MASS TRANSFER IN LAMINAR FLOWS AROUND SINGLE HOLLOW-FIBER MEMBRANES FOR HEMODIALYSIS

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Introduction

The laminar mass transfer resistances of the blood and dialysate channels have been reported for a flat-membrane dialyzer known as the B-G (Babb-Grimsrud) test dialyzer¹⁾, but less is known about hollow-fiber dialyzers. The laminar mass transfer coefficient, which is the reciprocal of the resistance, is usually estimated by dimensionless correlation of the Sherwood number Sh , the Reynolds number Re , the Schmidt number Sc , and the dimensionless length D/L in accordance with

$$Sh = 1.62 \{ReSc (D/L)\}^{1/3} \quad (1)$$

which was derived by Colburn²⁾ by analogy with laminar forced-convection heat transfer in a circular tube.

Okazaki and Yoshida⁹⁾ have reported that the laminar mass transfer resistances of the blood and dialysate channels in a flat-membrane test dialyzer and in two commercially available dialyzers of the flat-membrane and hollow-fiber types are proportional to each flow velocity raised to the power of 0.6. This result means that the Sherwood number is proportional to the 0.6th power of the Reynolds number, leading to the inapplicability of Eq.(1) to laminar mass transfer in the blood and dialysate channels.

Fukuda *et al.*⁵⁾ determined the laminar mass transfer coefficients of urea, creatinine and vitamin B₁₂ in the dialysate channel outside hollow fibers for 24 kinds of hollow-fiber hemodialyzers with different technical data using the so-called Wilson plots at varying dialysate velocities, and have reported the following empirical equation

$$Sh = 1.80 ReSc^{1/3} (D_e/L)^{1/2} \quad (2)$$

In the present paper, the authors determine dimensionless correlation for the external mass transfer of a single hollow fiber membrane fixed on the axis of a circular tube by means of an apparatus using optical fibers which we have reported.⁶⁾

1. Experimental

The authors have developed an apparatus using optical fibers to obtain time-dependent changes in the absorbancy of the solution in the stagnant hollow of a single hollow-fiber membrane fixed on the axis of a circular tube. The absorbancy measured by the apparatus is proportional to mean concentration in the hollow⁶⁾. The mean concentration is related to concentration profile in the hollow by the mathematical solution derived for unsteady-state diffusion in an infinitely long composite cylinder with skin resulting from a step-change in concentration in a large reservoir surrounding the cylinder^{10, 11)}.

Therefore, the values of solute permeability for the membrane k_M are obtainable through the mathematical solution of the time-dependent change obtained at high dialysate velocity at which the external mass transfer resistance of the membrane is negligible⁶⁾.

In this study, time-dependent changes in the absorbance were obtained with the same apparatus at varying dialysate velocities. The value of solute permeability for a single hollow-fiber membrane was first calculated by the previous procedure from the time-dependent change obtained at the highest dialysate flow rate. The external mass transfer coefficients of the laminar boundary layer developing around the membrane at each dialysate flow k_D rate were then calculated by the same mathematical analysis from the time-dependent changes and the value of solute permeability.

A single hollow-fiber membrane from 10 to 30 mm long was fixed on the axis of a circular tube 80 mm in length and 5.8 mm in inner diameter as shown in **Fig. 1**. The hollow of the membrane was stagnant because both ends of the membrane were sealed with quartz optical fibers using epoxy resin adhesive. **Table 1** summarizes technical data on hollow-fiber dialysis membranes of regenerated cellulose with substantially the same dimensions. Water permeability of the AM-FP membrane was about three times that of the AM-SD-M membrane.

Deaerated pure water and vitamin B₁₂ solution at a concentration of about 50 g/m³ in deaerated pure water were alternately fed into the circular tube as dialysate at flow rates Q_D ranging from 3.59 to 8,330 mm³/s at 310 K

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Table 1. Technical data on hollow-fiber dialysis membranes

Membrane (Manufacturer)	Material	Inner diameter* [μm]	Wall thickness* [μm]	Pure water permeability** [$\text{m}^3/(\text{m}^2\text{TsPa})$]	Water content*** [%]
AM-SD-M (Asahi Medical Co., Ltd.)	Regenerated cellulose	$188.0 \pm 15.0(100)$	$23.7 \pm 2.8(200)$	7.92	58.7
AM-FP (Asahi Medical Co., Ltd.)	Regenerated cellulose	$195.4 \pm 9.1(100)$	$30.5 \pm 3.6(200)$	24.2	72.8

* : Data taken under wet conditions at room temperature and expressed in mean $\pm$ standard deviation (number of data)

** : Data measured using a commercially available dialyzer of each membrane at 310 K

*** : Data measured at 310 K

Table 2. Solute permeability, intramembrane diffusion coefficient and equilibrium partition coefficient of AM-SD-M and AM-FP membranes

Membrane	Solute permeability k_M [$\mu\text{m/s}$]	Intramembrane diffusion coefficient D_M [$\mu\text{m}^2/\text{s}$]	Equilibrium partition coefficient K [-]
AM-SD-M	0.763	39.4	0.460
AM-FP	1.75	80.4	0.674

Fig. 1 Schematic of the dialyzer with a single hollow fiber for the measurement of the absorbance of the solution in the hollow

and time-dependent changes in the absorbance of the solution in the hollow of the membrane were obtained using the procedure which we have reported⁶⁾.

The diffusion coefficient D_0 of vitamin B₁₂ in pure water at 310 K, which was required for determining values of solute permeability, mass transfer coefficients and the Schmidt and Sherwood numbers, was estimated at $411 \mu\text{m}^2/\text{s}$, the average of two values in the literature^{7, 8)}.

2. Results and Discussion

2.1 Solute permeability, intramembrane diffusion coefficient and equilibrium partition coefficient

Steady state permeability P_S calculated from Eq.(4) in Ref.11 with the experimental data increased with increasing the Reynolds number Re in the circular tube in which a single hollow-fiber membrane was fixed and reached the asymptotes for each membrane with an Re above 1,000. Re is defined as

$$Re = \frac{D_e U_D}{\nu} \quad (3)$$

where D_e is the equivalent diameter of the dialysate channel of the outside of the membrane and is calculated from

$$D_e = \frac{D_h^2 - D^2}{D_h + D} = D_h - D \quad (4)$$

where D_h the inner diameter of the acrylic cylinder is 5.8 mm and D denotes the outer diameter of the membrane.

Solute permeability k_M was determined by averaging values of P_S with Re above 1,000 and intramembrane diffusion coefficient D_M and equilibrium partition coefficient K were calculated by the procedure reported earlier⁶⁾. Table 2 summarizes the values of k_M , D_M and K for the AM-SD-M and AM-FP membranes. The values were larger for the AM-FP membrane than for the AM-SD-M membrane, leading to correspondence to the water contents of the membranes.

2.2 Dimensionless correlation between the Sherwood, Schmidt and Reynolds numbers for external mass transfer of a single hollow-fiber membrane

The mass transfer coefficients k_D for the external laminar boundary layer of a single hollow-fiber membrane from 10 to 30 mm in length were determined with time-dependent changes in the absorbance at varying flow rates in the circular tube and with the values of k_M , D_M and K listed in Table 2. The Sherwood number Sh and the Schmidt number Sc of the dialysate channel outside the membrane are defined by the following equation using the equivalent diameter D_e as the characteristic length

$$Sh = \frac{k_D D_e}{D_0} \quad (5)$$

$$Sc = \frac{\nu}{D_0} \quad (6)$$

Values of $\{Sh/(Sc D_e/L)^{1/3}\}$ were plotted against the Reynolds number Re calculated from Eq. (3) on log-log graph paper as shown in Fig. 2. The plots for both membranes at Re of below 500 were in agreement with the

Fig. 2 External mass transfer characteristics of a single hollow fiber fixed on the axis of a circular tube expressed by dimensionless numbers

straight line calculated from Eq. (1) in spite of there being a significant difference in solute permeability and equilibrium partition coefficient of the membranes.

The analogy of heat transfer for external laminar boundary layer of a circular tube holds in mass transfer for external laminar boundary layer of a single hollow-fiber membrane determined by our optical apparatus. The difference in the flow conditions between the dialysate channel of hollow-fiber dialyzers and the exterior of a circular tube may cause a discrepancy in the values of the power of the Reynolds number in the dimensionless correlations determined for mass transfer, *i. e.*, Eqs. (1) and (2)^{3, 4}.

Deviation from the straight line on Fig. 2 of the plots for Re of above 500 may be due to vibration of a single hollow-fiber membrane caused by dialysate pulsation.

Conclusions

The dimensionless correlation on external laminar mass transfer of a single hollow-fiber membrane fixed on the axis of a circular tube is in agreement with that obtained by Colburn by analogy with laminar forced-convection heat

transfer in a circular tube, while that in the dialysate channel of a hollow-fiber hemodialyzer is not.

Nomenclature

D	= characteristic length, outer radius of a single hollow fiber membrane, inner diameter of a circular tube	[m]
D_0	= diffusion coefficient in solution	[m ² /s]
D_e	= equivalent diameter of dialysate channel	[m]
D_M	= intramembrane diffusion coefficient	[m ² /s]
K	= equilibrium partition coefficient	[-]
k_D	= external mass transfer coefficient of a single hollow-fiber membrane	[m/s]
k_M	= solute permeability of hollow-fiber membrane	[m/s]
L	= effective length of hollow fiber membrane	[m]
P_S	= solute permeability at steady state calculated from Eq. (4) in Ref. 11	[m/s]
Q_D	= volumetric flow rate of dialysate	[m ³ /s]
Re	= Reynolds number	[-]
Sc	= Schmidt number	[-]
Sh	= Sherwood number	[-]
U_D	= average dialysate velocity	[m/s]
ν	= kinematic viscosity	[m ² /s]

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