]
p	= $\{p^* - p_0^* + \rho g(z^* - z_0^*)\}d_e/\gamma$	[-]
R	= σ_2/σ_1	[-]
Re	= $d_e V_1^*/\nu_1$	[-]
r^*	= radial distance in cylindrical coordinates	[m]
r_o^*	= radius of outer boundary	[m]
r, r_o	= $r^*/d_e, r_o^*/d_e$	[-]
S	= $\varepsilon_2/\varepsilon_1$	[-]
t	= tangential unit vector	[-]
u^*	= radial component of velocity vector	[m.s ⁻¹]
V^*	= electric potential	[V]
V	= $V^*/(\gamma d_e/4\pi \varepsilon_1)^{1/2}$	[-]
V_1^*	= terminal velocity	[m.s ⁻¹]

v^*	= velocity vector	[m.s ⁻¹]
v'	= axial component of velocity vector	[m.s ⁻¹]
u, V, v, v'	= $u^* \mu_1/\gamma, V^* \mu_1/\gamma, v^* \mu_1/\gamma, v' \mu_1/\gamma$	[-]
z^*	= axial distance in cylindrical coordinates	[m]
z	= z^*/d_e	[-]
α	= ρ_2/ρ_1	[-]
Γ	= Lagrangian quadratic basis function	[-]
γ	= interfacial tension	[mN.m ⁻¹]
ε	= permittivity	[F.m ⁻¹]
ε_r	= dielectric constant	[-]
θ	= polar angle	[rad]
κ	= μ_2/μ_1	[-]
λ	= reference pressure difference	[-]
μ	= viscosity	[Pa.s]
ν	= kinematic viscosity	[m ² .s ⁻¹]
ρ	= density	[kg.m ⁻³]
σ	= electric conductivity	[S.m ⁻¹]
τ^*	= hydrodynamic stress tensor	[Pa]
$\bar{\tau}^{E^*}$	= Maxwell's stress tensor	[Pa]
$\bar{\tau}, \bar{\tau}^E$	= $\bar{\tau}^* d_e/\gamma, \bar{\tau}^{E^*} d_e/\gamma$	[-]
χ	= Lagrangian quadratic basis function	[-]
ϕ	= Lagrangian biquadratic basis function	[-]
ψ	= linear basis function	[-]

<subscripts>

0	= reference value
1	= continuous phase
2	= drop

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