PRESSURE DROP THROUGH PACKED BED

JUN SUG LEE AND KOHEI OGAWA

Department of Chemical Engineering, Tokyo Institute of Technology, Tokyo 152

Key Words: Fluid Mechanics, Packed Bed, Pressure Drop, Flow Characteristics, Ergun's Equation

The pressure drop ΔP per unit length in a stream of fluid through a packed bed is conventionally represented by the following equation.

$$\Delta P = 4 f_b \frac{\rho u^2}{2} \frac{1}{d_p} \quad (1)$$

The modified friction coefficient, f_b , is known to be a function of Reynolds number, Re , and bed voidage, ϵ . Many investigators have tried to derive empirical correlations of f_b with Re and ϵ . However, many parameters; particle shape, particle surface roughness, particle size distribution, manner of particle packing and similar definable parameters of a packed bed, have made the task difficult. At present Ergun's correlation²⁾

$$4 f_b = \frac{(1-\epsilon)^2}{\epsilon^3} \left(300 Re^{-1} + \frac{3.5}{1-\epsilon} \right) \quad (2)$$

is well recognized for predicting the pressure drop through a bed of spherical or non-spherical material. In the case of non-spherical material, sphericity should be considered to be added to Eq.(2). Though this correlation has the apparent advantages of simple type and definite physical meaning, it gives higher pressure drop than experiment results at high Re .

The purpose of this study is to present a more general and more accurate correlation of f_b with Re and ϵ based on the flow characteristics in a packed bed than that obtained experimentally in the previous paper⁶⁾ as follows.

(1) The dimensionless number L_R , that expresses the size of opening among the particles and is obtained by a statistical approach³⁾, has a direct connection with bed voidage ϵ and the parameters of a packed bed; $L_R = f_1(\epsilon)$.

(2) Intensity of the turbulence fluctuations of velocity gradient at the surface of a packed sphere $\kappa'/\bar{\kappa}$ can be related to Re as $\kappa'/\bar{\kappa} = Re^m$ and the power m that represents flow behavior among the particles is affected by L_R ; $m = f_2(L_R) = f_2(\epsilon)$. The correlation between m and L_R could be expressed as follows.

$$m = 0.43 L_R^{1.09} \quad (3)$$

(3) The overall effect of parameters of a packed bed is represented as flow behavior of a packed bed and it can be simply integrated by voidage, ϵ . Additionally, there exist

three types of flow behavior in a packed bed; mixing, plug and transient flow.

1. Pressure Drop through a Packed Bed

The consideration of fluid flow past a single sphere supports the view of flow through a packed bed. The pressure drop through a packed bed consisting of N spheres is represented as follows.

$$\Delta P = NF \xi f(\epsilon) / A \quad (4)$$

where, N is represented as $6A(1-\epsilon)/(\pi d_p^3)$ by using the cross-sectional area A of the bed and F is the drag force on a single sphere by fluid and is represented as $C_d(\rho u^2/2)(\pi d_p^2/4)$. The voidage function, $f(\epsilon)$, represents the mutual effect of spheres and was proposed to be expressed as $25(1-\epsilon)/(3\epsilon^3)$ by Steinour⁷⁾. The correction factor ξ is introduced to account for the effect of parameters except voidage into consideration. Therefore, Eq.(4) can be rewritten as follows.

$$\Delta P = 12.5 C_d \xi \frac{(1-\epsilon)^2}{\epsilon^3} \frac{\rho u^2}{2} \frac{1}{d_p} \quad (5)$$

Eq.(5) is represented simply also as Eq.(1) by applying the following relation.

$$4 f_b = 12.5 C_d \xi \frac{(1-\epsilon)^2}{\epsilon^3} \equiv 12.5 C'_d \frac{(1-\epsilon)^2}{\epsilon^3} \quad (6)$$

The effect of voidage on modified friction coefficient, f_b , is clear from Eq.(6). All of the effects of Re and other parameters are included in a modified coefficient for a packed bed, $C'_d (= C_d \xi)$. Up to this time, the effect of Reynolds number on C_d for a sphere is known to be expressed by the following equation.

$$C_d = c_1 Re^{-1} + c_2 \quad (7)$$

The effects of other parameters of a packed bed can be represented by the flow behavior in a packed bed. By supposing that the effect of flow behavior is represented in the form of Re^{-n} , similar to the case of m in the second result of the previous paper described above, C'_d for a packed bed can be represented by the following equation.

$$C'_d = c_1 Re^{-1} + c_2 Re^{-n} + c_3 \quad (8)$$

The power n in the second term can be considered to

* Received September 16, 1993. Correspondence concerning this article should be addressed to K. Ogawa.

Fig. 1 Pressure drop at low Reynolds number

depend on voidage as in the case of m ; $n = f_3(\epsilon)$. Additionally the term tends to weaken as the voidage increase because the effect of other parameters is not important as voidage increases. Based on the results of Martin's experiment⁵⁾, the power n can be expressed by the following relational equation by using the voidage.

$$n = 0.352 + 0.1 \epsilon + 0.275 \epsilon^2 \quad (9)$$

The coefficients in Eq.(8), c_1 , c_2 and c_3 are determined by using the data for a packed bed⁸⁾. Therefore the correlation equation of modified friction coefficient is obtained as follows.

$$4 f_b = \frac{12.5 (1 - \epsilon)^2}{\epsilon^3} (29.32 Re^{-1} + 1.56 Re^{-n} + 0.1) \quad (10)$$

Finally, pressure drop through a packed bed with voidage, ϵ , is determined as follows.

$$\Delta P = \frac{12.5 (1 - \epsilon)^2}{\epsilon^3} (29.32 Re^{-1} + 1.56 Re^{-n} + 0.1) \frac{\rho u^2}{2} \frac{1}{d_p} \quad (11)$$

2. Reliability of Correlation Equation

Comparison between the correlation equations and the data¹⁾ which Ergun used at low Re , are shown in Fig. 1. The results of both correlation equations for pressure drop agree well with the data for practical purposes. Comparison between the correlation equations and data⁴⁾ at high Re are shown in Fig. 2. High Re and complex particle shape make the accuracy of correlation equation low

Fig. 2 Pressure drop at high Reynolds number

for low values of Re . But the correlation equation of this study gives more exact results than Ergun's correlation equation and more practical results. Therefore, the correlation equation of this study is superior to Ergun's equation over a wide range of Re . The assumption that the effect of other parameters is expressed in the form of Re^{-n} in Eq.(7) produce a good result.

Based on the experimental results of fluid characteristics in a packed bed of uniform spheres, a new coefficient correlation for pressure drop is obtained. It gives better agreement with experimental data than Ergun's correlation equation over a wide range of Re and has a simple, physical meaning.

Nomenclature

A	= section area of packed bed	[m ²]
C_d	= drag coefficient	[-]
d_p	= diameter of a particle	[m]
F	= drag force	[N]
f_b	= modified friction coefficient	[-]
L_R	= index of void size	[-]
m	= slope of $\log(\kappa'/\bar{\kappa})$ vs. $\log Re$	[-]
N	= number of particle	[m ⁻¹]
n	= index of eq.(8)	[-]
Re	= Reynolds number ($= d_p u / \nu$)	[-]
u	= superficial fluid velocity	[m/s]
ΔP	= pressure drop per unit length	[Pa/m]
ϵ	= voidage fraction	[-]
κ'	= fluctuation intensity ($= \sqrt{(\kappa' - \bar{\kappa})^2}$)	[1/s]
$\bar{\kappa}$	= time-mean value of velocity gradient	[1/s]
ρ	= density	[kg/m ³]
ξ	= correction factor	[-]

Literature Cited

- 1) Burke, S.P. and W.B. Plummer: *Ind. Eng. Chem.*, **20**, 1196-1200 (1928)
- 2) Ergun, S.: *Chem. Eng. Prog.*, **48**, 89-94 (1952)
- 3) Gotoh, K., W.S. Jodrey and E.M. Tory: *Powder Technol.*, **21**, 285-287 (1987)
- 4) Leva, M.: *Chem. Eng. Prog.*, **43**, 549-554 (1949)
- 5) Martin, J.J., W.L. McCabe and C.C. Monrad: *Chem. Eng. Prog.*, **47**, 91-94 (1951)
- 6) Ogawa, K. and J.S. Lee: *Kagaku Kogaku Ronbunshu*, **19**, 134-137 (1993)
- 7) Steinour, H.H.: *Ind. Eng. Chem.*, **36**, 618-624 (1944)
- 8) Zenz, F.A. and D.F. Othmer: "Fluidization and Fluid-Particle Systems", p.178-183, Reinhold Publishing Corporation, New York (1960)